Quantization (10B)

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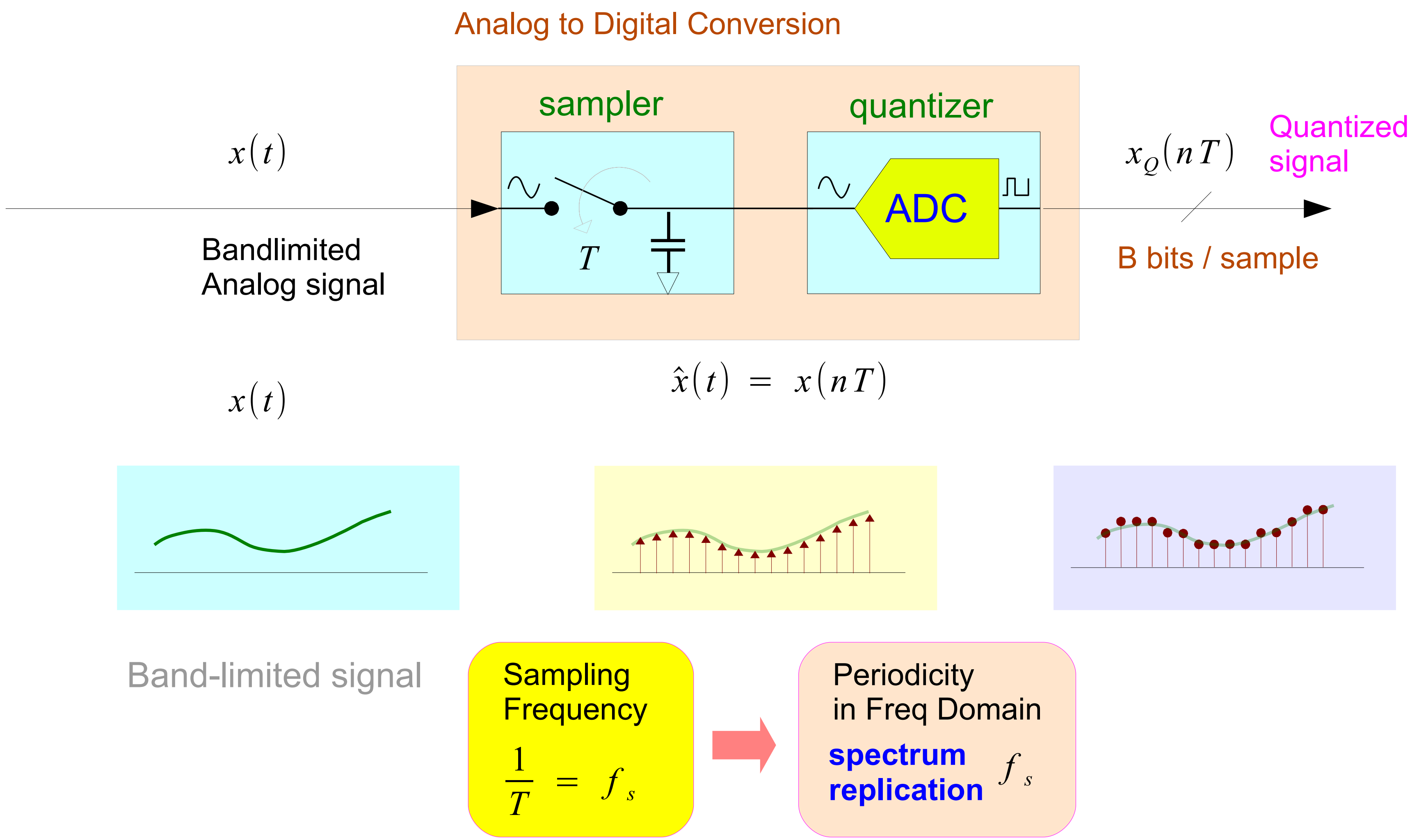
Analog to Digital Conversion



Quantization Resolution

Quantization Resolution (Width)

$$Q = \frac{R}{2^B}$$

(range / level)

