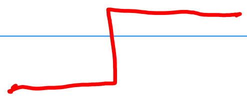
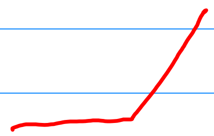
Time Responses (H.1)
System Types

20150529

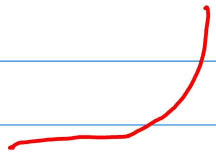
step fn $u(t) \iff \frac{1}{s}$

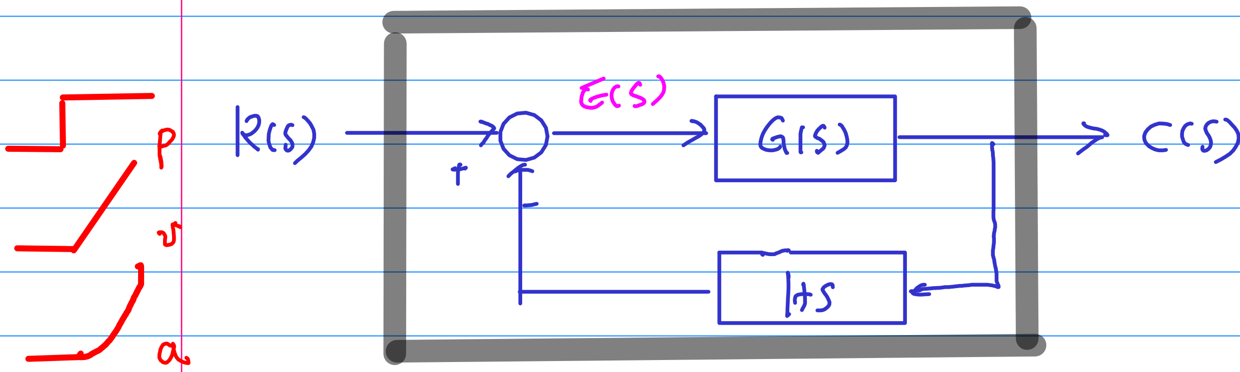


ramp fn $t\,u(t) \iff \frac{1}{s^2}$



parabola fn $t^2 u(t) \iff \frac{2}{s^3}$

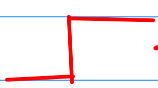


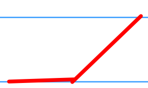


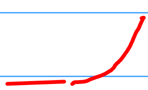
$$\frac{G(s)}{1+G(s)\,H(s)} \qquad \frac{1}{1+G(s)\,H(s)} \times G(s)$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)\,H(s)}$$

$$E(s) = \frac{1}{1+G(s)\,H(s)}\, R(s)$$

$u(t)$  $\frac{1}{s}$ $\qquad E_p(s) = \dfrac{1}{1+G(s)\,H(s)} \cdot \dfrac{1}{s}$

$t\,u(t)$  $\frac{1}{s^2}$ $\qquad E_v(s) = \dfrac{1}{1+G(s)\,H(s)} \cdot \dfrac{1}{s^2}$

$\frac{t^2}{2}u(t)$  $\frac{1}{s^3}$ $\qquad E_a(s) = \dfrac{1}{1+G(s)\,H(s)} \cdot \dfrac{1}{s^3}$

Steady State : $\boxed{t \to \infty}$

$$y'' + ay' + b = x(t)$$

$$y_h = c_1 e^{-m_1 t} + c_2 e^{-m_2 t} \qquad m_1 > 0,\ m_2 > 0$$

$t \to \infty \quad y_h \to 0$

$y_p =$ $x(t)$와 비슷한 모양 : undetermined coefficient

$f(t) \Longleftrightarrow F(s)$

- **Initial value theorem:**

$$f(0^+) = \lim_{s\to\infty} sF(s).$$

- **Final value theorem:**

$$f(\infty) = \lim_{s\to 0} sF(s),$$ if all poles of $sF(s)$ are in the left half-plane.

