Stability (H.1)

20150611

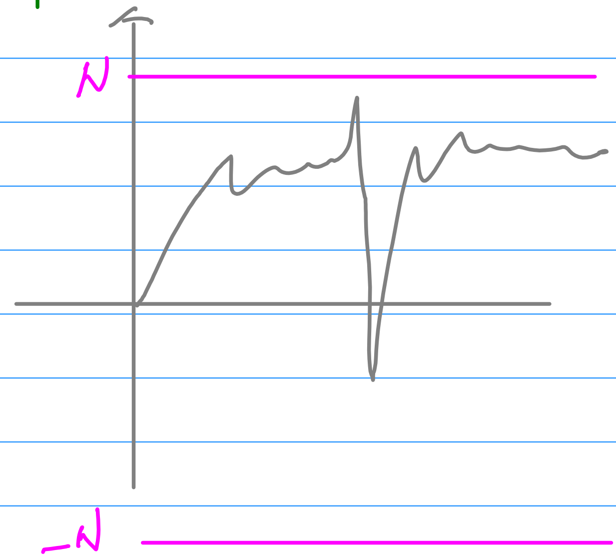
BIBO

Bounded Input Bounded Output

$$u(t) \longrightarrow \boxed{g(t)} \longrightarrow y(t) = g(t) * u(t)$$

$|y(t)| \leq N$

$|u(t)| \leq M$

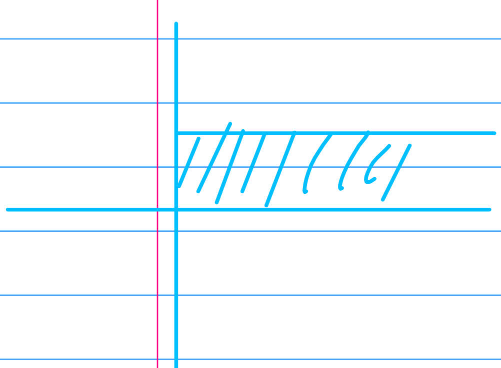


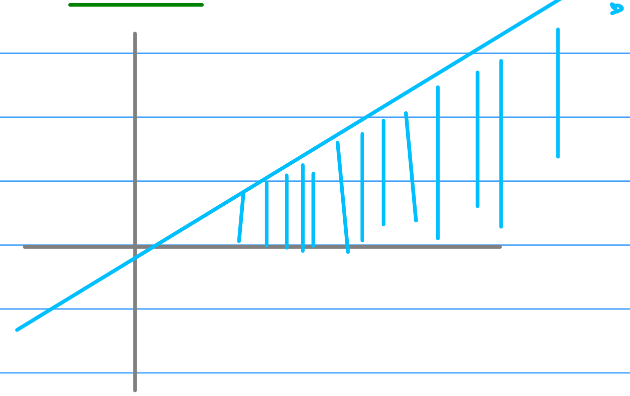
$$|u(t)| \leq M \rightsquigarrow |y(t)| \leq N$$

unstable

BIBO stable

?





선형시스템 ~ transfer function으로 표현 가능

(시간 상 특성
주파수상 특성) 파악 가능

Laplace Transform

$G(s)$ ...

$s = \sigma + j\omega$

$j\omega$ $(\sigma = 0)$

$G(j\omega)$

Fourier Transform

frequency Response

impulse response

$$u(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

Causal System $\quad h(t) = 0 \quad t < 0$

causal signal $\quad y(t) = 0 \quad t < 0$

$u(t) = 0 \quad t < 0$

$y(t) = g(t) * u(t)$ $\quad u(t-\tau) = 0 \quad t-\tau < 0$

$\Rightarrow \tau < t$

$$= \int_0^\infty u(t-\tau) \cdot g(\tau)\, d\tau$$

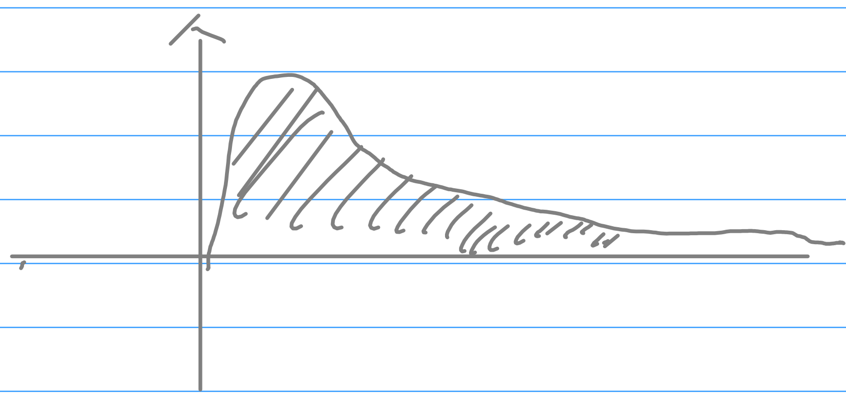
$$|y(t)| = \left| \int_0^\infty u(t-\tau) \cdot g(\tau)\, d\tau \right|$$

$$\leq \int_0^\infty \underbrace{|u(t-\tau)|}_{<M} \cdot |g(\tau)|\, d\tau$$ Bounded Input

