Angle (1A)

- Angles in Degree
- Angles in Radian
- Conversion between Degree and Radian
- Co-terminal Angles

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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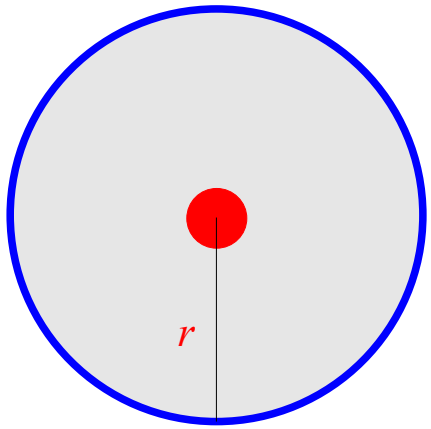
Angle and Arc Length

Circumference

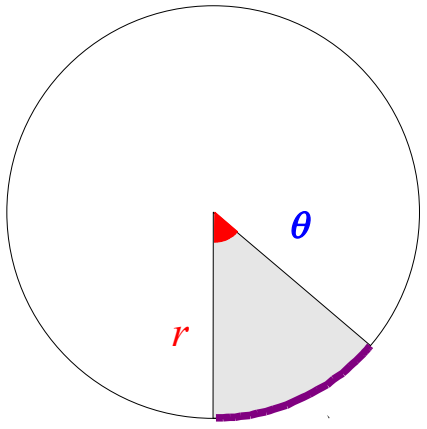


$2\pi r$

$$Angle \propto Arc\ Length$$

$$\left(\theta \propto \frac{l}{2\pi r}\right)$$

Arc length



l

l : *Arc length*

Ratio : $0 \le \frac{l}{2\pi r} \le 1$

$2\pi r$: *Circumference*

Arc Length Ratio

$$\theta \propto \frac{l}{2\pi r}$$

l : *Arc length*

$2\pi r$: *Circumference*

Angle in degree

$$0 \le \theta_d \le 360$$

$$0 \cdot 360 \le \frac{l}{2\pi r} \cdot 360 \le 1 \cdot 360$$

Ratio

$$0 \le \frac{l}{2\pi r} \le 1$$

Angle in radian

$$0 \le \theta_r \le 2\pi$$

$$0 \cdot 2\pi \le \frac{l}{2\pi r} \cdot 2\pi \le 1 \cdot 2\pi$$

Degree and Radian Scales

$$\theta \propto \frac{l}{2\pi r}$$

l : *Arc length*

$2\pi r$: *Circumference*

Angle in degree

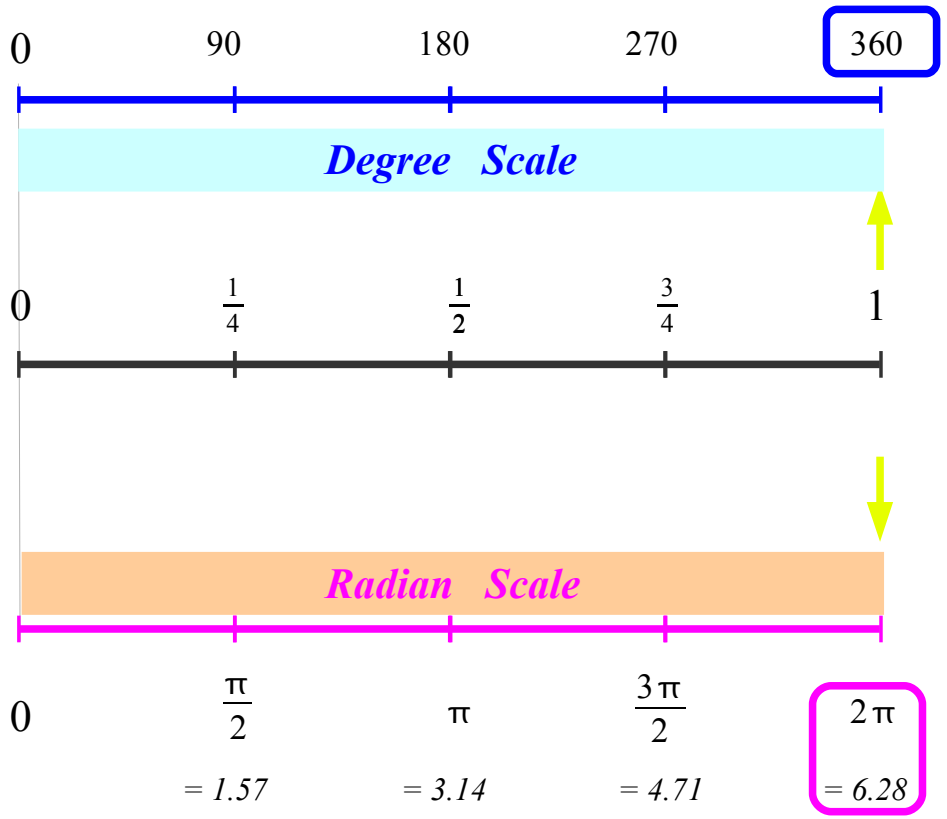
$$0 \le \theta_d \le 360$$

Ratio

$$0 \le \frac{l}{2\pi r} \le 1$$

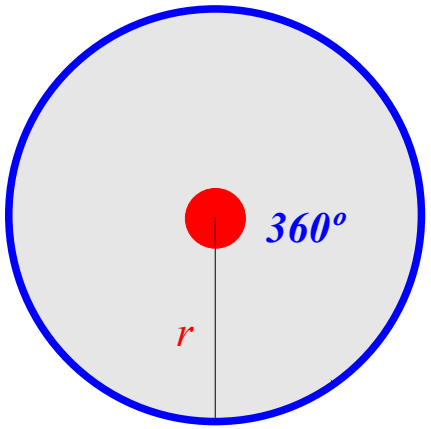
Angle in radian

$$0 \le \theta_r \le 2\pi$$

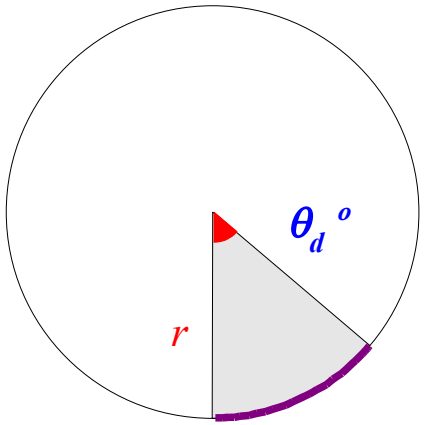


Measuring Angle in Degree

Circumference $2\pi r$



Arc length l



Ratio

$$0 \le \frac{l}{2\pi r} \le 1$$

Angle in degree

$$0 \le \theta_d \le 360$$

$2\pi r$ *Circumference* → *360°*

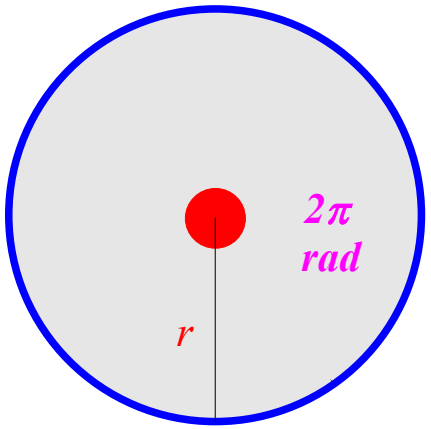
l *Arc length* → θ

$$2\pi r : l = 360 : \theta_d$$

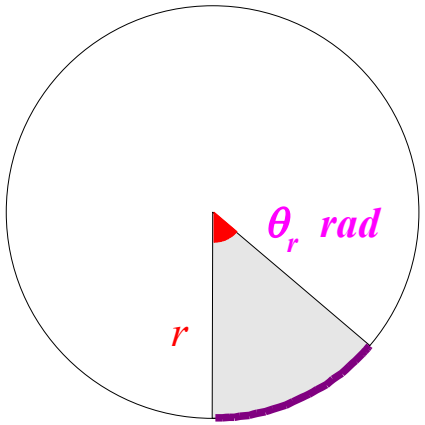
$$\theta_d = \frac{l}{2\pi r} \cdot 360$$

