

t-Testing (Group)

Young W. Lim

2019-08-21 Wed

Outline

- 1 Based on
- 2 t Test for correlated groups
- 3 z test for independent groups
- 4 t test for independent groups

"Understanding Statistics in the Behavioral Sciences" R. R. Pagano

I, the copyright holder of this work, hereby publish it under the following licenses: GNU head Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled GNU Free Documentation License.

CC BY SA This file is licensed under the Creative Commons Attribution ShareAlike 3.0 Unported License. In short: you are free to share and make derivative works of the file under the conditions that you appropriately attribute it, and that you distribute it only under a license compatible with this one.

t Test for single samples and correlated groups (1)

t Test for Single Samples

t Test for Correlated Groups

$$t_{obt} = \frac{\bar{X}_{obt} - \mu}{s/\sqrt{N}}$$

$$t_{obt} = \frac{\bar{D}_{obt} - \mu_D}{s_D/\sqrt{N}}$$

$$t_{obt} = \frac{\bar{X}_{obt} - \mu}{\sqrt{\frac{SS}{N(N-1)}}}$$

$$t_{obt} = \frac{\bar{D}_{obt} - \mu_D}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

$$SS = \sum X^2 - \frac{(\sum X)^2}{N}$$

$$SS_D = \sum D^2 - \frac{(\sum D)^2}{N}$$

t Test for single samples and correlated groups (2)

D	difference score
\bar{D}_{obt}	mean of the sample difference scores
μ_D	mean of the population of difference scores
S_D	standard deviation of the sample difference scores
N	number of difference scores
SS_D	$= \Sigma(D - \bar{D})^2$ sum of squares of sample difference scores

degree of freedom (df)

- the degree of freedom for any statistic is the number of scores that are free to vary in calculating static

the mean of the difference between sample means

- the mean of the sampling distribution of the difference between sample means

$$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$$

the s.d. of the difference between sample means

- standard deviation of the sampling distribution of the difference between sample means
alternatively, standard error of the difference between sample means

$$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\sigma_{\bar{X}_1}^2 + \sigma_{\bar{X}_2}^2}$$

- $\sigma_{\bar{X}_1}^2$ variance of the sampling distribution of the mean for samples of size n_1 taken for the first population
- $\sigma_{\bar{X}_2}^2$ variance of the sampling distribution of the mean for samples of size n_2 taken for the second population

- equation for z_{obt} independent groups design

$$z_{obt} = \frac{(\bar{X}_1 - \bar{X}_2) - \mu_{\bar{X}_1 - \bar{X}_2}}{\sigma_{\bar{X}_1 - \bar{X}_2}}$$

z and t equations compared

$$z_{obt} = \frac{(X_1 - X_2) - \mu\bar{x}_1 - \bar{x}_2}{\sigma\bar{x}_1 - \bar{x}_2} = \frac{(X_1 - X_2) - \mu\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$t_{obt} = \frac{(X_1 - X_2) - \mu\bar{x}_1 - \bar{x}_2}{s\bar{x}_1 - \bar{x}_2} = \frac{(X_1 - X_2) - \mu\bar{x}_1 - \bar{x}_2}{\sqrt{s_w^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

- s_w^2 weighted estimate of σ^2
- $s_{\bar{x}_1 - \bar{x}_2}$ estimate of $\sigma_{\bar{x}_1 - \bar{x}_2}$
estimate of standard error of the difference between sample means

size of effect using Cohen's d (1)

- general equation for size of effect

$$d = \frac{\text{mean difference}}{\text{population standard deviation}}$$

- conceptual equation for size of effect, correlated groups t test

$$d = \frac{\bar{D}_{obt}}{\sigma_D}$$

size of effect using Cohen's d (2)

- computational equation for size of effect, correlated group test

$$\hat{d} = \frac{|\bar{D}_{obt}|}{s_D}$$

- \hat{d} : estimate of d
- $|\bar{D}_{obt}|$: the absolute value of the mean of the sample difference scores
- s_D : the standard deviation of the sample difference scores

Cohen's criteria for interpreting the value of \hat{d}

Value of \hat{d}	Interpretation of \hat{d}
0.00 ~ 0.20	small effect
0.21 ~ 0.79	medium effect
0.80 ~	large effect

- Single Sample

$$t_{obt} = \frac{\bar{X}_{obt} - \mu}{\sqrt{\frac{SS}{N(N-1)}}}$$

- Correlated Groups

$$t_{obt} = \frac{\bar{D}_{obt} - 0}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

- Independent Groups

$$t_{obt} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{SS_1 + SS_2}{n(n-1)}}}$$

Correlated and Independent Groups Designs

- Correlated Groups

$$t_{obt} = \frac{\bar{D}}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

- Independent Groups

$$t_{obt} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{SS_1 + SS_2}{n(n-1)}}}$$

Constructing 95% confidence interval for μ

- Single Sample Experiment
- 95% Confidence Interval for μ

$$\mu_{lower} = \bar{X}_{obt} - s_{\bar{X}} t_{0.025}$$

$$\mu_{upper} = \bar{X}_{obt} + s_{\bar{X}} t_{0.025}$$

where

$$s_{\bar{X}} = \frac{s}{\sqrt{n}}$$

Constructing 95% confidence interval for $\mu_1 - \mu_2$

- Two Sample Experiment
- 95% Confidence Interval for $\mu_1 - \mu_2$

$$\mu_{lower} = (\bar{X}_1 - \bar{X}_2) - s_{\bar{X}_1 - \bar{X}_2} t_{0.025}$$

$$\mu_{upper} = (\bar{X}_1 - \bar{X}_2) + s_{\bar{X}_1 - \bar{X}_2} t_{0.025}$$

where

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{\left(\frac{SS_1 + SS_2}{n_1 + n_2 - 2} \right) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$