R **full-scale Range**

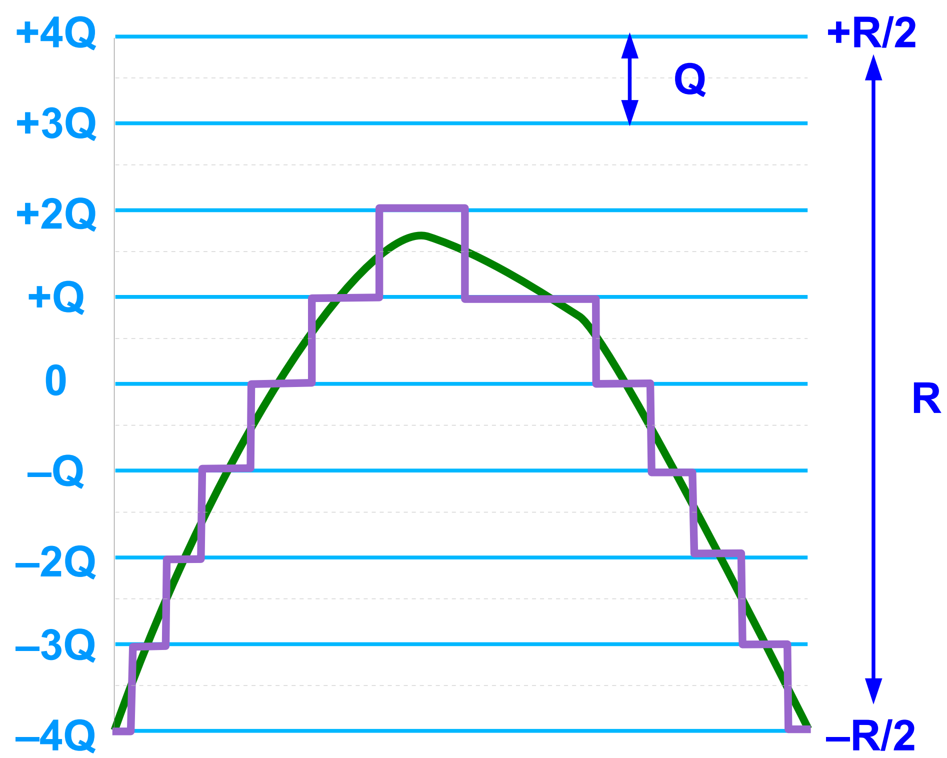
$$-\frac{R}{2} \le X_Q(nT) < \frac{R}{2}$$ **Bipolar ADC**

$$0 \le X_Q(nT) < R$$ **Unipolar ADC**

B bits → 2^B **quantization levels**

$$2^B = \frac{R}{Q}$$

$$\frac{\text{range}}{\text{(range / level)}}$$



Quantization Levels: SNR

B bits $\quad 2^B$ **quantization levels**

$$2^B = \frac{R}{Q}$$

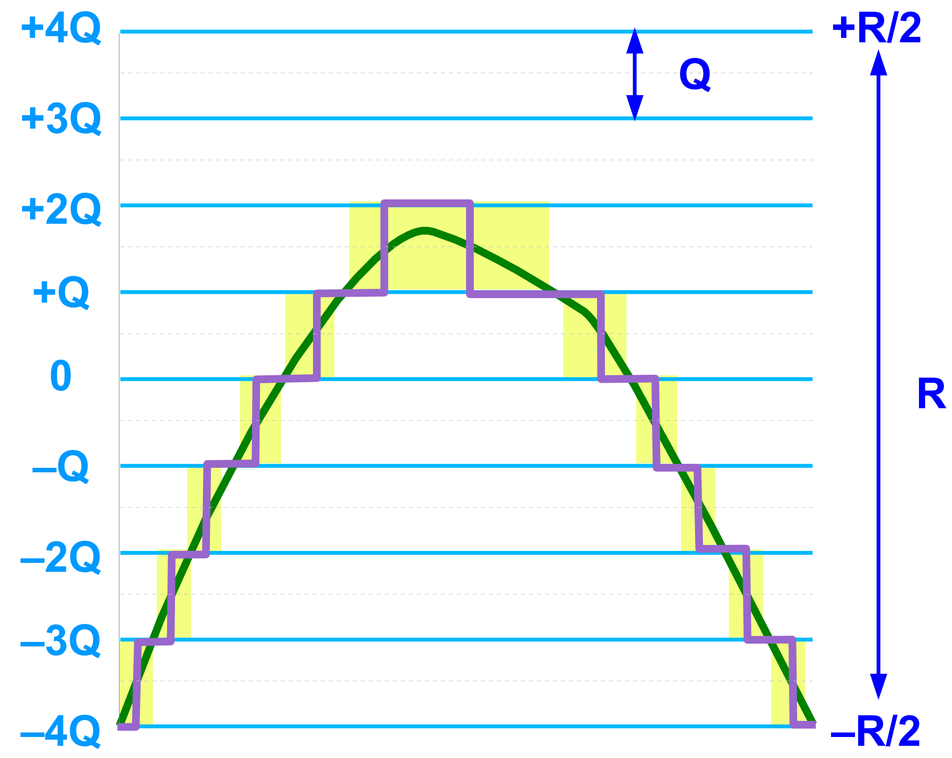
$\frac{\text{range}}{\text{(range / level)}}$