Measuring Angle in Radian

Circumference $2\pi r$



Arc length l



Ratio

$$0 \le \frac{l}{2\pi r} \le 1$$

Angle in radian

$$0 \le \theta_r \le 2\pi$$

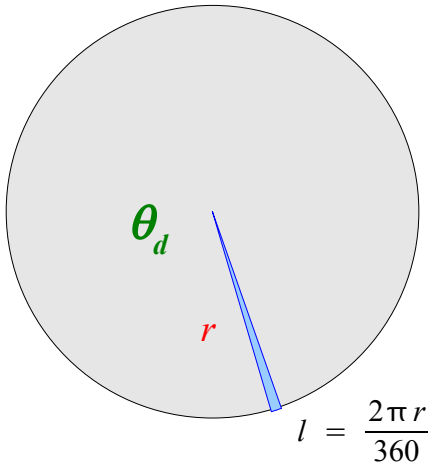
$2\pi r$ *Circumference* → 2π

l *Arc length* → θ

$$2\pi r : l = 2\pi : \theta_r \Rightarrow \theta_r = \frac{l}{2\pi r} \cdot 2\pi$$

Unit Degree and Unit Radian

Unit Degree



Degree

$$\theta_d \ deg = \boxed{\frac{l}{2\pi r}} \cdot 360 \ deg$$

$$= \frac{l}{r} \cdot \frac{180}{\pi} \ deg$$

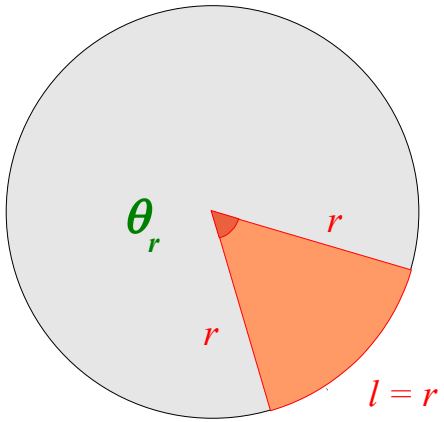
1 *degree*

If l = (circumference / 360)

$$l = \frac{2\pi r}{360}$$

The unit degree is represented for the sake of explanation

Unit Radian



Radian

$$\theta_r \ rad = \boxed{\frac{l}{2\pi r}} \cdot 2\pi \ rad$$

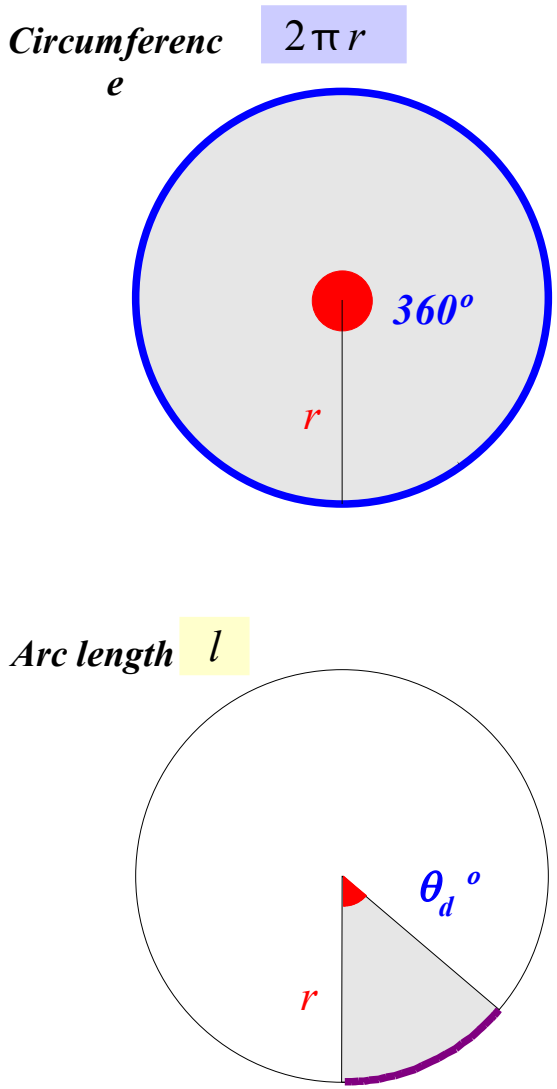
$$= \frac{l}{r} \ rad$$

1 *radian*

If l = radius

$$l = r$$

Degree ⇒ Radian



Degree

$$\theta_d \ \ deg = \frac{l}{2\pi r} \cdot 360 \ \ deg$$

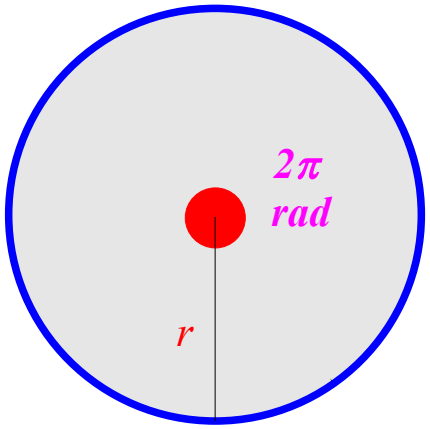
$$\theta_d \ \ deg \times \frac{\pi}{180} = \frac{l}{r} \cdot \frac{180}{\pi} \ \ deg \times \frac{\pi}{180}$$

Radian

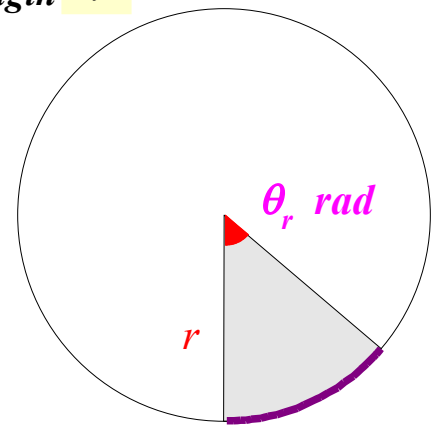
$$\theta_d \cdot \frac{\pi}{180} \ \ rad = \frac{l}{r} \ \ rad$$

Radian ⇒ Degree

Circumference $2\pi r$



Arc length l



Radian

$\theta_r \ rad = \frac{l}{r} \ rad$

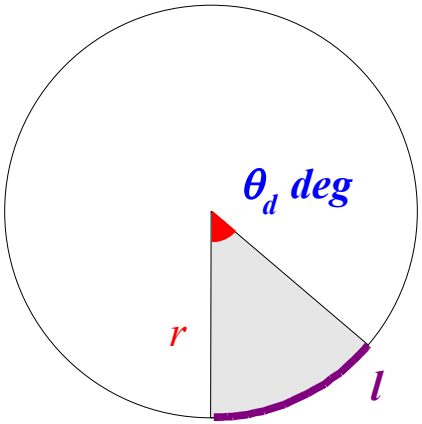
$\theta_r \ rad \times \frac{180}{\pi} = \frac{l}{r} \ rad \times \frac{180}{\pi}$