$$\leq \int_0^\infty M \cdot |g(\tau)|\, d\tau$$

$$\leq M \int_0^\infty |g(\tau)|\, d\tau \leq N$$ Bounded Output ও। হয়ে যাবে

$$\int_0^\infty |g(\tau)|\, d\tau \leq \frac{N}{M} < \infty$$



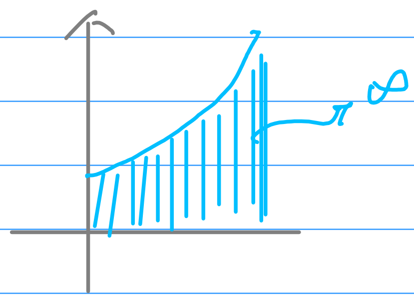
Conditions for BIBO Stability

$$\int_0^\infty |g(z)|\,dz < \infty$$

$$\boxed{g(t)} \Longleftrightarrow \boxed{G(s)}$$

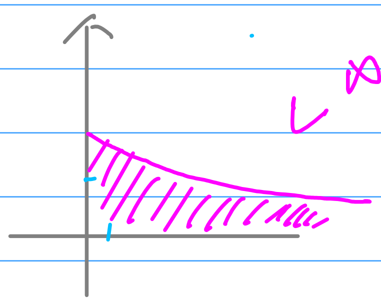
impulse response

Tranfer function



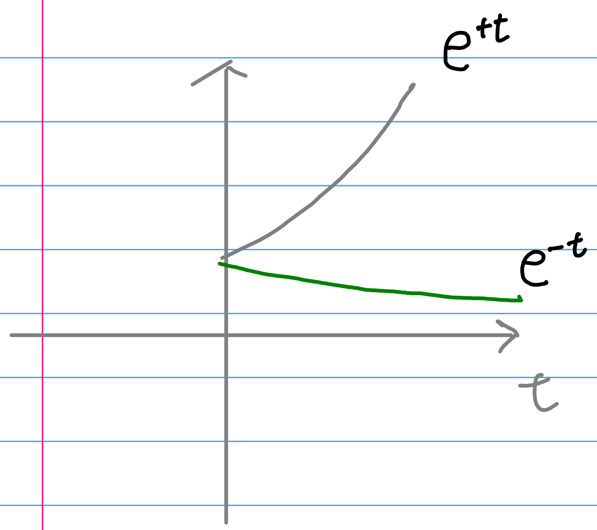
$$\int_0^\infty |g(z)|\,dz \to \infty$$

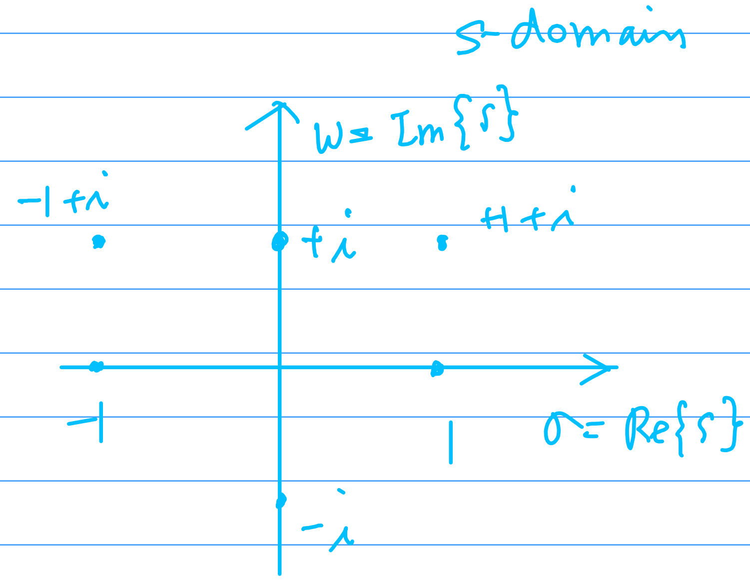
∴ BIBO stable X



$$\int_0^\infty |g(z)|\,dz < \infty$$

∴ BIBO stable





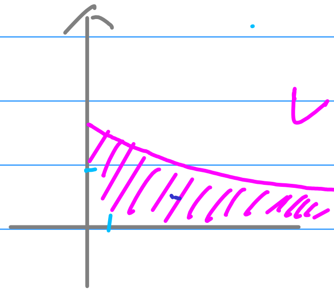
$$g(t) = e^{+t} \iff G(s) = \frac{1}{(s-1)}$$

pole +1
RHP

$$g(t) = e^{-t} \iff G(s) = \frac{1}{(s+1)}$$

pole -1
LHP

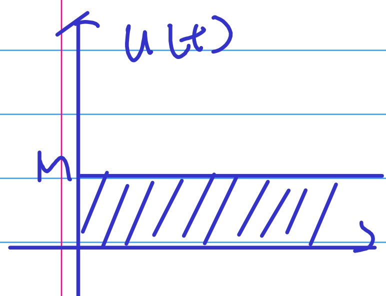
any pole in RHP $\rightarrow$ unstable



$< \infty$

$$\int_0^\infty |g(\tau)|\, d\tau < \infty$$

$\therefore$ BIBO stable



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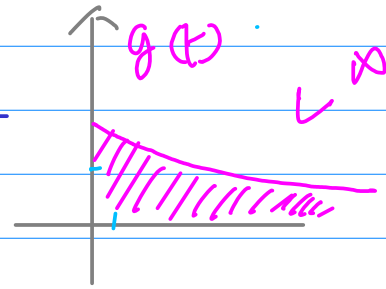
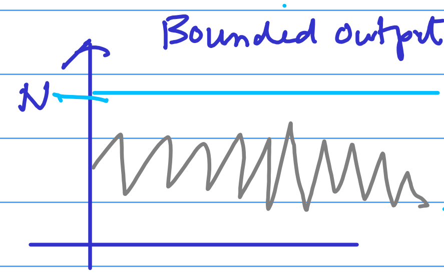
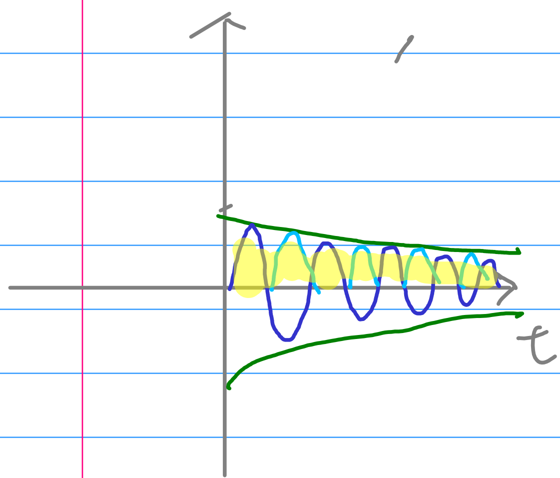
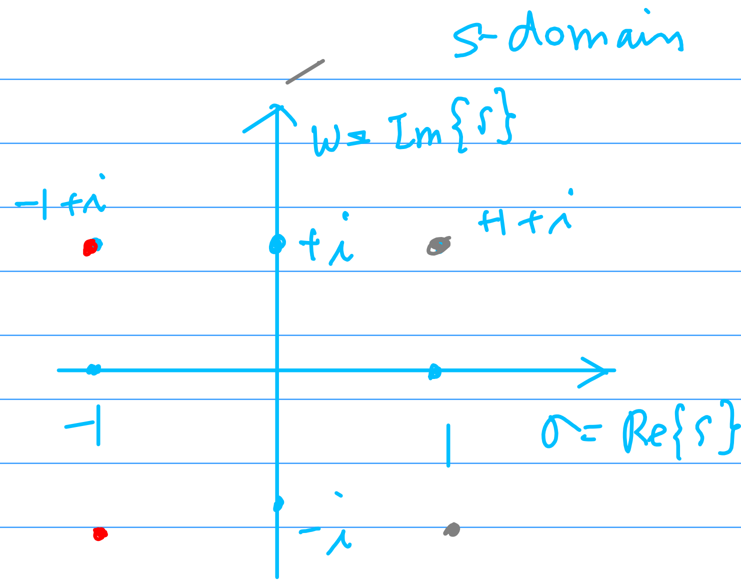








$$G(s) = \frac{1}{(s-(-1+i))(s-(-1-i))}$$

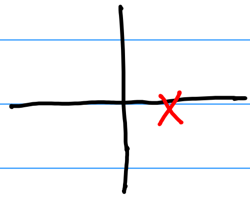
$$= \frac{1}{((s+1)-i)((s+1)+i)}$$

$e^{-t}\sin t$ $\Longleftrightarrow$ $= \dfrac{1}{(s+1)^2+1} = \dfrac{1}{s^2+2s+2}$

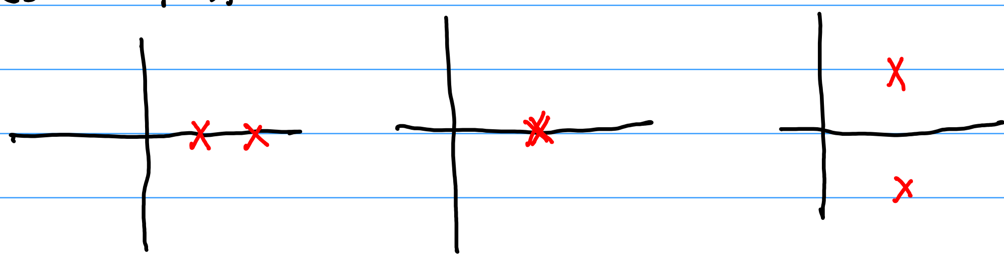
$\sin t$	$\Longleftrightarrow$	$\dfrac{1}{s^2+1}$

$\sin kt$	$\Longleftrightarrow$	$\dfrac{k}{s^2+k^2}$
$\cos kt$	$\Longleftrightarrow$	$\dfrac{s}{s^2+k^2}$

