t-Testing (Group)

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Outline

Based on

"Understanding Statistics in the Behavioral Sciences" R. R. Pagano

t Test for single samples and correlated groups (1)

t Test for Single Samples	t Test for Correlated Groups
$t_{obt} = \frac{\overline{X}_{obt} - \mu}{s/\sqrt{N}}$	$t_{obt} = \frac{\overline{D}_{obt} - \mu_D}{s_D/\sqrt{N}}$
$t_{obt} = \frac{\overline{X}_{obt} - \mu}{\sqrt{\frac{SS}{N(N-1)}}}$	$t_{obt} = \frac{\overline{D}_{obt} - \mu_D}{\sqrt{\frac{SS_D}{N(N-1)}}}$
$SS = \Sigma X^2 - \frac{(\Sigma X)^2}{N}$	$SS_D = \Sigma D^2 - \frac{(\Sigma D)^2}{N}$

t Test for single samples and correlated groups (2)

D	difference score
$\overline{D}_{obt}$	mean of the sample difference scores
μ_D	mean of the population of difference scores
S_D	standard deviation of the sample difference scores
N	number of difference scores
SS_D	$= \Sigma(D - \overline{D})^2$ sum of squares of sample difference scores

degree of freedom (df)

- the degree of freedom for any statistic is
  the number of scores that are free to vary in calculating static

the mean of the difference between sample means

- the mean of
  the sampling distribution of the difference between sample means

$$\mu_{\overline{X}_1 - \overline{X}_2} = \mu_1 - \mu_2$$

the s.d. of the difference between sample means

- standard deviation of
  the sampling distribution of the differece between sample means
  alternatively, standard error of the difference between sample means

$$\sigma_{\overline{X}_1 - \overline{X}_2} = \sqrt{\sigma^2_{\overline{X}_1} + \sigma^2_{\overline{X}_2}}$$

- $\sigma^2_{\overline{X}_1}$ variance of the sampling distribution of the mean
  for samplesof size n_1 taken for the first population
- $\sigma^2_{\overline{X}_1}$ variance of the sampling distribution of the mean
  for samplesof size n_1 taken for the second population

Hormone X Experiment

- equation for z_{obt} independent groups design

$$z_{obt} = \frac{(\overline{X}_1 - \overline{X}_2) - \mu\overline{x}_1 - \overline{x}_2}{\sigma\overline{x}_1 - \overline{x}_2}$$

z and t equations compared

$$z_{obt} = \frac{(X_1 - X_2) - \mu\overline{x}_1 - \overline{x}_2}{\sigma\overline{x}_1 - \overline{x}_2} = \frac{(X_1 - X_2) - \mu\overline{x}_1 - \overline{x}_2}{\sqrt{\sigma^2(\frac{1}{n_1} + \frac{1}{n_2})}}$$

$$t_{obt} = \frac{(X_1 - X_2) - \mu\overline{x}_1 - \overline{x}_2}{s\overline{x}_1 - \overline{x}_2} = \frac{(X_1 - X_2) - \mu\overline{x}_1 - \overline{x}_2}{\sqrt{s_w^2(\frac{1}{n_1} + \frac{1}{n_2})}}$$

- s_w^2 weighted estimate of σ^2
- $s_{\overline{x}_1 - \overline{x}_2}$ estimate of $\sigma_{\overline{x}_1 - \overline{x}_2}$
  estimate of standard error of the difference between sample means

size of effect using Cohen's d (1)

- general equation for size of effect

$$d = \frac{mean\ difference}{population\ standard\ deviation}$$

- conceptual equation for size of effect, correlated groups t test

$$d = \frac{\overline{D}_{obt}}{\sigma_D}$$

size of effect using Cohen's d (2)

- computational equation for size of effect, correlated group test

$$\hat{d} = \frac{|\overline{D}_{obt}|}{s_D}$$

- $\hat{d}$: estimate of d
- $|\overline{D}_{obt}|$: the absolute value of the mean of the sample difference scores
- s_D : the standard deviation of the sample difference scores

Cohen's criteria for interpreting the value of $\hat{d}$

Value of $\hat{d}$	Interpretation of $\hat{d}$
0.00 ~ 0.20	small effect
0.21 ~ 0.79	medium effect
0.80 ~	large effect

Power of the t-test

- Single Sample

$$t_{obt} = \frac{\overline{X}_{obt} - \mu}{\sqrt{\frac{SS}{N(N-1)}}}$$

- Correlated Groups

$$t_{obt} = \frac{\overline{D}_{obt} - 0}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

- Independent Groups

$$t_{obt} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{SS_1+SS_2}{n(n-1)}}}$$

Correlated and Independent Groups Designs

- Correlated Groups

$$t_{obt} = \frac{\overline{D}}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

- Independent Groups

$$t_{obt} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{SS_1+SS_2}{n(n-1)}}}$$

Constructing 95% confidence interval for μ

- Single Sample Experiment
- 95% Confidence Interval for μ

$$\mu_{lower} = \overline{X}_{obt} - s_{\overline{X}} t_{0.025}$$

$$\mu_{upper} = \overline{X}_{obt} + s_{\overline{X}} t_{0.025}$$

where

$$s_{\overline{X}} = \frac{s}{\sqrt{n}}$$

Constructing 95% confidence interval for $\mu_1 - \mu_2$

- Two Sample Experiment
- 95% Confidence Interval for $\mu_1 - \mu_2$

$$\mu_{lower} = (\overline{X}_1 - \overline{X}_2) - s_{\overline{X}_1 - \overline{X}_2} t_{0.025}$$

$$\mu_{lower} = (\overline{X}_1 - \overline{X}_2) + s_{\overline{X}_1 - \overline{X}_2} t_{0.025}$$

where

$$s_{\overline{X}_1 - \overline{X}_2} = \sqrt{\left(\frac{SS_1 + SS_2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$