Degree

$\theta_r \cdot \frac{180}{\pi} \ deg = \frac{l}{2\pi r} \cdot 360 \ deg$

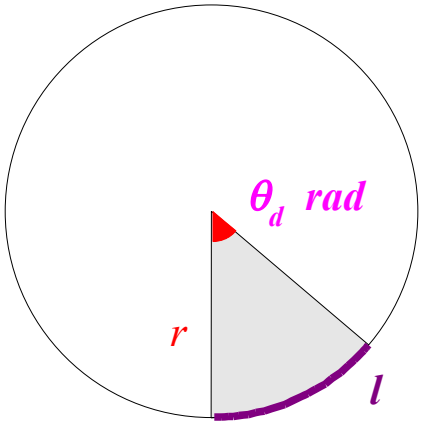
Degree ⇔ Radian

The Same Angle



$$\theta_d \ deg = \frac{l}{2\pi r} \cdot 360 \ deg$$

$$\theta_r \ rad = \frac{l}{2\pi r} \cdot 2\pi \ rad$$

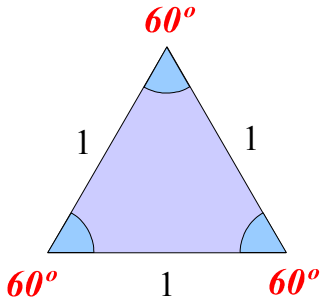


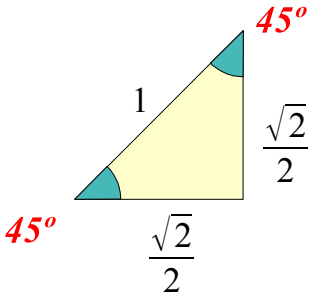
θ_d *degree* = θ_r *radian*

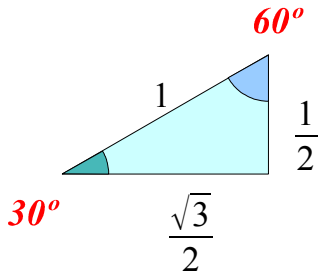
Degree $\theta_d \ deg$ → $\theta_d \cdot \frac{\pi}{180} \ rad$ **Radian**

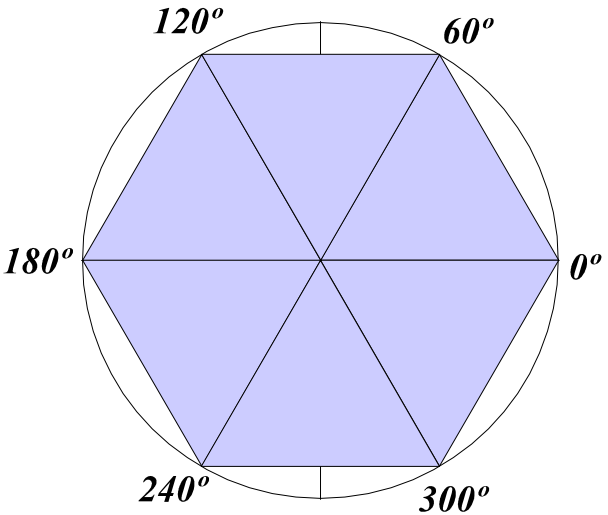
Degree $\theta_r \cdot \frac{180}{\pi} \ deg$ ← $\theta_r \ rad$ **Radian**