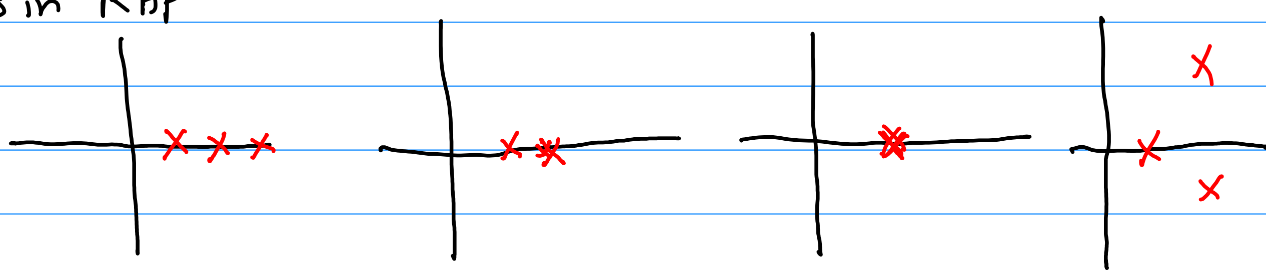
① pole in RHP

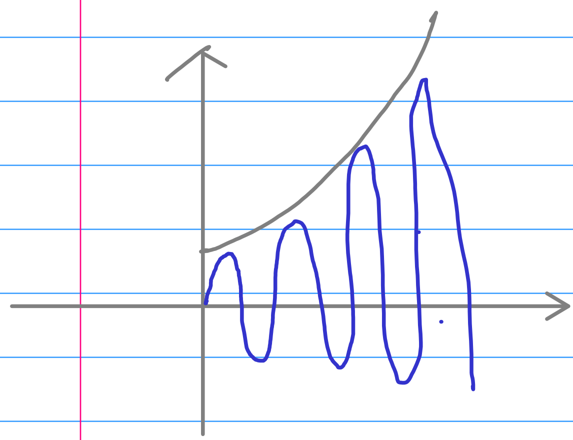


② poles in RHP

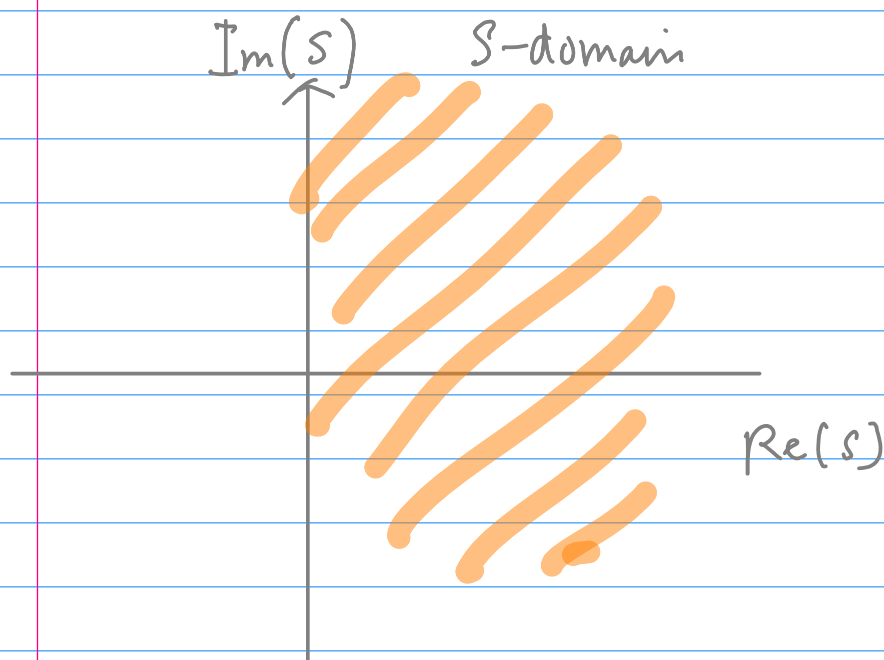


③ poles in RHP



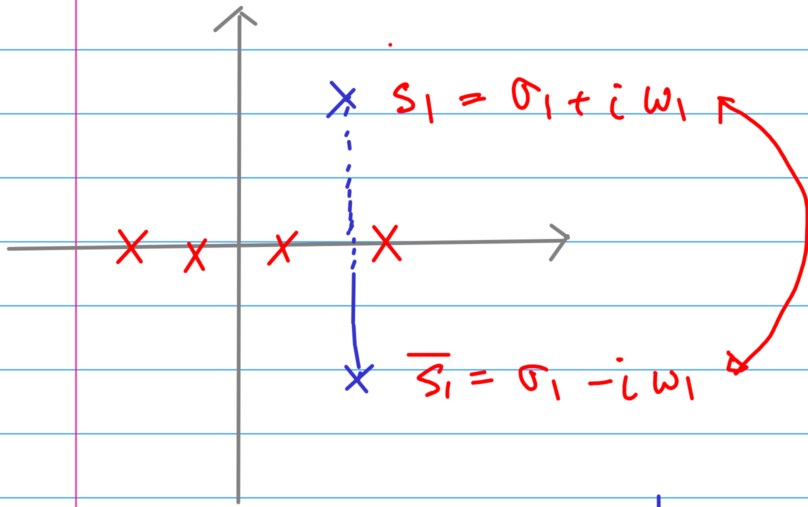


$$e^{+t}\sin t \iff G(s) = \frac{1}{(s-(+1+i))(s-(+1-i))} = \frac{1}{((s-1)-i)((s-1)+i)} = \frac{1}{(s-1)^2+1} = \frac{1}{s^2-2s+2}$$



any pole in RHP causes a system unstable

Complex Conjugate Roots



$$G(s) = \frac{1}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} = \frac{1}{F(s)}$$

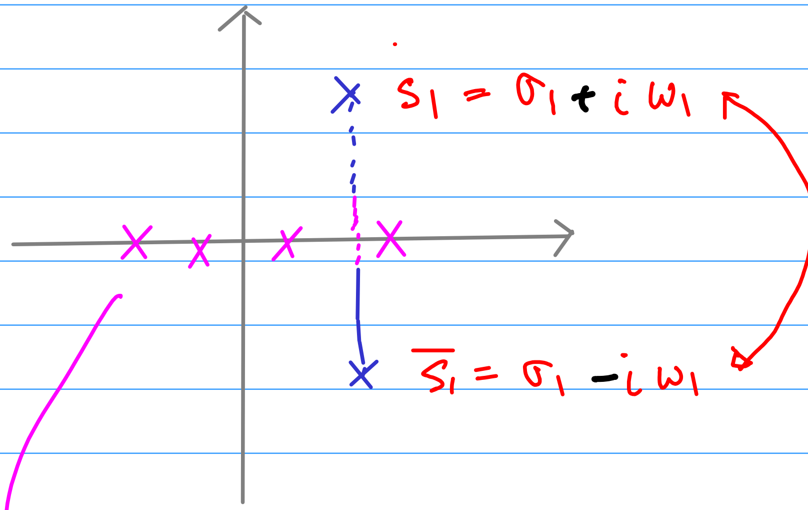
generally $a_n, a_{n-1}, \cdots, a_1, a_0$ all real

