

Group & Phase Velocities (2B)

- 3-D Group & Phase Velocities

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Phase Velocity (1)

At any point in space \mathbf{x}

$$s(\mathbf{x}, t) = A e^{j(\omega t - \mathbf{k} \cdot \mathbf{x})}$$

The wave oscillates with a temporal frequency ω

During one period of oscillation

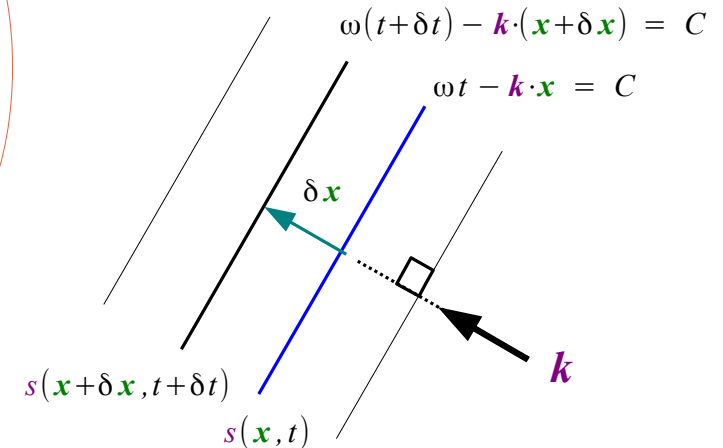
$$T = \frac{2\pi}{\omega}$$

In the direction of \mathbf{k} (spatial frequency)

The wave propagates forward

By one wavelength

$$\lambda = \frac{2\pi}{|\mathbf{k}|}$$



Phase Velocity (2)

$$s(\mathbf{x}, t) = A e^{j(\omega t - \mathbf{k} \cdot \mathbf{x})}$$

The speed of propagation

The speed at which planes of constant phase $\mathbf{k} \cdot \mathbf{x} = c$

Phase Velocity

$$|\mathbf{v}_p| = \frac{\lambda}{T} = \frac{\omega}{|\mathbf{k}|}$$

← distance
← time

If the directions are the same \mathbf{v}_p and \mathbf{k}

$$\frac{\mathbf{k}}{|\mathbf{k}|} \cdot |\mathbf{v}_p| = \frac{\omega}{|\mathbf{k}|} \cdot \frac{\mathbf{k}}{|\mathbf{k}|}$$

$$\mathbf{v}_p = \frac{\omega \mathbf{k}}{|\mathbf{k}|^2}$$

Phase Velocity (2)