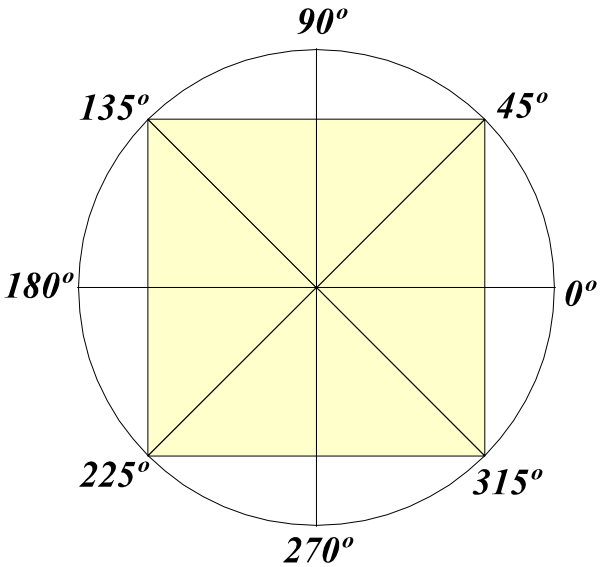
Well-known Angles in Degree

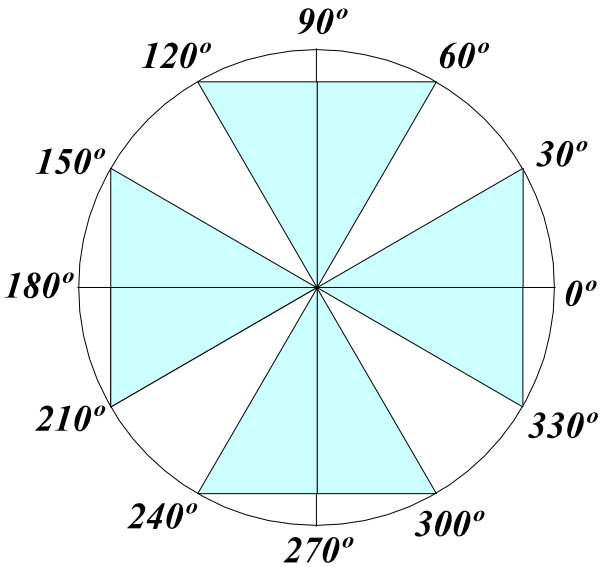




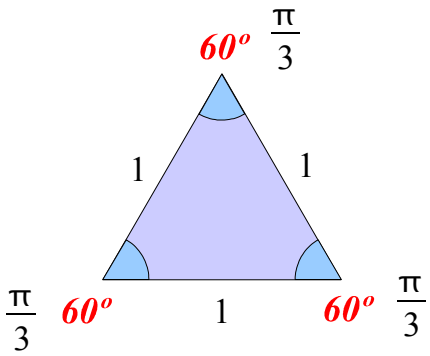


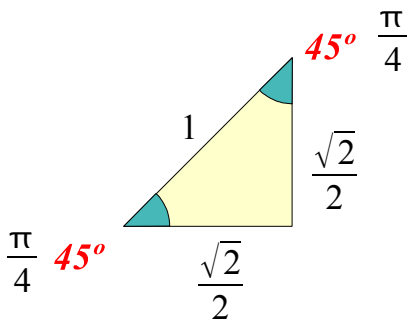


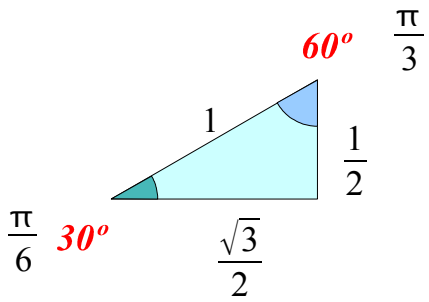


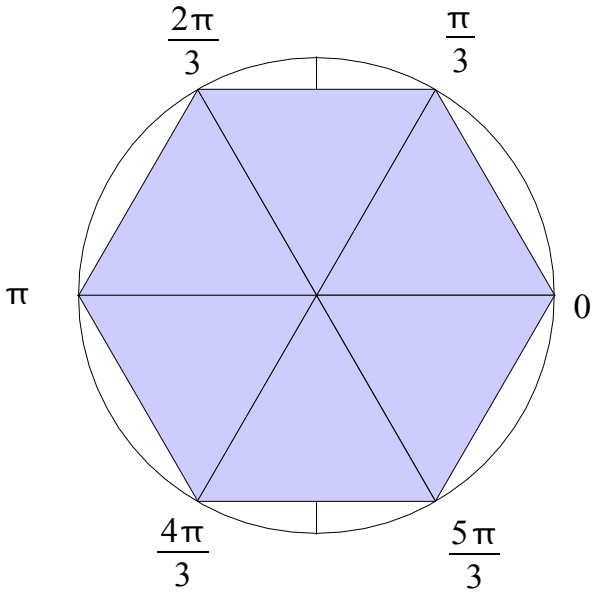


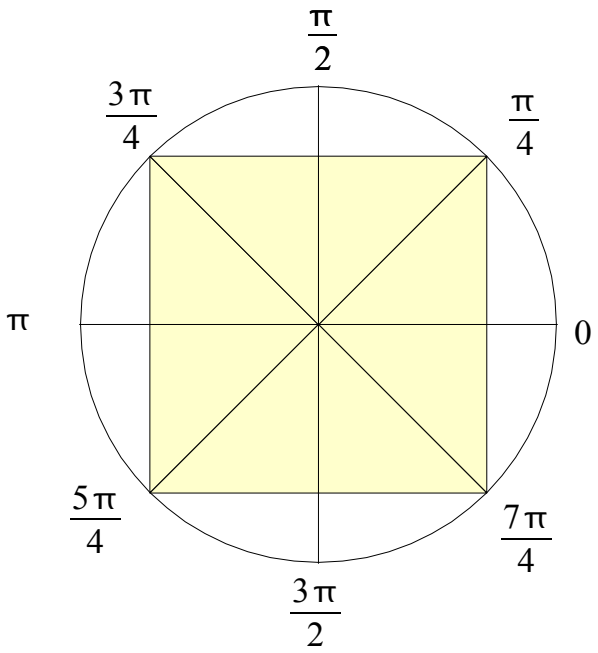
Well-known Angles in Radian

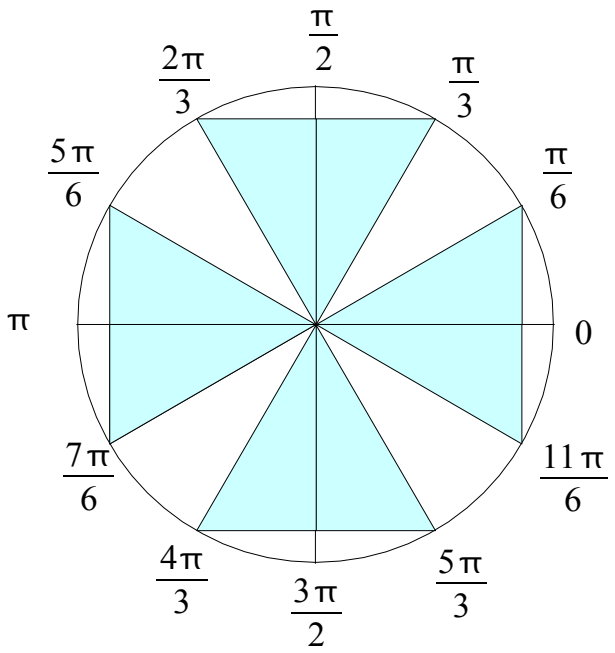




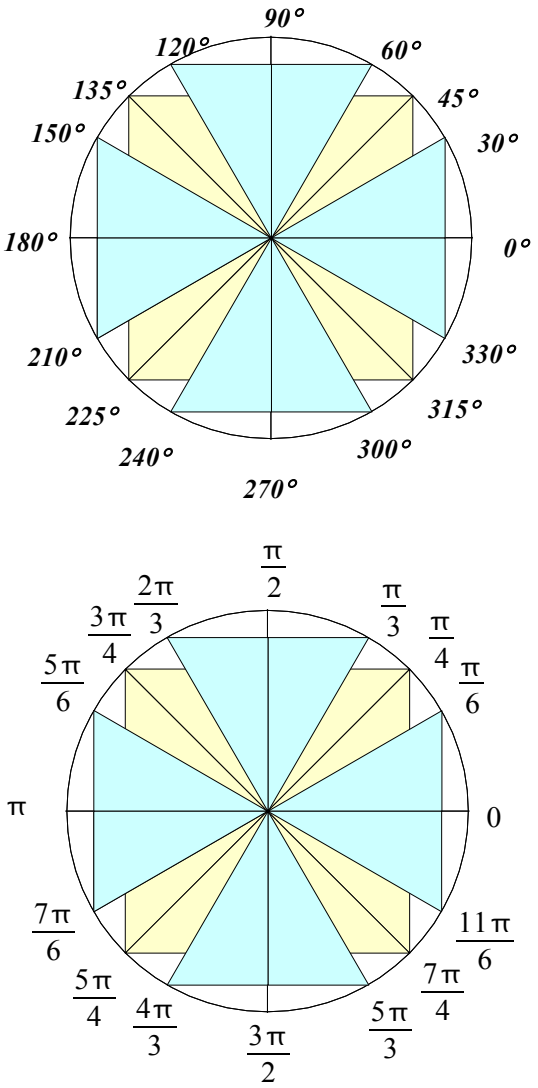






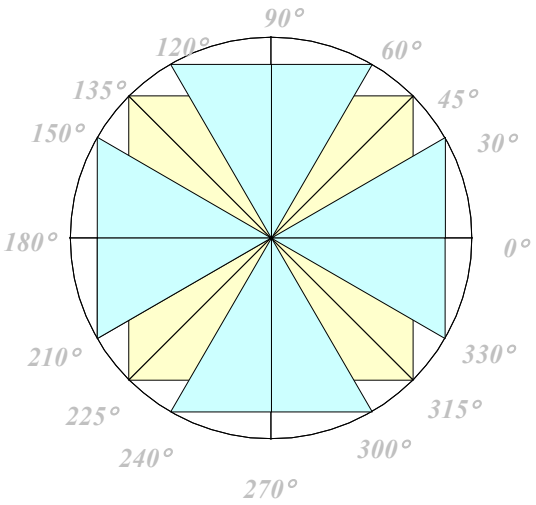


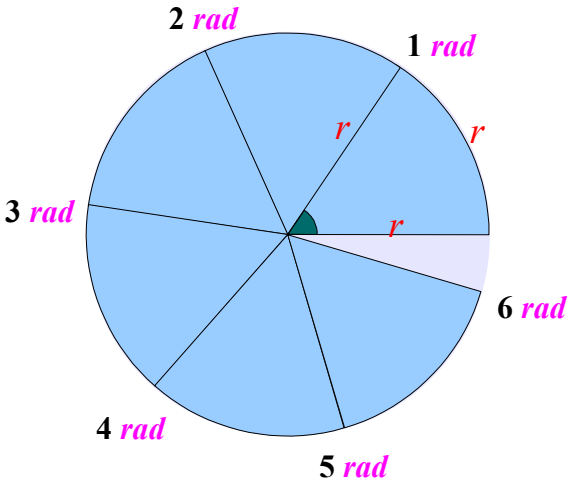
Well-known Angles (Degree → Radian)



degree	radian				
0°	0	=	0	=	0.000
30°	π/6	=	π/6	=	0.524
45°	π/4	=	π/4	=	0.785
60°	π/3	=	π/3	=	1.047
90°	π/2	=	π/2	=	1.571
120°	π/6 + π/2	=	2π/3	=	2.094
135°	π/4 + π/2	=	3π/4	=	2.356
150°	π/3 + π/2	=	5π/6	=	2.618
180°	π/2 + π/2	=	π	=	3.142
210°	π/6 + π π	=	7π/6	=	3.665
225°	π/4 + π π	=	5π/4	=	3.927
240°	π/3 + π π	=	4π/3	=	4.189
270°	π/2 + π π	=	3π/2	=	4.712
300°	π/6 + π + π/2	=	5π/3	=	5.236
315°	π/4 + π + π/2	=	7π/4	=	5.498
330°	π/3 + π + π/2	=	11π/6	=	5.760
360°	π/2 + π + π/2	=	2π	=	6.283