$\Rightarrow$ $s_1 = \sigma_1 + i\omega_1$ 이

$a_n s_1^n + a_{n-1} s_1^{n-1} + \cdots + a_1 s_1 + a_0 = 0$ 이면

$\bar{s}_1 = \sigma_1 - i\omega_1$ 이

$a_n \bar{s}_1^n + a_{n-1} \bar{s}_1^{n-1} + \cdots + a_1 \bar{s}_1 + a_0 = 0$



$$\frac{b}{s+a}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Uparrow$$

$$\frac{1}{(s-s_1)(s-\bar{s}_1)}$$

$$= \frac{1}{[s-(\sigma+i\omega)][s-(\sigma-i\omega)]}$$

$$= \frac{1}{(s-\sigma)^2 + \omega^2}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-b \pm \sqrt{b^2 - ac}}{a}$$

$$s = -\zeta\omega_n \pm \sqrt{\zeta^2\omega_n^2 - \omega_n^2}$$

$$= -\zeta\omega_n \pm \sqrt{(\zeta^2 - 1)\,\omega_n^2}$$

$$= -\zeta\omega_n \pm \sqrt{(\zeta^2 - 1)}\;\omega_n$$

만약 $\zeta < 1$ $\quad \zeta^2 < 1$

$$= -\zeta\omega_n \pm i\sqrt{(1-\zeta^2)}\;\omega_n$$

$$s^2+2\zeta\omega_n s+\omega_n^2 \Rightarrow \left(s^2+2\zeta\omega_n s+\zeta^2\omega_n^2\right)-\zeta^2\omega_n^2+\omega_n^2$$

$$= (s+\zeta\omega_n)^2 + (1-\zeta^2)\,\omega_n^2$$

$$= (s+\zeta\omega_n)^2 + \left(\sqrt{1-\zeta^2}\,\omega_n\right)^2$$

$$a^2 \quad + \quad b^2$$

$$= \left((s+\zeta\omega_n)+i\sqrt{(1-\zeta^2)}\,\omega_n\right)\left((s+\zeta\omega_n)-i\sqrt{(1-\zeta^2)}\,\omega_n\right)$$

$$(a+ib)\quad(a-ib)$$

$$\frac{1}{\left((s+\zeta\omega_n)+i\sqrt{(1-\zeta^2)}\,\omega_n\right)\left((s+\zeta\omega_n)-i\sqrt{(1-\zeta^2)}\,\omega_n\right)}$$

$$s=-\zeta\omega_n+i\sqrt{(1-\zeta^2)}\,\omega_n \quad , \quad -\zeta\omega_n-i\sqrt{1-\zeta^2}\,\omega_n$$

$$= \frac{-1}{2i\sqrt{(1-\zeta^2)}\,\omega_n}\left[\frac{1}{\left((s+\zeta\omega_n)+i\sqrt{(1-\zeta^2)}\,\omega_n\right)}-\frac{1}{\left((s+\zeta\omega_n)-i\sqrt{(1-\zeta^2)}\,\omega_n\right)}\right]$$

$$= \frac{1}{2i\sqrt{(1-\zeta^2)}\,\omega_n}\left[\frac{1}{\left(s+\left(\zeta\omega_n-i\sqrt{(1-\zeta^2)}\,\omega_n\right)\right)}-\frac{1}{\left(s+\left(\zeta\omega_n+i\sqrt{(1-\zeta^2)}\,\omega_n\right)\right)}\right]$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 \Rightarrow \left(s^2 + 2\zeta\omega_n s + \zeta^2\omega_n^2\right) - \zeta^2\omega_n^2 + \omega_n^2$$

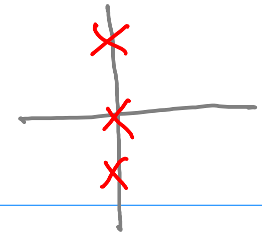
$$= (s + \zeta\omega_n)^2 + (1-\zeta^2)\omega_n^2$$

$$= \frac{1}{2i\sqrt{(1-\zeta^2)}\,\omega_n}\left[\frac{1}{\left(s + \left(\zeta\omega_n - i\sqrt{(1-\zeta^2)}\,\omega_n\right)\right)} - \frac{1}{\left(s + \left(\zeta\omega_n + i\sqrt{(1-\zeta^2)}\,\omega_n\right)\right)}\right]$$

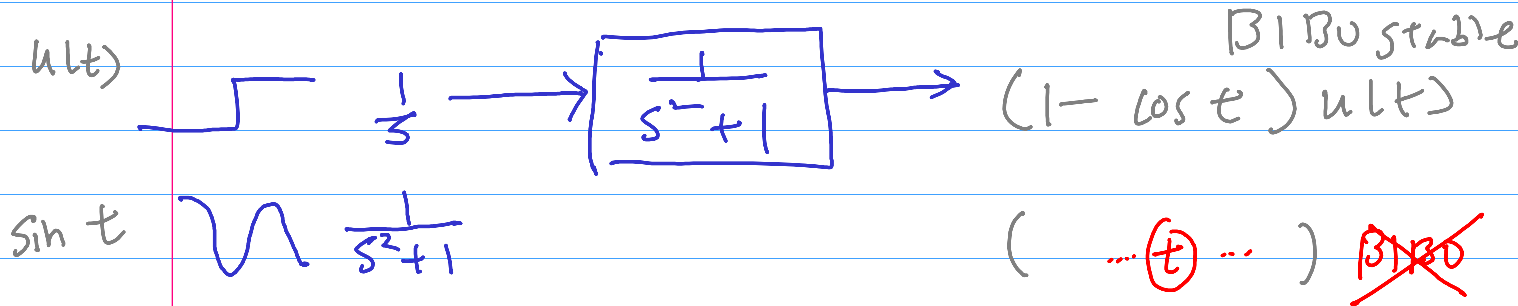
$$\Updownarrow \qquad \Updownarrow$$

$$\frac{1}{2i\sqrt{(1-\zeta^2)}\,\omega_n}\left[e^{-\zeta\omega_n t}e^{+i\sqrt{(1-\zeta^2)}\,\omega_n t} - e^{-\zeta\omega_n t}e^{-i\sqrt{(1-\zeta^2)}\,\omega_n t}\right]$$

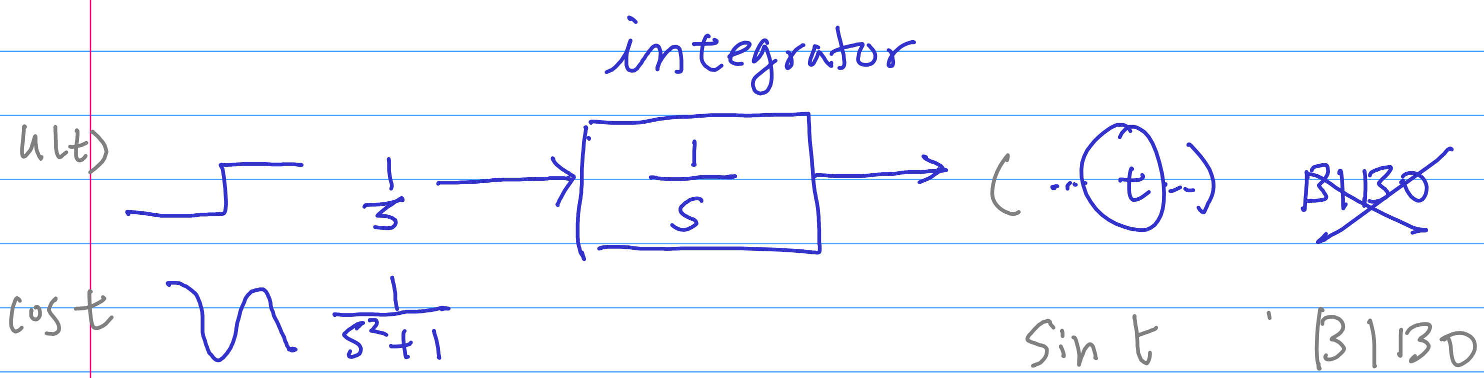
Marginal Stability



when poles +jk -jk (on the imaginary axis)



when pole at the origin



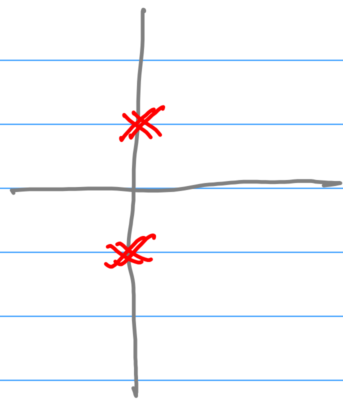
depending on inputs
sometimes bounded outputs
sometimes unbounded outputs