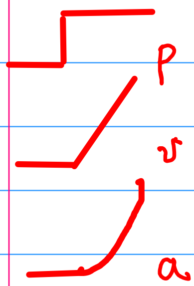
time $t \in [0, \infty)$

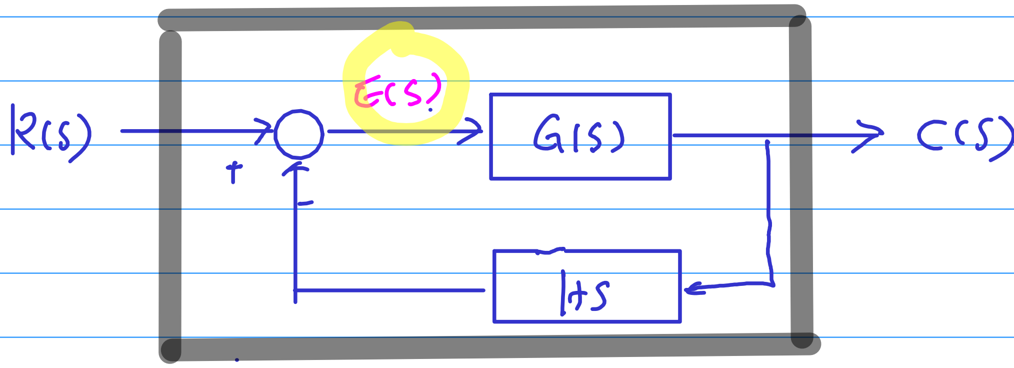
$f(0)$ — initial value

$f(\infty)$ — final value — steady state

$$f(\infty) = \lim_{s\to 0} s\,F(s)$$: Steady State

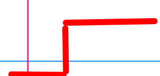
$$f(0) = \lim_{s\to\infty} s\,F(s)$$



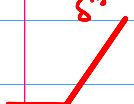


$$E(s) = \frac{1}{1+G(s)H(s)} R(s)$$

$$e(\infty) = \lim_{s\to 0} s \cdot E(s) = \lim_{s\to 0} \frac{s R(s)}{1+G(s)H(s)}$$

$u(t)$ $\frac{1}{s}$  p

$$e_p(\infty) = \lim_{s\to 0} \frac{s}{1+G(s)H(s)} \frac{1}{s}$$

$t\,u(t)$ $\frac{1}{s^2}$  v

$$e_v(\infty) = \lim_{s\to 0} \frac{s}{1+G(s)H(s)} \frac{1}{s^2}$$

$\frac{t^2}{2}u(t)$ $\frac{1}{s^3}$  a

$$e_a(\infty) = \lim_{s\to 0} \frac{s}{1+G(s)H(s)} \frac{1}{s^3}$$

$u(t)$ $\quad \frac{1}{s}$ $\quad p$

$$e_p(\infty) = \lim_{s\to 0} \frac{1}{1+G(s)H(s)}$$

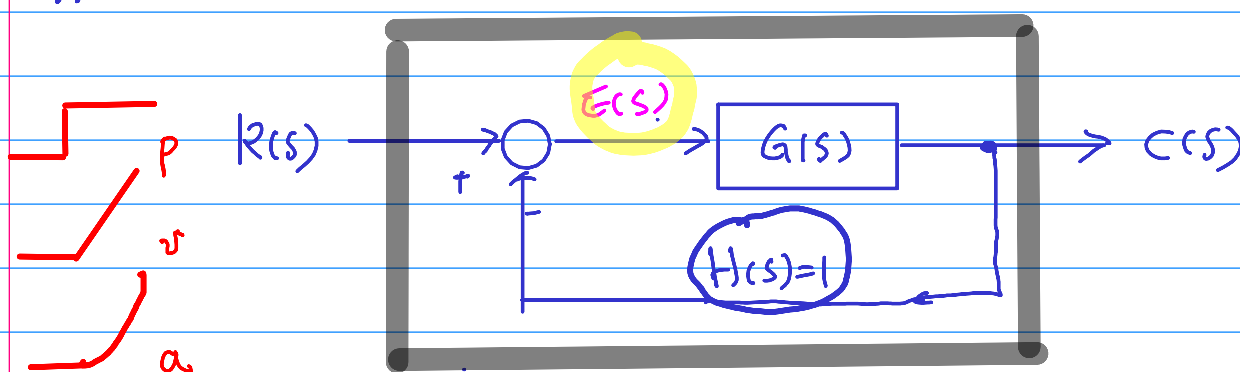
$t\,u(t)$ $\quad \frac{1}{s^2}$ $\quad v$