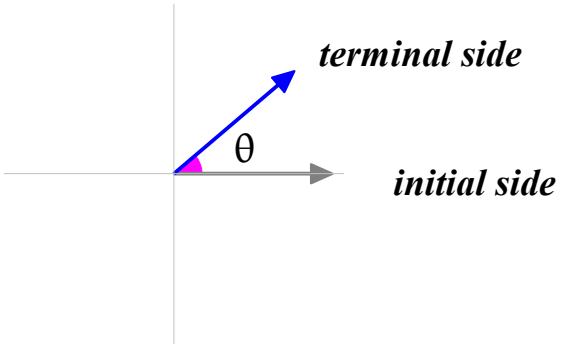
Well-known Angles (Radian → Degree)





radian	degree	
0	0.0 x 180 / π =	0°
1	1.0 x 180 / π =	57.3°
2	2.0 x 180 / π =	114.6°
3	3.0 x 180 / π =	171.9°
4	4.0 x 180 / π =	229.2°
5	5.0 x 180 / π =	286.5°
6	6.0 x 180 / π =	343.8°
1.57	π/2 x 180 / π =	90°
3.14	π x 180 / π =	180°
4.71	3π/2 x 180 / π =	270°
6.28	2π x 180 / π =	360°
0.79	π/4 x 180 / π =	45°
2.36	3π/4 x 180 / π =	135°
3.93	5π/4 x 180 / π =	225°
5.50	7π/4 x 180 / π =	315°

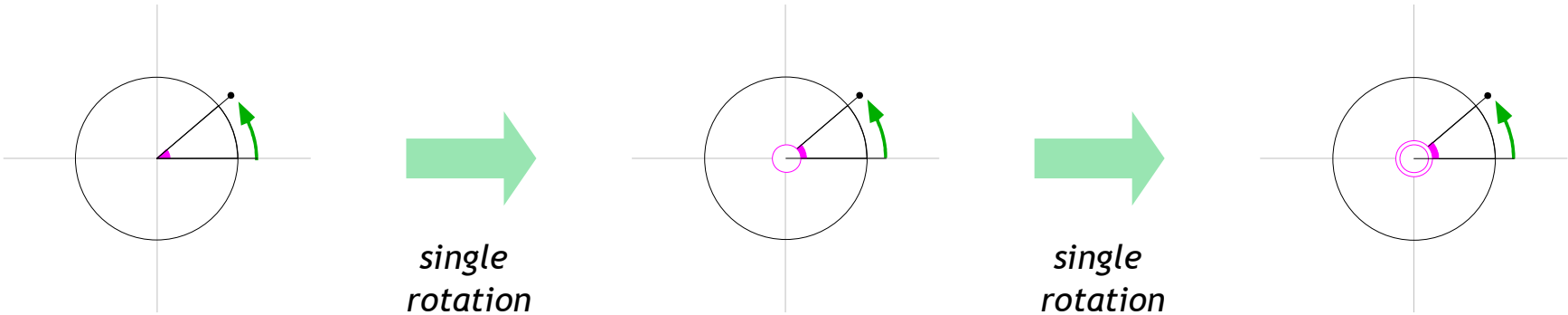
Co-terminal Angle (Multiple Rotations)



Co-terminal Angles

- the same terminal side
- the same initial side
- different rotations

$$\theta_1 = \theta \qquad \theta_2 = 1\ rotation + \theta \qquad \theta_3 = 2\ rotations + \theta$$



$$f(\theta_1) = f(\theta_2) = f(\theta_3) = f(\theta) \qquad f \in \{\sin\theta,\ \cos\theta,\ \tan\theta\}$$

Multiple Rotations

Allow multiple rotations

l : *Arc length* $0 \le l \le 2\pi r$

d : ***Distance*** *traveled along circumference* $d \ge 0$

$\theta_1 = \theta$

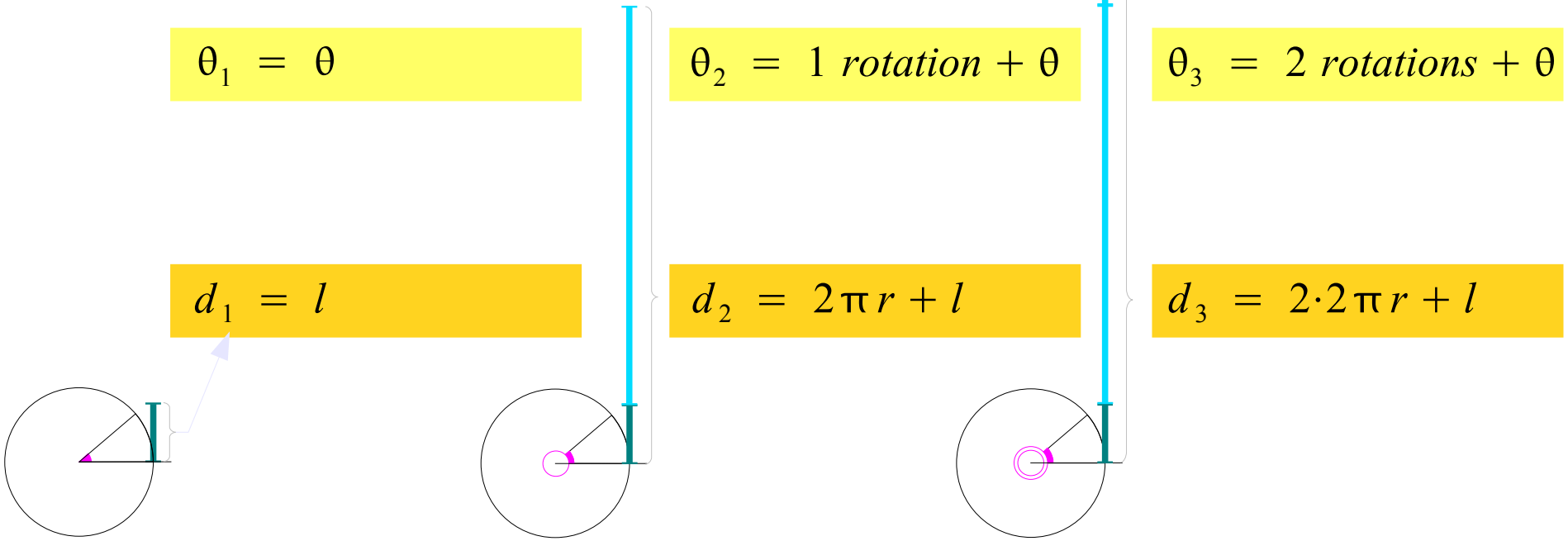
$\theta_2 = 1\ rotation + \theta$

$\theta_3 = 2\ rotations + \theta$

$d_1 = l$

$d_2 = 2\pi r + l$

$d_3 = 2 \cdot 2\pi r + l$



Multiple Rotations in Degree

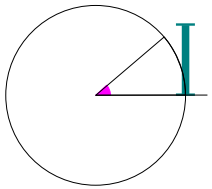
$$\theta = \frac{l}{2\pi r} \times 360$$