$$s(\mathbf{x}, t) = A e^{j(\omega t - \mathbf{k} \cdot \mathbf{x})}$$

The speed of propagation

The speed at which planes of constant phase $\mathbf{k} \cdot \mathbf{x} = c$

Phase Velocity

$$|\mathbf{v}_p| = \frac{\lambda}{T} = \frac{\omega}{|\mathbf{k}|}$$

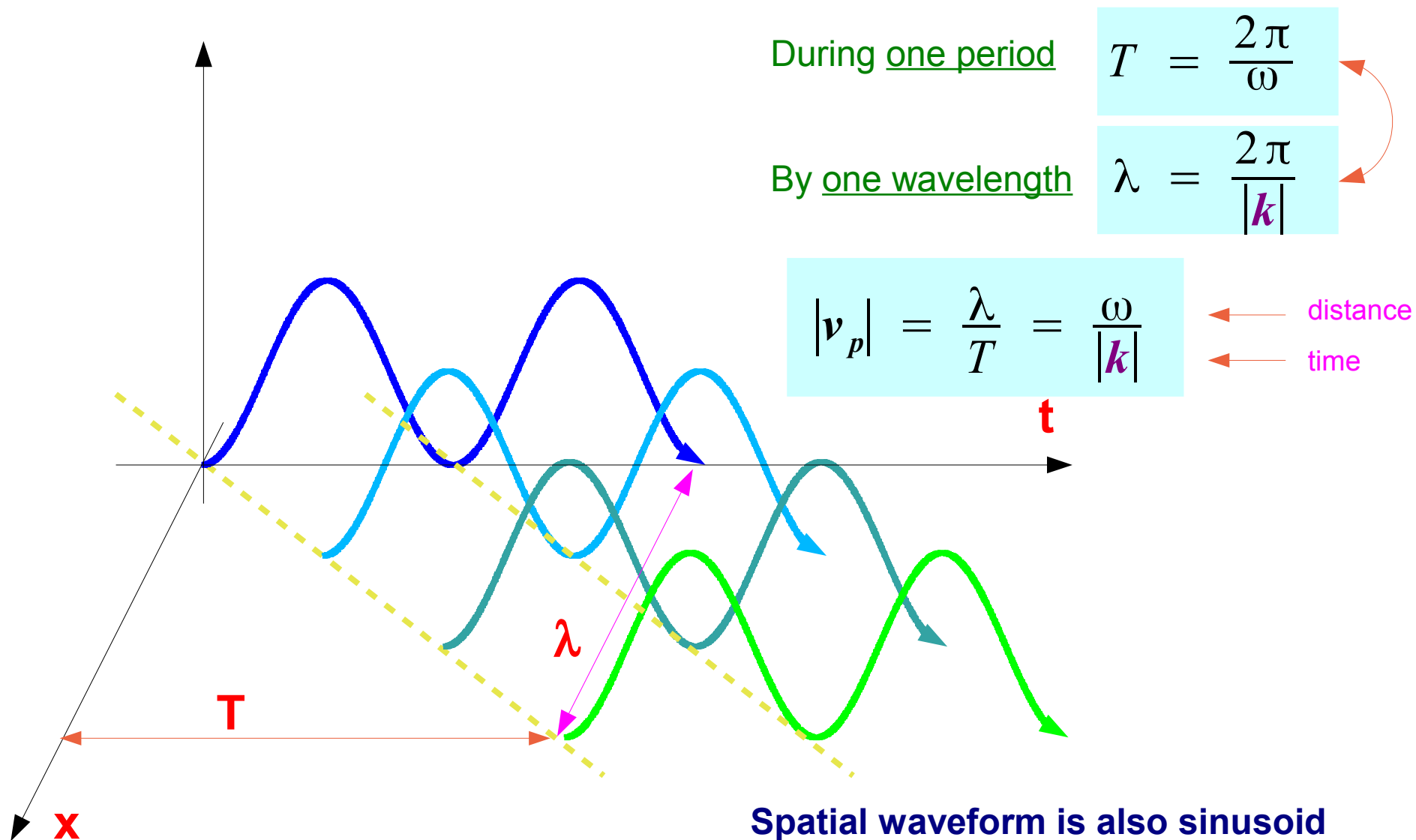
← distance
← time

If the directions are the same \mathbf{v}_p and \mathbf{k}

$$\frac{\mathbf{k}}{|\mathbf{k}|} \cdot |\mathbf{v}_p| = \frac{\omega}{|\mathbf{k}|} \cdot \frac{\mathbf{k}}{|\mathbf{k}|}$$

$$\mathbf{v}_p = \frac{\omega \mathbf{k}}{|\mathbf{k}|^2}$$

Planes of Constant Phase



Acoustic Phonon Dispersion

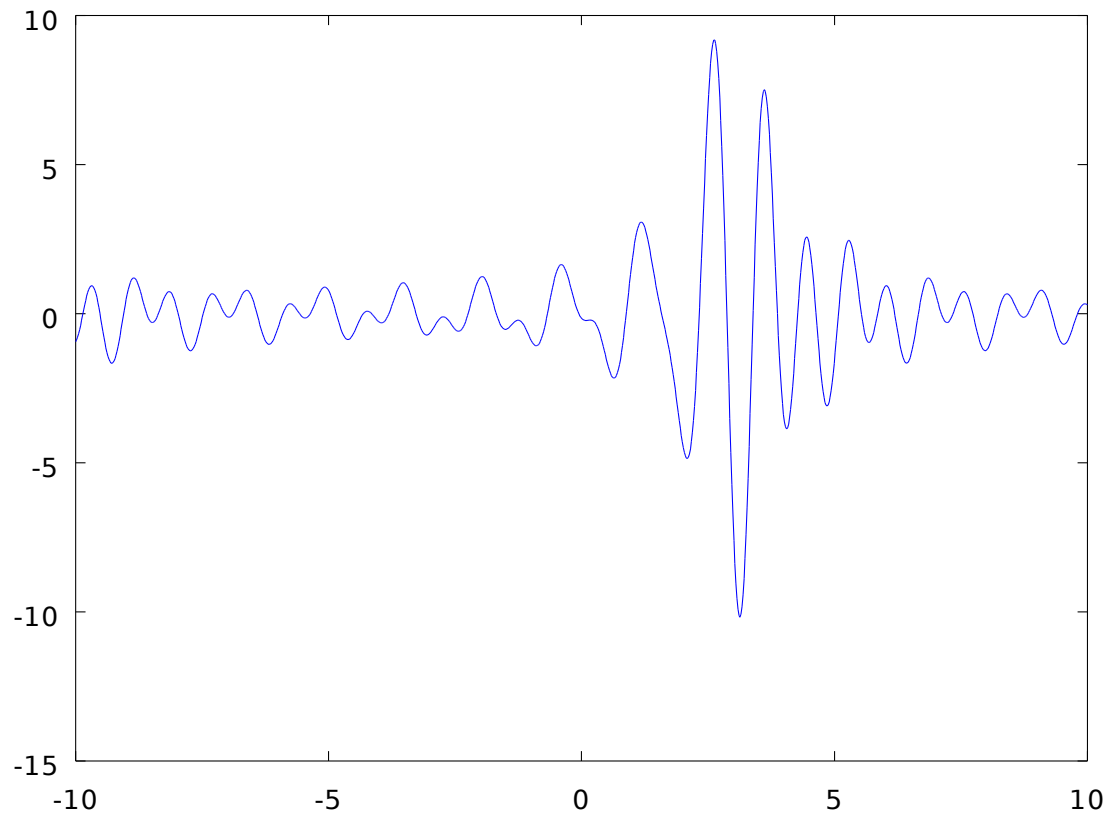
Acoustic Phonon Dispersion

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Acoustic Phonon Dispersion



```
x = linspace(-10, +10, 1000);
```

```
y = zeros(1, 1000);
```

```
for k= 1.0:0.1:2.0
```

```
    y = y + cos(4*k*(x-k));
```

```
end
```

```
plot(x, y);
```

References

- [1] <http://en.wikipedia.org/>
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- [3] <http://www.mathpages.com/>, Phase, Group, and Signal Velocity
- [4] R. Barlow, www.hep.man.ac.uk/u/roger/PHYS10302/lecture15.pdf
- [5] P. Hofmann, www.philiphofmann.net/book_material/notes/groupphasevelocity.pdf