$$e_v(\infty) = \lim_{s\to 0} \frac{1}{1+G(s)H(s)} \cdot \frac{1}{s}$$

$\frac{t^2}{2}u(t)$ $\quad \frac{1}{s^3}$ $\quad a$

$$e_a(\infty) = \lim_{s\to 0} \frac{1}{1+G(s)H(s)} \cdot \frac{1}{s^2}$$

* Unit feedback System



$u(t)$ $\quad p$

$$e_p(\infty) = \lim_{s\to 0} \frac{1}{1+G(s)} = \lim_{s\to 0} \frac{1}{1+G(s)}$$

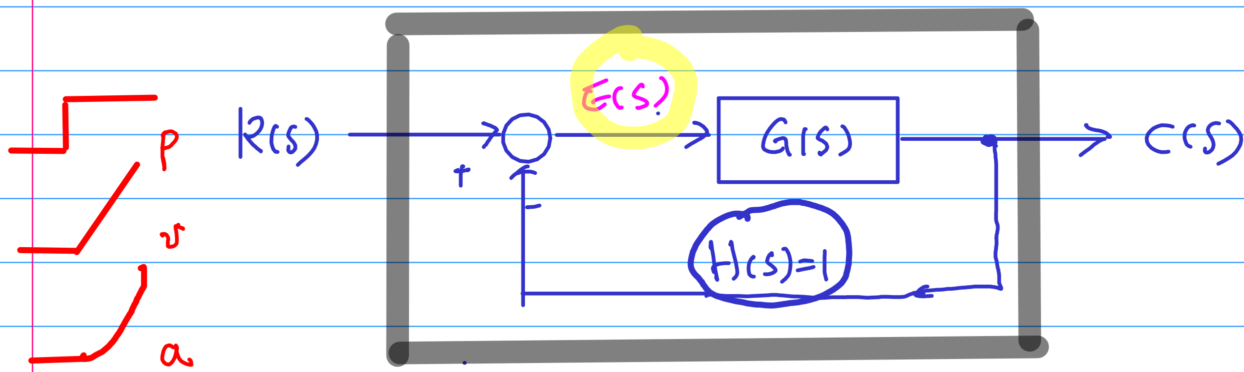
$t\,u(t)$ $\quad v$

$$e_v(\infty) = \lim_{s\to 0} \frac{1}{s+sG(s)} = \lim_{s\to 0} \frac{1}{sG(s)}$$

$\frac{t^2}{2}u(t)$ $\quad a$

$$e_a(\infty) = \lim_{s\to 0} \frac{1}{s^2+s^2G(s)} = \lim_{s\to 0} \frac{1}{s^2G(s)}$$

* Unit feedback System



u(t) p $\quad e_p(\infty) = \lim_{s\to 0} \frac{1}{1+G(s)} = \frac{1}{1+K_p}$

t u(t) v $\quad e_v(\infty) = \lim_{s\to 0} \frac{1}{sG(s)} = \frac{1}{K_v}$

$\frac{t^2}{2}u(t)$ a $\quad e_a(\infty) = \lim_{s\to 0} \frac{1}{s^2G(s)} = \frac{1}{K_a}$