$$d_1 = l$$

$$\theta_1 = \frac{d_1}{2\pi r} \times 360$$

$$= \frac{l}{2\pi r} \times 360$$

$$\theta_1 = \theta$$



$$\theta = \frac{l}{2\pi r} \times 360$$

$$d_2 = 2\pi r + l$$

$$\theta_2 = \frac{d_2}{2\pi r} \times 360$$

$$= (1 + \frac{l}{2\pi r}) \times 360$$

$$\theta_2 = 360 + \theta$$

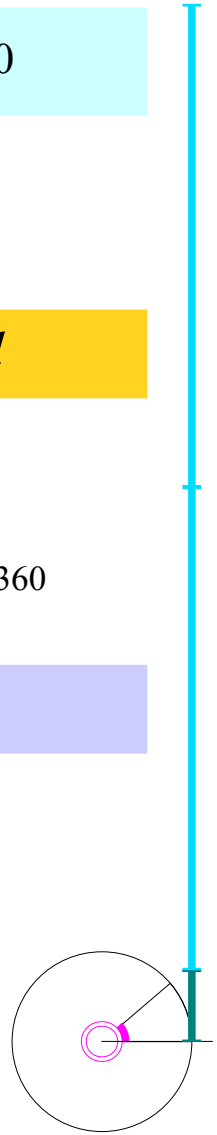
$$\theta = \frac{l}{2\pi r} \times 360$$

$$d_3 = 4\pi r + l$$

$$\theta_3 = \frac{d_3}{2\pi r} \times 360$$

$$= (2 + \frac{l}{2\pi r}) \times 360$$

$$\theta_3 = 2 \cdot 360 + \theta$$



Multiple Rotations in Radian

$$\theta = \frac{l}{2\pi r} \times 2\pi$$

$$\theta = \frac{l}{2\pi r} \times 2\pi$$

$$\theta = \frac{l}{2\pi r} \times 2\pi$$

$$d_1 = l$$

$$d_2 = 2\pi r + l$$

$$d_3 = 4\pi r + l$$

$$\theta_1 = \frac{d_1}{2\pi r} \times 2\pi$$

$$= \frac{l}{2\pi r} \times 2\pi$$

$$\theta_2 = \frac{d_2}{2\pi r} \times 2\pi$$

$$= (1 + \frac{l}{2\pi r}) \times 2\pi$$

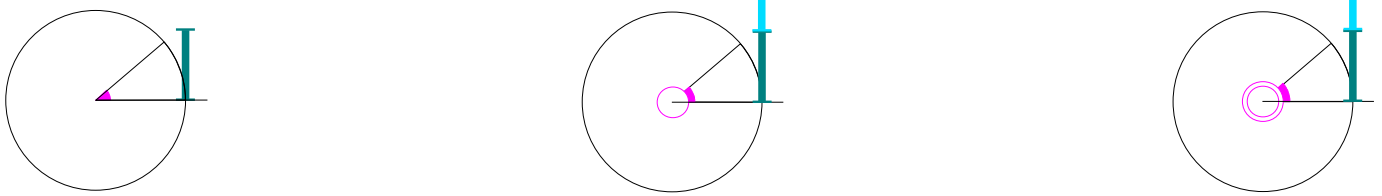
$$\theta_3 = \frac{d_3}{2\pi r} \times 2\pi$$

$$= (2 + \frac{l}{2\pi r}) \times 2\pi$$

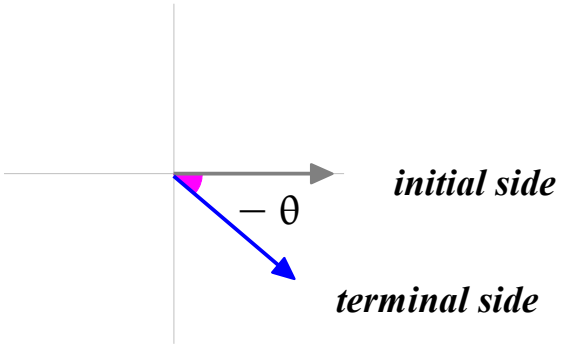
$$\theta_1 = \theta$$

$$\theta_2 = 2\pi + \theta$$

$$\theta_3 = 2 \cdot 2\pi + \theta$$



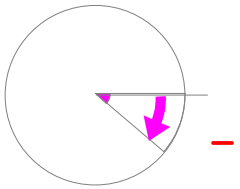
Co-terminal Angle (Reverse Rotations)



Co-terminal Angles

- the same terminal side
- the same initial side
- different rotations

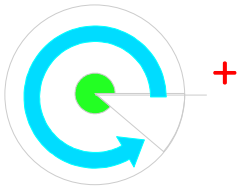
$\theta_1 = -\theta$ $\qquad$ $\theta_2 = 1\ rotation - \theta$



Negative Angle

clockwise

single rotation



Positive Angle

counter-clockwise

$$f(\theta_1) = f(\theta_2) = f(\theta) \qquad f \in \{\sin\theta,\ \cos\theta,\ \tan\theta\}$$

Reverse Rotations

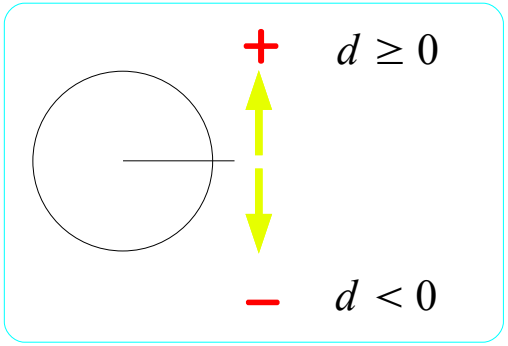
Consider the direction of rotations

l : *Arc length*

$0 \le l \le 2\pi r$

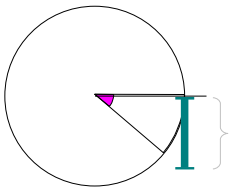
d : *Displacement* (*directed distance*)

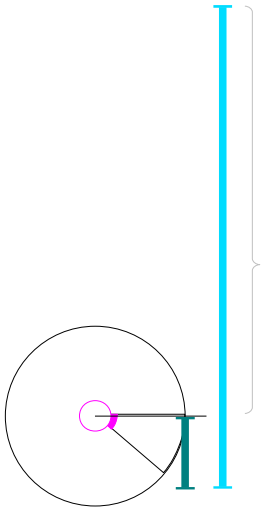
$d < 0$ *or* $d \ge 0$



$\theta_1 = -\theta$

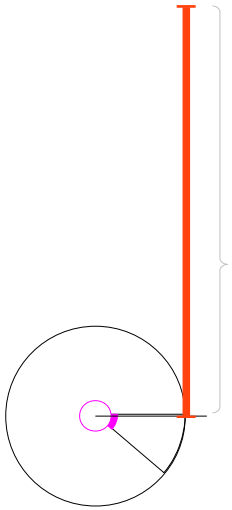
$d_1 = -l$





$\theta_2 = 1\ rotation - \theta$

$d_2 = 2\pi r - l$



Reverse Rotations in Degree

$$\theta = \frac{l}{2\pi r} \times 360$$

$$\theta = \frac{l}{2\pi r} \times 360$$

$$d_1 = -l$$

$$d_2 = 2\pi r - l$$

$$\theta_1 = \frac{d_1}{2\pi r} \times 360$$

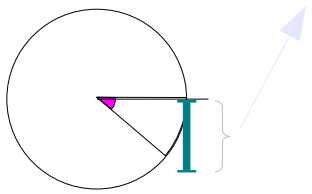
$$= \frac{-l}{2\pi r} \times 360$$

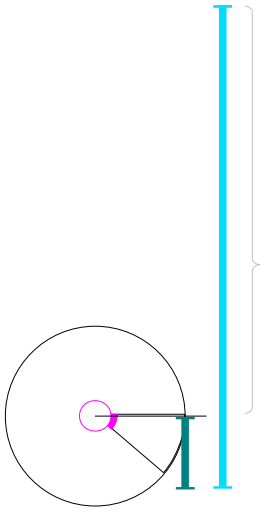
$$\theta_2 = \frac{d_2}{2\pi r} \times 360$$