BIBO
~~BIBO~~

marginally stable

definitely unstable

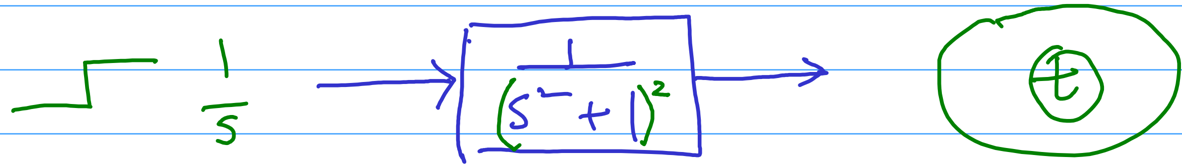
when $\boxed{\text{repeated}}$ pole on the imaginary axis



$(s^2+1)^2$ $\quad +i, +i$

$-i, -i$

Unbound



~~BIBO~~

BIBO Unstable

1) a pole with positive real part

2) a repeated pole on the imaginary axis

impulse response는 ZSR

zero state response

all initial condition = 0

$y'' + 5y' + 6y = x$

$y \longleftrightarrow Y(s)$

$y' \longleftrightarrow sY(s) - y(0)$

$y'' \longleftrightarrow s(Y(s) - y(0)) - y'(0)$

$\left(s^2 Y(s) - s\,y(0) - y'(0)\right) + 5\left(sY(s) - y(0)\right) + 6Y(s) = X(s)$

$(s^2 + 5s + 6)\,Y(s) = \underbrace{s\,y(0) + y'(0) + 5y(0)}_{y(0),\ y'(0)\ \text{초기치}} + \underbrace{X(s)}_{\text{input } x(t)}$

$$Y(s) = \frac{s\,y(0) + y'(0) + 5y(0)}{s^2 + 5s + 6} + \frac{X(s)}{s^2 + 5s + 6}$$

$(s+2)(s+3)$

$s = -2,\ -3$

$$\frac{N_0(s)}{D(s)} + \frac{N(s)}{D(s)}$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$y_{zi}(t) \qquad\qquad y_{zs}(t)$$

e^{-2t} $\quad$ e^{-3t}

Routh-Hurwitz Criterion

$$G(s) = \frac{N(s)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

denominator

$$F(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

RHP에 있는 근의 개수 알려주는 technique

≥ 1 ---> ~~B1130~~ ~~stable~~

$\boxed{s^4}$	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	b_1	b_2	b_3
s^1	c_1	c_2	c_3
$\boxed{s^0}$	d_1	d_2	d_3

given (rows s^4, s^3)

to be computed (rows s^2, s^1, s^0)

check the sign changes in the first column

# of sign changes = # of poles in RHS
(unstable poles)

this does not include poles on the imaginary axis
(marginal poles)

$$F(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

$$\begin{array}{c|ccc}
s^4 & a_4 & a_2 & a_0 \\
s^3 & a_3 & a_1 & 0 \\
\hline
s^2 & b_1 & b_2 & b_3 \\
s^1 & c_1 & c_2 & c_3 \\
s^0 & d_1 & d_2 & d_3
\end{array}
\qquad \begin{array}{c} 0 \\ 0 \end{array}$$

$$b_1 = \frac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} \qquad b_2 = \frac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} \qquad b_3 = \frac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$$

$$\begin{array}{cccc}
s^4 & a_4 & a_2 & a_0 \\
s^3 & a_3 & a_1 & 0 \\
\hline
s^2 & b_1 & b_2 & b_3 \\
s^1 & c_1 & c_2 & c_3 \\
s^0 & d_1 & d_2 & d_3
\end{array}$$

$$c_1 = \frac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} \qquad c_2 \; \frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & b_3 \end{vmatrix}}{b_1} = 0 \qquad c_3 = \frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$$

$$\begin{array}{cccc}
s^4 & a_4 & a_2 & a_0 \\
s^3 & a_3 & a_1 & 0 \\
\hline
s^2 & b_1 & b_2 & b_3 \\
s^1 & c_1 & c_2 & c_3 \\
s^0 & d_1 & d_2 & d_3
\end{array}$$

$$d_1 = \frac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}}{c_1} \qquad d_2 = \frac{-\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix}}{c_1} = 0 \qquad d_3 = \frac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$$

$$s^4 + 10s^3 + 35s^2 + 50s + 24 = 0$$

s^4	1	35	24
s^3	10	50	0
s^2	30	24	0
s^1	42	0	0
s^0	24		

$$b_1 = \frac{-\begin{vmatrix} 1 & 35 \\ 10 & 50 \end{vmatrix}}{10} \quad b_2 = \frac{-\begin{vmatrix} 1 & 24 \\ 10 & 0 \end{vmatrix}}{10} \quad b_3 = \frac{-\begin{vmatrix} 1 & 0 \\ 10 & 0 \end{vmatrix}}{10} = 0$$

$$\frac{-(50-350)}{10}$$

$+30$ $\quad$ 24

$$c_1 = \frac{-\begin{vmatrix} 10 & 50 \\ 30 & 24 \end{vmatrix}}{30} \quad c_2 = \frac{-\begin{vmatrix} 10 & 0 \\ 30 & 0 \end{vmatrix}}{30} = 0 \quad c_3 = \frac{-\begin{vmatrix} 10 & 0 \\ 30 & 0 \end{vmatrix}}{30} = 0$$

$-(240-1500)$

$$\frac{1260}{30}$$

42 $\quad$ 0 $\quad$ 0

$$c_1 = \frac{-\begin{vmatrix} 30 & 24 \\ 42 & 0 \end{vmatrix}}{42} \quad c_2 = \frac{-\begin{vmatrix} 30 & 0 \\ 42 & 0 \end{vmatrix}}{42} = 0 \quad c_3 = \frac{-\begin{vmatrix} 30 & 0 \\ 42 & 0 \end{vmatrix}}{42} = 0$$

24

$$s^5 + 6s^4 + 5s^3 + 30s^2 + 4s + 24 = 0$$

s^5	1	5	4	
s^4	6	30	24	$\Rightarrow (6s^4 + 30s^2 + 24)'$
s^3	24	60	0	$\Leftarrow 24s^3 + 60s + 0$
s^2	15.	24	0	
s^1				
s^0				

$$b_1 = \frac{-\begin{vmatrix} 1 & 5 \\ 6 & 30 \end{vmatrix}}{6}, \quad b_2 = \frac{-\begin{vmatrix} 1 & 4 \\ 6 & 24 \end{vmatrix}}{6}, \quad b_3 = \frac{-\begin{vmatrix} 1 & 0 \\ 6 & 0 \end{vmatrix}}{6}$$

$30 - 30 = 0$ $\qquad$ $24 - 24 = 0$ $\qquad$ $0 = 0$

$$c_1 = \frac{-\begin{vmatrix} 6 & 30 \\ 24 & 60 \end{vmatrix}}{24}, \quad c_2 = \frac{-\begin{vmatrix} 6 & 24 \\ 24 & 0 \end{vmatrix}}{24}, \quad c_3 = \frac{-\begin{vmatrix} 6 & 0 \\ 24 & 0 \end{vmatrix}}{24} = 0$$