Position $\quad K_p = \lim_{s\to 0} G(s)$

velocity $\quad K_v = \lim_{s\to 0} sG(s)$

acceleration $\quad K_a = \lim_{s\to 0} s^2G(s)$

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

$$\boxed{G(s) = \frac{(s+z_1)(s+z_2)\cdots\cdots}{s^n(s+p_1)(s+p_2)\cdots\cdots\ \cdots}}$$

$n = 0$ $\quad G(s) = \dfrac{(s+z_1)(s+z_2)\cdots\cdots}{(s+p_1)(s+p_2)\cdots\cdots\ \cdots}$ $\quad$ type 0 system

$n = 1$ $\quad G(s) = \dfrac{(s+z_1)(s+z_2)\cdots\cdots}{s\,(s+p_1)(s+p_2)\cdots\cdots\ \cdots}$ $\quad$ type 1 system

$n = 2$ $\quad G(s) = \dfrac{(s+z_1)(s+z_2)\cdots\cdots}{s^2(s+p_1)(s+p_2)\cdots\cdots\ \cdots}$ $\quad$ type 2 system

$n = 3$ $\quad G(s) = \dfrac{(s+z_1)(s+z_2)\cdots\cdots}{s^3(s+p_1)(s+p_2)\cdots\cdots\ \cdots}$

Type 0 System

$\cancel{\frac{1}{s} : \text{Integrator}}$

$n=0$ $\quad G(s) = \dfrac{(s+z_1)(s+z_2)\cdots\cdots}{(s+p_1)(s+p_2)\cdots\cdots\cdots}$ type 0 System

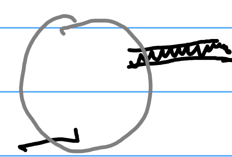
$$\lim_{s\to 0} G(s) = \frac{z_1 \cdot z_2 \cdots\cdots}{p_1 \cdot p_2 \cdots\cdots}$$

Position $\quad K_p = \lim_{s\to 0} G(s) = \dfrac{z_1\, z_2 \cdots\cdots}{p_1\, p_2 \cdots\cdots}$

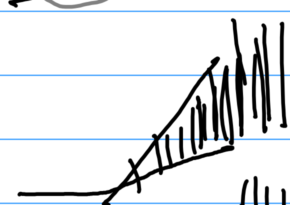
velocity $\quad K_v = \lim_{s\to 0} s \cdot G(s) = \lim_{s\to 0} s \cdot \dfrac{z_1\, z_2 \cdots\cdots}{p_1\, p_2 \cdots\cdots} \Rightarrow 0$

acceleration $\quad K_a = \lim_{s\to 0} s^2 \cdot G(s) = \lim_{s\to 0} s^2 \cdot \dfrac{z_1\, z_2 \cdots\cdots}{p_1\, p_2 \cdots\cdots} \Rightarrow 0$

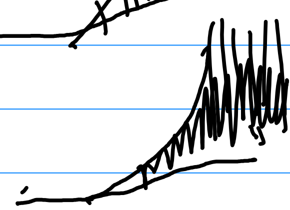
$u(t)$ p $\quad e_p(\infty) = \dfrac{1}{1+K_p}$ 상수.



$t\,u(t)$ v $\quad e_v(\infty) = \dfrac{1}{K_v} \rightarrow \infty$



$\frac{t^2}{2} u(t)$ a $\quad e_a(\infty) = \dfrac{1}{K_a} \rightarrow \infty$



Type 1 System $\frac{1}{S}$: Integrator

$n = 1$ $G(s) = \dfrac{(s+z_1)(s+z_2)\cdots\cdots}{s\,(s+p_1)(s+p_2)\cdots\cdots}$ Type 1 System

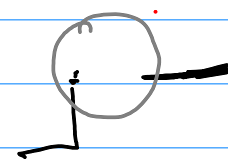
$$\lim_{s\to 0} G(s) = \frac{z_1 \cdot z_2 \cdots\cdots}{p_1 \cdot p_2 \cdots\cdots}$$