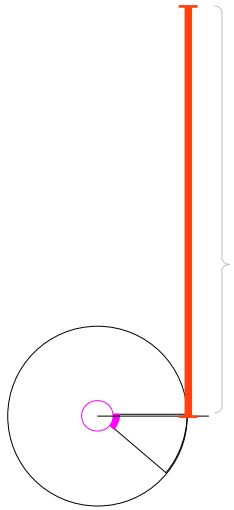
$$= (1 - \frac{l}{2\pi r}) \times 360$$

$$\theta_1 = -\theta$$

$$\theta_2 = 360 - \theta$$







Reverse Rotations in Radian

$$\theta = \frac{l}{2\pi r} \times 2\pi$$

$$\theta = \frac{l}{2\pi r} \times 2\pi$$

$$d_1 = -l$$

$$d_2 = 2\pi r - l$$

$$\theta_1 = \frac{d_1}{2\pi r} \times 2\pi$$

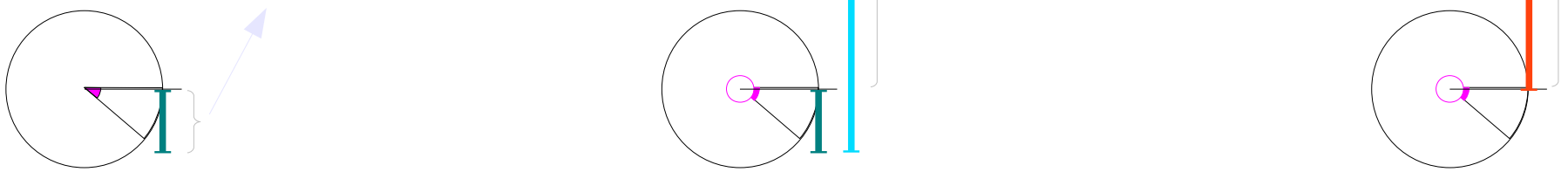
$$= \frac{-l}{2\pi r} \times 2\pi$$

$$\theta_2 = \frac{d_2}{2\pi r} \times 2\pi$$

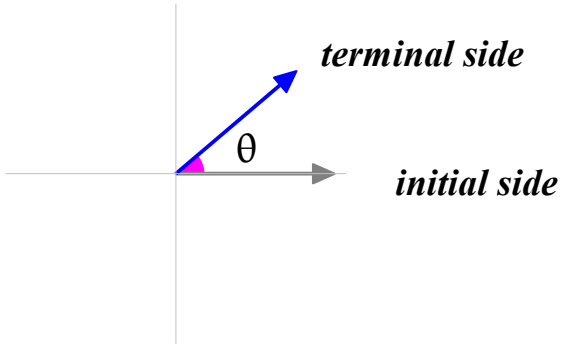
$$= (1 - \frac{l}{2\pi r}) \times 2\pi$$

$$\theta_1 = -\theta$$

$$\theta_2 = 2\pi - \theta$$

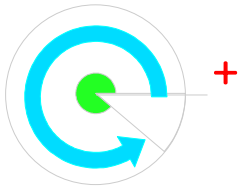


Angles in the Standard Position



Angles in the standard position

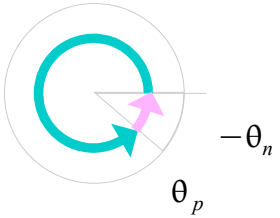
- *Vertex* is in the origin of a rectangular coordinate system
- *Initial side* lies along the positive x-axis



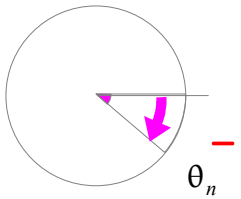
Positive Angle

$\theta_p > 0$

counter-clockwise



$\theta_p - \theta_n = 360$



Negative Angle

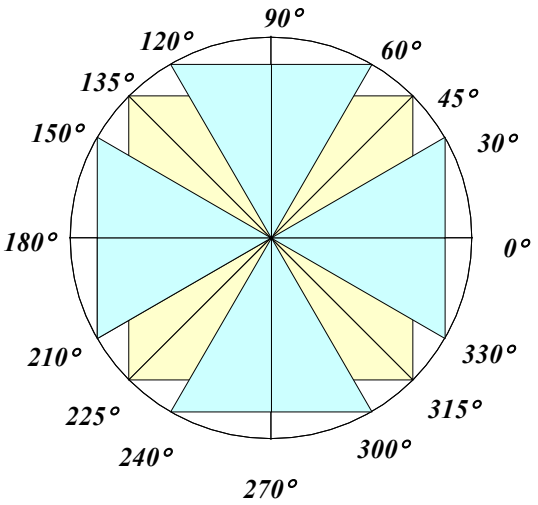
$\theta_n < 0$

clockwise

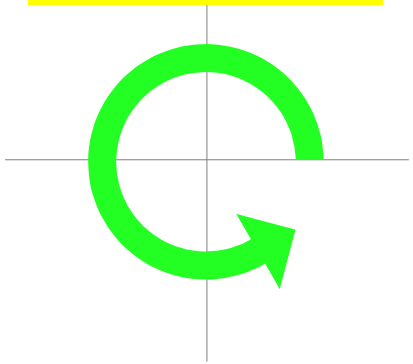
$\theta_p = \theta_n + 360 \quad (\theta_p > 0)$

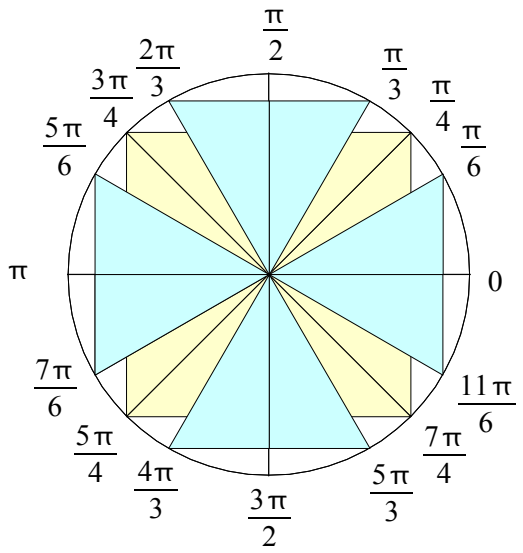
$\theta_n = \theta_p - 360 \quad (\theta_n < 0)$