$360 - 720$ $\qquad$ $\frac{24 \cdot \cancel{24}}{\cancel{24}}$

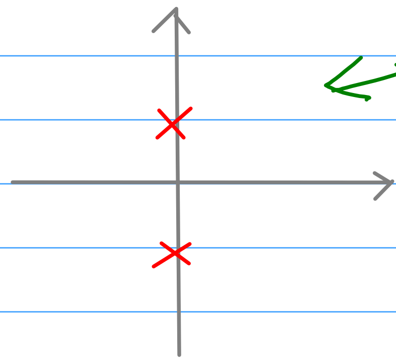
$\frac{360}{24} = 15$ $\qquad$ 24

Check the 2 special cases

$\boxed{\text{(a) when all elements in a row are zero (zero row)}}$

(b) when the first element is zero (non-zero row)

1

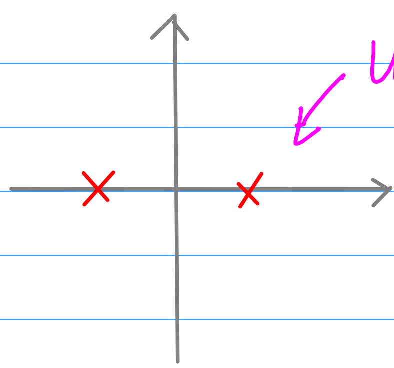


marginally stable

(s^2+1)

1, 2, 3 radially symmetric

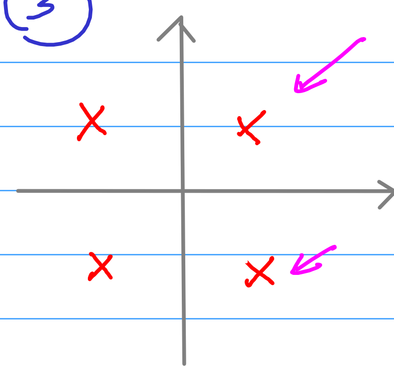
2



unstable

(s^2-1)

3

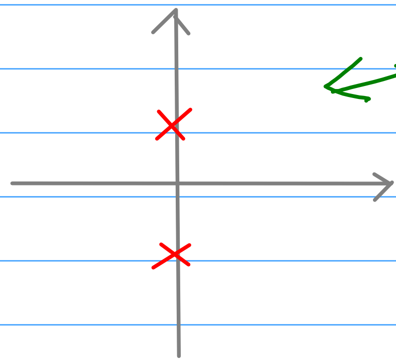


unstable

$$((s-1)^2+1)((s+1)^2+1)$$

$$= (s^2-2s+2)(s^2+2s+2)$$

$$= (s^4+4)$$

① 

$\leftarrow$ marginally stable

$$\begin{array}{c|cc} s^2 & 1 & 1 \\ s^1 & 0 & 0 \end{array}$$

$\leftarrow$ aux eq

$\leftarrow$ zero row

(s^2+1)

$(s^2+1)' = 2s$

$$\begin{array}{c|cc} s^2 & 1 & 1 \\ s^1 & \boxed{0 \quad 0} \end{array} \Rightarrow \begin{array}{c|cc} s^2 & 1 & 1 \\ s^1 & \boxed{2 \quad 0} \\ s^0 & +1 & \end{array}$$

$$\frac{-\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix}}{2} = +1$$

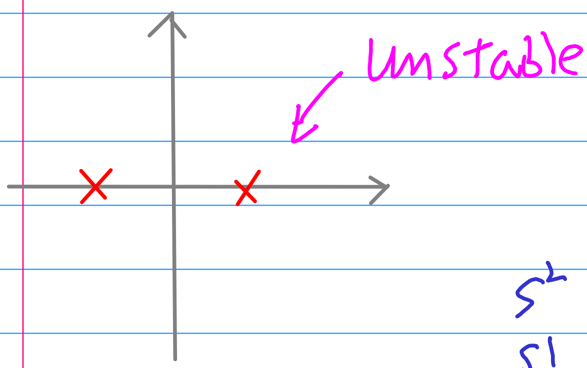
no sign change

\# poles in LHP = 0 $\Leftarrow$ \# poles in RHP = 0

Symmetric

$\therefore$ 2 poles on the imaginary axis

(2)



$(s^2 - 1)$

$$\begin{array}{c|cc} s^2 & 1 & -1 \\ s^1 & 0 & 0 \end{array}$$

← aux eq

← zero row

$$\begin{array}{c|cc} s^2 & 1 & -1 \\ s^1 & \boxed{0 \quad 0} & \end{array} \Rightarrow \begin{array}{c|cc} s^2 & 1 & -1 \\ s^1 & 2 & 0 \\ s^0 & -1 & \end{array}$$

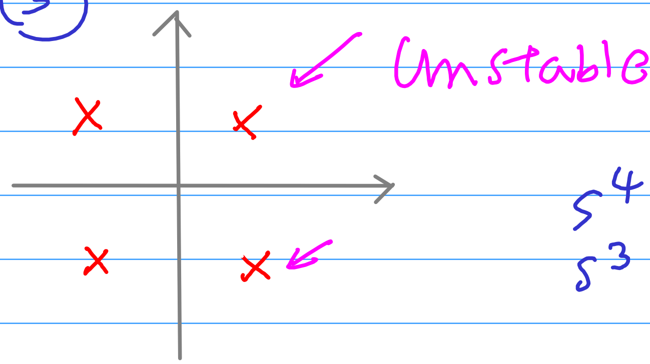
$$\frac{-\begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix}}{2} = -1$$

# of poles in LHP = 1 ⇐ # of poles in RHP = 1

symmetric

no poles on the imaginary axis

③



$$\begin{array}{c|ccc} s^4 & 1 & 0 & 4 \\ s^3 & 0 & 0 & 0 \end{array}$$

$\leftarrow$ aux eq

$\leftarrow$ zero row

$$((s-1)^2+1)((s+1)^2+1)$$
$$= (s^2-2s+2)(s^2+2s+2)$$
$$= (s^4+4)$$

$$\begin{array}{c|ccc} s^4 & 1 & 0 & 4 \\ s^3 & 0 & 0 & 0 \end{array} \Rightarrow \begin{array}{c|ccc} s^4 & 1 & 0 & 4 \\ s^3 & 4 & 0 & 0 \\ s^2 & 0 & 4 & \end{array}$$

$(s^4+4)' = 4s^3$

non-zero row
zero first element

$\rightarrow$ use ε

$$\frac{-\begin{vmatrix} 1 & 0 \\ 4 & 0 \end{vmatrix}}{4} = 0 \qquad \frac{-\begin{vmatrix} 1 & 4 \\ 4 & 0 \end{vmatrix}}{4} = 4$$

Check the 2 special cases

(a) when all elements in a row are zero (zero row)

(b) when the first element is zero (non-zero row)