Position $K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{1}{s}\,\frac{z_1 z_2 \cdots\cdots}{p_1 p_2 \cdots\cdots} \Rightarrow \infty$

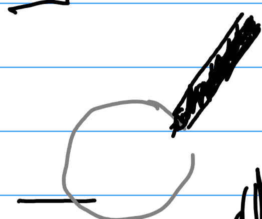
velocity $K_v = \lim_{s\to 0} s\cdot G(s) = \lim_{s\to 0} \cdot \frac{z_1 z_2 \cdots\cdots}{p_1 p_2 \cdots\cdots}$

acceleration $K_a = \lim_{s\to 0} s^2 \cdot G(s) = \lim_{s\to 0} s \cdot \frac{z_1 z_2 \cdots\cdots}{p_1 p_2 \cdots\cdots} \Rightarrow 0$

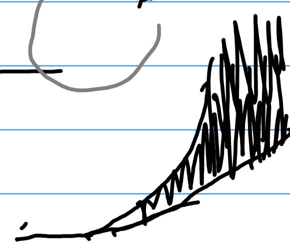
$u(t)$ _p_ $e_p(\infty) = \dfrac{1}{1+K_p} \rightarrow 0$



$t\,u(t)$ _v_ $e_v(\infty) = \dfrac{1}{K_v}$



$\frac{t^2}{2} u(t)$ _a_ $e_a(\infty) = \dfrac{1}{K_a} \rightarrow \infty$



Type (2) System

$\frac{1}{s}$: Integrator (2개)

$n=2$ $\quad G(s) = \dfrac{(s+z_1)(s+z_2)\cdots\cdots}{s^2(s+p_1)(s+p_2)\cdots\cdots}$ Type (2) System

$$\lim_{s\to 0} G(s) = \frac{z_1\cdot z_2\cdots\cdots}{p_1\cdot p_2\cdots\cdots}$$

Position $\quad K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{1}{s^2}\frac{z_1\, z_2\cdots\cdots}{p_1\; p_2\cdots\cdots} \Rightarrow \infty$

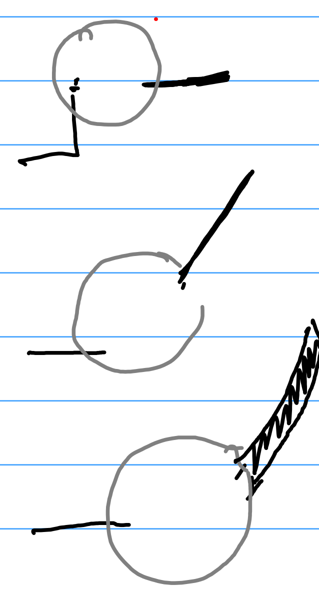
Velocity $\quad K_v = \lim_{s\to 0} s\cdot G(s) = \lim_{s\to 0} \frac{1}{s}\frac{z_1\, z_2\cdots\cdots}{p_1\; p_2\cdots\cdots} \Rightarrow \infty$

acceleration $\quad K_a = \lim_{s\to 0} s^2\cdot G(s) = \lim_{s\to 0} \cdot\frac{z_1\, z_2\cdots\cdots}{p_1\; p_2\cdots\cdots}$

$u(t)$ p $\quad e_p(\infty) = \dfrac{1}{1+K_p} \rightarrow 0$

$t\,u(t)$ v $\quad e_v(\infty) = \dfrac{1}{K_v} \rightarrow 0$

$\frac{t^2}{2}u(t)$ a $\quad e_a(\infty) = \dfrac{1}{K_a}$



system type	error constant			steady state error		
	K_p	K_v	K_a	$e_p(\infty)$	$e_v(\infty)$	$e_a(\infty)$
$n=0$	c	0	0	$1/(1+K_p)$	∞	∞
$n=1$	∞	c	0	0	$1/K_v$	∞
$n=2$	∞	∞	c	0	0	$1/K_a$
$n=3$	∞	∞	∞	0	0	0