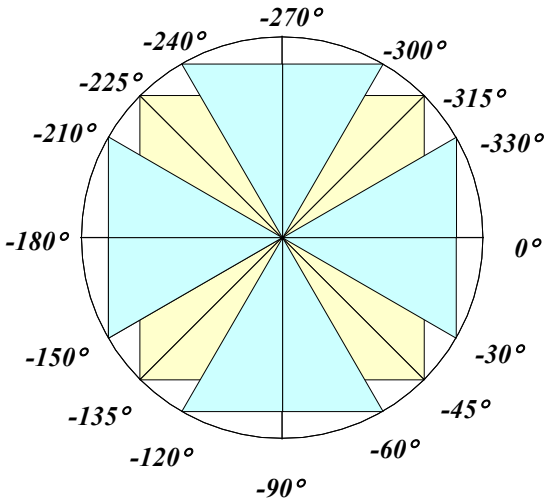
Positive and Negative Angles



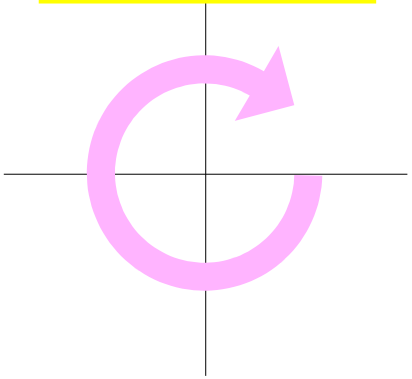
Positive Angle

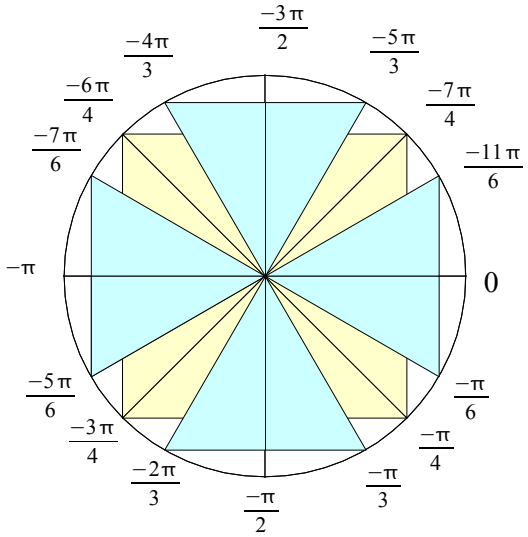






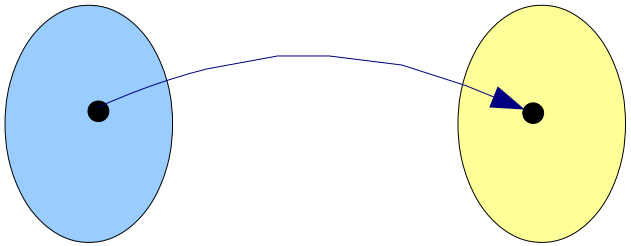
Negative Angle



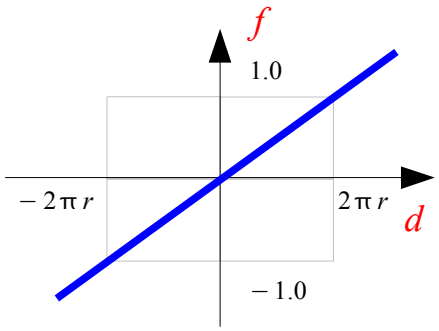


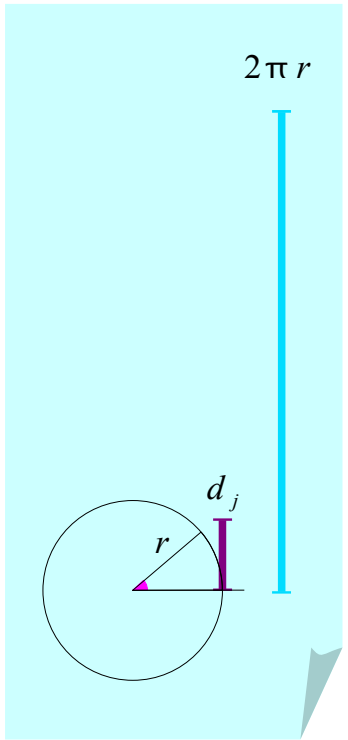
Arc Length Ratio

$$f(d_i) = \frac{d_i}{2\pi r}$$

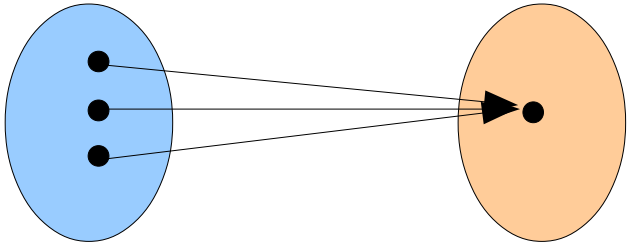


d_i $\qquad$ $\dfrac{d_i}{2\pi r}$





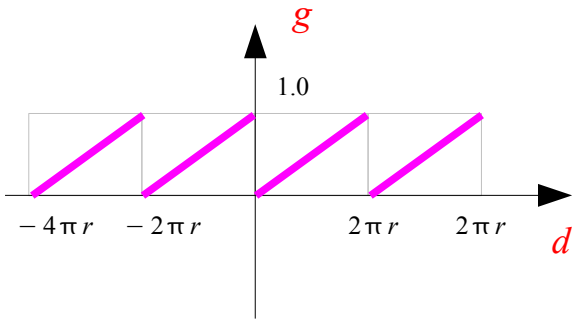
$$g(d_i) = \textbf{Remainder}\left(\frac{d_i}{2\pi r}\right)$$



$d_j \pm k \cdot 2\pi r$

$(0 \le d_j \le 2\pi r)$

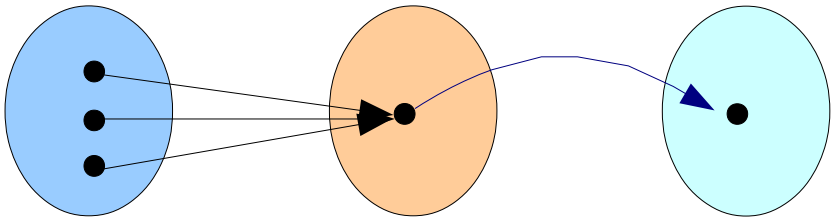
$\dfrac{d_j}{2\pi r}$



Co-terminal Angle & Arc Length Ratio

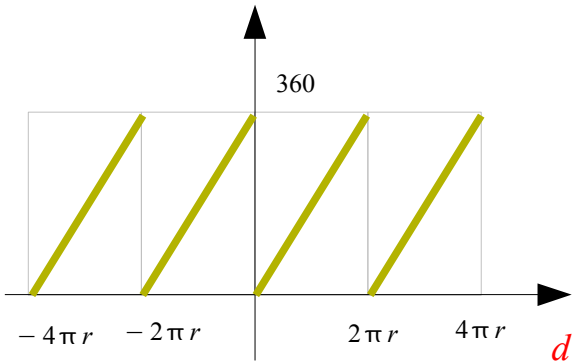
$g(d_i) = \text{Remainder}\left(\frac{d_i}{2\pi r}\right)$

Degree



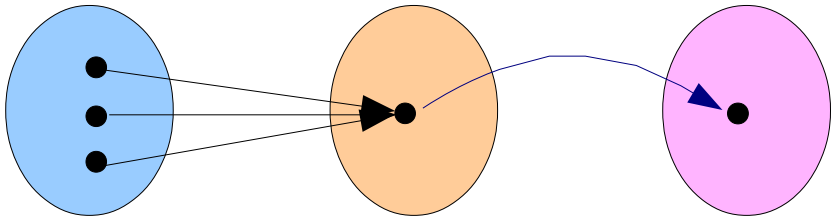
d_i $\quad g(d_i)$ $\quad 360 \cdot g(d_i)$

Degree



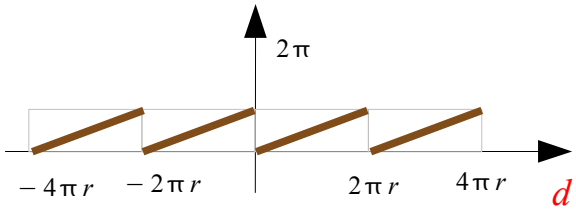
$g(d_i) = \text{Remainder}\left(\frac{d_i}{2\pi r}\right)$

Radian



d_i $\quad g(d_i)$ $\quad 2\pi \cdot g(d_i)$

Radian



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[3] Blitzer, R. “Algebra & Trigonometry.” 3rd ed, Prentice Hall
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[5] 홍성대, “기본/실력 수학의 정석,”성지출판