Use ε small number but non-zero

$$-\frac{\begin{vmatrix}4 & 0\\ \varepsilon & 4\end{vmatrix}}{\varepsilon}$$

$$-\frac{\begin{vmatrix}\varepsilon & 4\\ \frac{-16}{\varepsilon} & 0\end{vmatrix}}{\frac{-16}{\varepsilon}} \qquad \frac{\frac{-16}{\varepsilon}}{\frac{-16}{\varepsilon}}$$

s^4	1	0	4	+	+
s^3	4	0	0	+	+
s^2	ε	4		+	−
s^1	$\frac{-16}{\varepsilon}$	0		−	+
s^0	1			+	+

regardless of the sign of epsilon
there are two sign changes

Symmetric

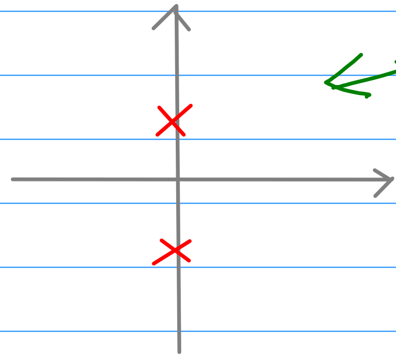
=> 2 poles in RHP

=> 2 poles in LHP

=> 0 pole on the imaginary axis

Epsilon in the marginally stable case

①



← marginally stable

$$\begin{array}{c|cc} s^2 & 1 & 1 \\ s^1 & 0 & 0 \end{array}$$

s^2 row ← aux eq

s^1 row ← zero row

(s^2+1)

$$\begin{array}{c|cc} s^2 & 1 & 1 \\ s^1 & \boxed{0 \quad 0} & \end{array} \Rightarrow \begin{array}{c|cc} s^2 & 1 & 1 \\ s^1 & \varepsilon & 0 \\ s^0 & 1 & \end{array}$$

$\varepsilon > 0$	$\varepsilon < 0$
+	+
+	−
+	+

0 ↓ 2 sign changes

sometimes stable,

sometimes <u>not</u>

$\Rightarrow$ <u>marginal</u>

$$-\frac{\begin{vmatrix} 1 & 1 \\ \varepsilon & 0 \end{vmatrix}}{\varepsilon} = +\frac{\varepsilon}{\varepsilon}$$

can identify # of polse on the imaginary axis

# of polse in LHP

# of polse in RHP

Zero Row Case

같은 내용

polse on the imaginary axis

should check $\varepsilon > 0$, $\varepsilon < 0$

Zero First Element Case

all real poles case

$$F(s) = \boxed{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} = 0$$

$$\Downarrow$$

$$s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

if all poles are real $\Rightarrow$

$$= (s - p_1)(s - p_2)(s - p_3)(s - p_4)$$

$$a_3 = -(p_1 + p_2 + p_3)$$
$$a_2 = +(p_1 p_2 + p_1 p_3 + p_1 p_4 + p_2 p_3 + p_2 p_4 + p_3 p_4)$$
$$a_1 = -(p_1 p_2 p_3 + p_1 p_2 p_4 + p_1 p_3 p_4 + p_1 p_3 p_4)$$
$$a_0 = +(p_1 p_2 p_3 p_4)$$

$p_1, p_2, p_3, p_4 < 0$

$p_i < 0 \longrightarrow a_3 > 0$

$p_i p_j > 0 \longrightarrow a_2 > 0$

$p_i p_j p_k < 0 \longrightarrow a_1 > 0$

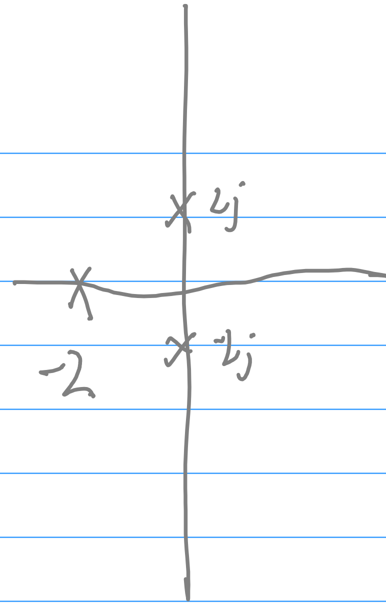
$p_i p_j p_k p_l > 0 \longrightarrow a_0 > 0$

marginal stability

$(s+2)(s^2+4)=0$

$s^3+4s+2s^2+8=0$

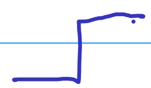
$s^3+2s^2+4s+8=0$



s^3	$1+$ 4
s^2	2^+ 8
s^1	ε 0
s^0	8_+

$$G(s)=\frac{1}{s^2+4}=\frac{1}{2}\,\frac{2}{s^2+2^2}$$

Step response

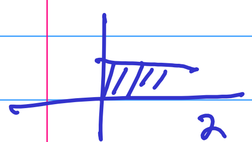


R(1)

$\frac{1}{s}$ $\longrightarrow$ $\boxed{\frac{1}{s^2+4}}$ $\longrightarrow$ $\frac{1}{s(s^2+4)}$

$= \frac{1}{4}\left(\frac{1}{s} - \frac{s}{s^2+4}\right)$

$u(t) \longrightarrow \boxed{} \longrightarrow \frac{1}{4}\left(u(t) - \cos 2t\right) < 2$



$\frac{2}{s^2+4}$

$\sin(2t) \longrightarrow \boxed{\frac{1}{s^2+4}} \longrightarrow \frac{2}{(s^2+4)^2} \sim (\; t \;)$

$\frac{1}{2}\left(\frac{2}{s^2+4}\right)\left(\frac{2}{s^2+4}\right)$

$\frac{1}{2}\left(\sin 2t\right) * \left(\sin 2t\right)$

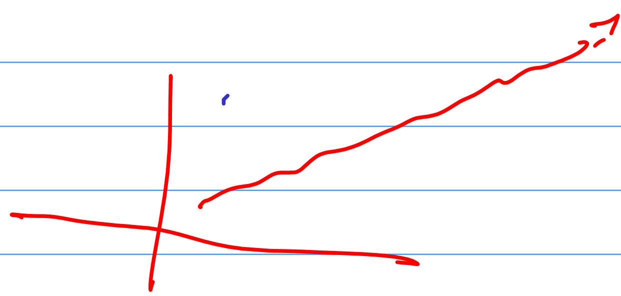
$=$

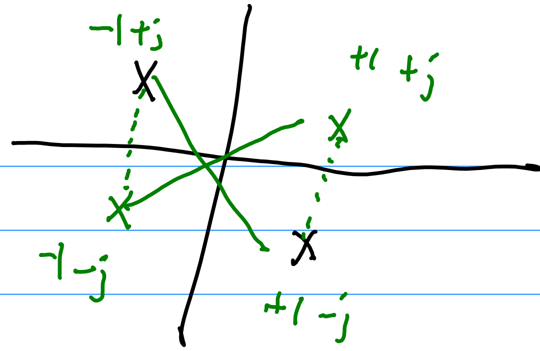
paul's online math note

Laplace + convolution

$$h(t) = (f*g)(t)$$
$$= \frac{1}{a^2}\int_0^t \sin(at - a\tau)\sin(a\tau)\,d\tau$$
$$= \frac{1}{2a^3}\left(\sin(at) - at\cos(at)\right)$$

ould have gotten by using #11 from the table.





$(s-(-1+i))(s-(-1-i))\,(s-(1+j))(s-(1-j))$

$((s+1)+i)((s+1)-i)((s-1)-i)((s-1)+i)$

$((s+1)^2+1)((s-1)^2+1)$

$(s^2+2s+2)(s^2-2s+2)$

(s^2+2s+2)
(s^2-2s+2)

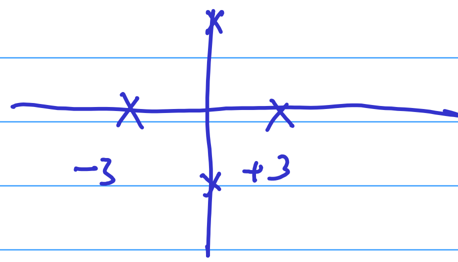
$2s^2+\cancel{4s}+4$
$-2s^3-4s^2-\cancel{4s}$
$s^4+\cancel{2s^3}+2s^2$

$s^4+0+0+0\quad 4$

s^4	1	0	4
s^3	0	0	0

$(s^4+4)(s-1)$ $\qquad$ s^5+4s-s^4-4

$$\begin{array}{c|ccc} s^5 & 1 & 0 & 4 \\ s^4 & -1 & 0 & -4 \\ \hline s^3 & 0 & 0 & 0 \end{array} \quad (s-1)$$



$s^2-9=0.$

$$\begin{array}{c|cc} s^2 & 1 & -9 \\ s^1 & 0 & 0 \end{array}$$

$$\begin{array}{cc c|cc} + & + & s^2 & 1 & -9 \\ + & - & s^1 & \varepsilon & 0 \\ - & - & s^0 & -9 & \dfrac{\begin{vmatrix} 0 & 0 \\ +9\varepsilon \end{vmatrix}}{-\varepsilon} = -9 \end{array}$$

$(s^2-9)(s^2+1)$

$s^4-9s^2+s^2-9$

$$\begin{array}{c|ccc} s^4 & 1 & -8 & -9 \\ s^3 & \varepsilon & 0 & 0 \\ s^2 & -8 & -9 & 0 \\ - & \frac{n2}{\varepsilon} & & \end{array}$$

$$s^4 + 2s^3 + s^2 + 2s + 3 = 0$$

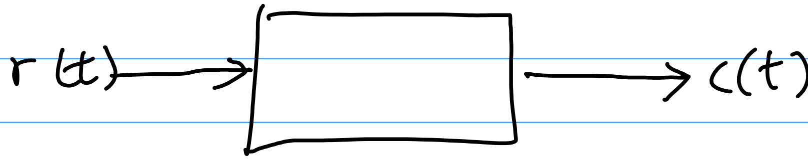
s^4	1	1	3
s^3	2	2	0
s^2	ε	3	0
s^1	$2-\frac{6}{\varepsilon}$	0	0
s^0	3	0	0

$$\frac{-\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix}}{2} \qquad \frac{-\begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix}}{2} \qquad \frac{6}{2} = 3$$

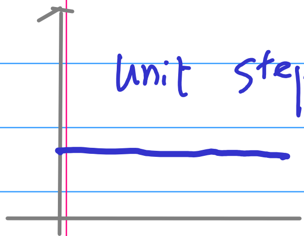
$$\frac{-\begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix}}{2}$$

+	1	+
+	2	+
+	ε	+
+	$2-\frac{6}{\varepsilon}$	−
+	3	+

RHP ⇒ 2 ✗



unit step function

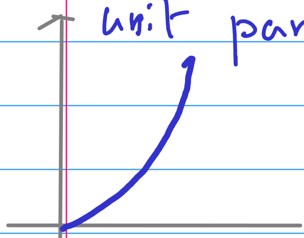


$u(t) \Longleftrightarrow \frac{1}{s}$

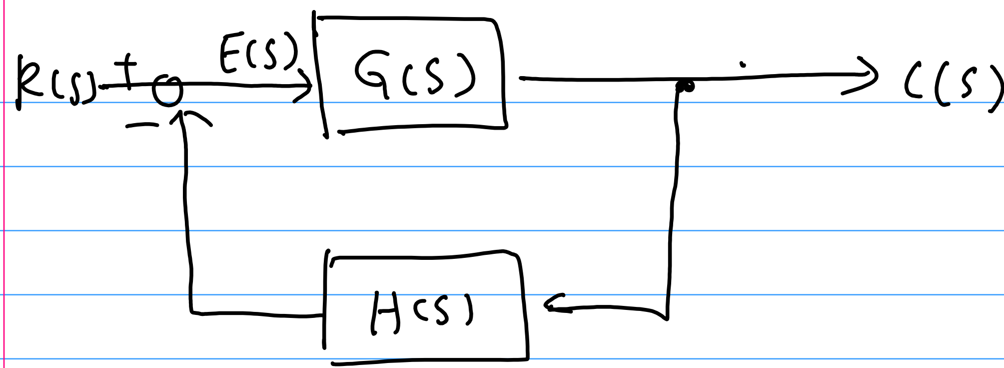
unit ramp function

$t\,u(t) \Longleftrightarrow \frac{1}{s^2}$

unit parabola function



$t^2 u(t) \Longleftrightarrow \frac{2}{s^3}$



$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

$E(s)$

$$C(s) = \frac{G(s)\,R(s)}{1+G(s)H(s)}$$

Steady State Response

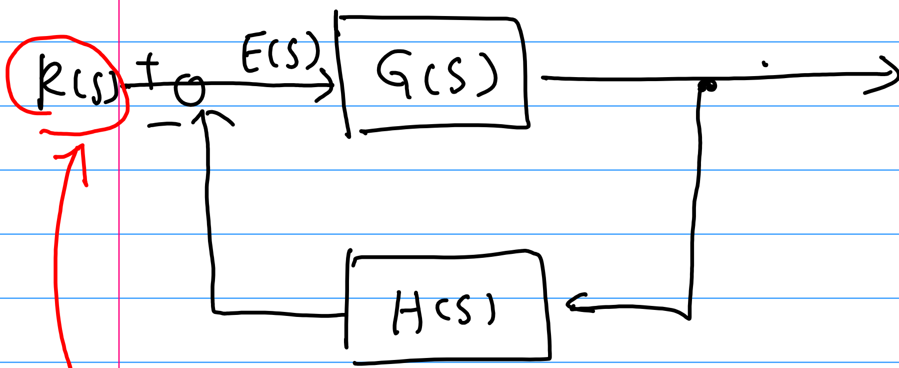
$\lim_{t\to\infty} y(t) \Rightarrow$ Steady state response

Stable $\quad e^{-m.t} \quad e^{-m.t} \longrightarrow 0$

$$C(s) = \frac{G(s)\,R(s)}{1+G(s)H(s)}$$

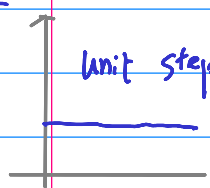
$$= \frac{G(s)\,R(s)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$e^{-m.t} \qquad e^{-m.t}\sin$



$$C(s) = \frac{G(s)\,R(s)}{1 + G(s)H(s)}$$

unit step function

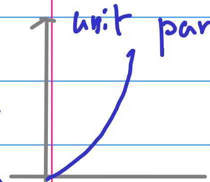


$u(t) \Longleftrightarrow \frac{1}{s}$

unit ramp function

$t\,u(t) \Longleftrightarrow \frac{1}{s^2}$

unit parabola function



$t^2\,u(t) \Longleftrightarrow \frac{2}{s^3}$

Step Rsp LT

$$\frac{G(s)}{s\left(1 + G(s)H(s)\right)}$$

Ramp Rsp LT

$$\frac{G(s)}{s^2\left(1 + G(s)H(s)\right)}$$

Parabola Rsp LT

$$\frac{G(s)}{s^3\left(1 + G(s)H(s)\right)}$$

$$G(s) = \frac{b}{s+a} = \frac{Y(s)}{X(s)}$$

$$(s+a)Y(s) = bX(s)$$

$$sY(s) + aY(s) = bX(s)$$

$$y'(t) + a\,y(t) = b\,x(t)$$

$$G(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0}$$

$$y'' + a_1 y' + a_0 y = b_1 x + b_0$$

impulse rsp $\leftarrow y(t)$ $\quad x = \delta(t)$

step rsp $\leftarrow y(t)$ $\quad x = u(t)$

ramp rsp $\leftarrow y(t)$ $\quad x = tu(t)$

impulse response $= \mathcal{L}^{-1}\{G(s)\}$

step response $= \mathcal{L}^{-1}\left\{\frac{G(s)}{s}\right\}$

ramp response $= \mathcal{L}^{-1}\left\{\frac{G(s)}{s^2}\right\}$