R ➡ **Signal Range**

Q ➡ **Noise Range**

$$SNR = 20\log_{10}\left(\frac{R}{Q}\right) = 6B \ \ dB$$

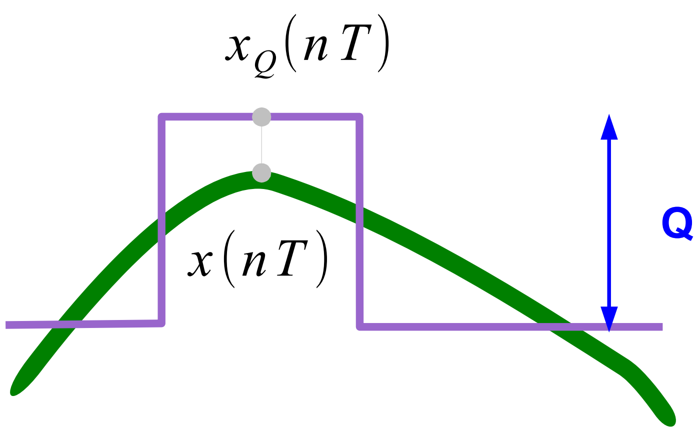
$$20\log_{10}2^B = B \cdot 20\log_{10}2 = 6B \ \ dB$$



Quantization Error

$$e(nT) = x_Q(nT) - x(nT)$$

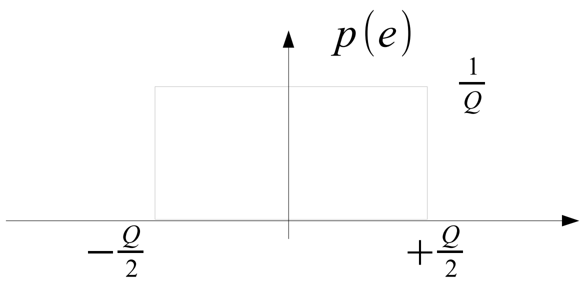
$$e_{rms} = \sqrt{\overline{e^2}} = \frac{Q}{\sqrt{12}}$$



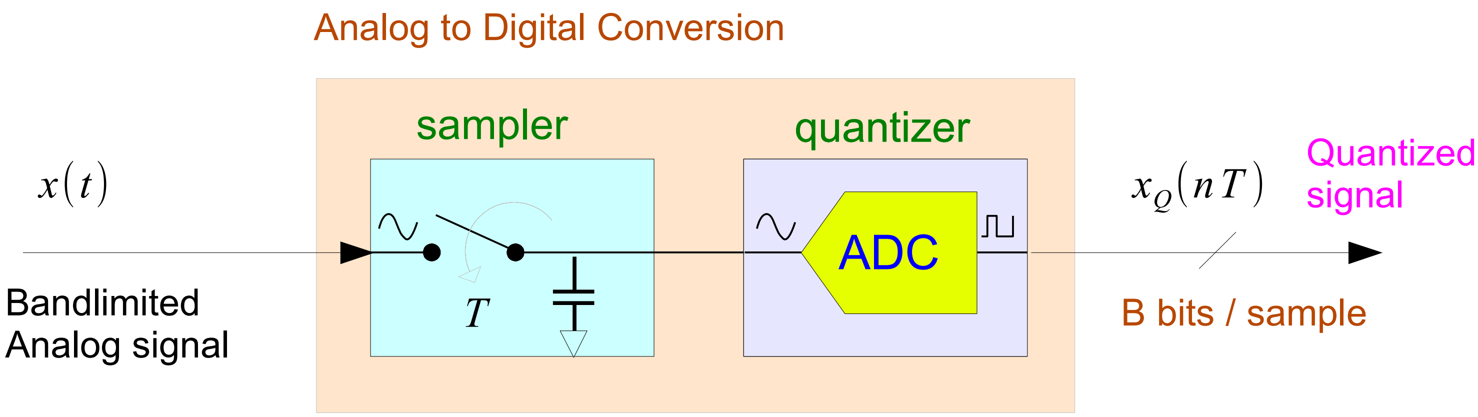
$$e = x_Q - x \qquad -\frac{Q}{2} \le e \le +\frac{Q}{2}$$

$$\bar{e} = \frac{1}{Q}\int_{-Q/2}^{+Q/2} e\, de = 0$$

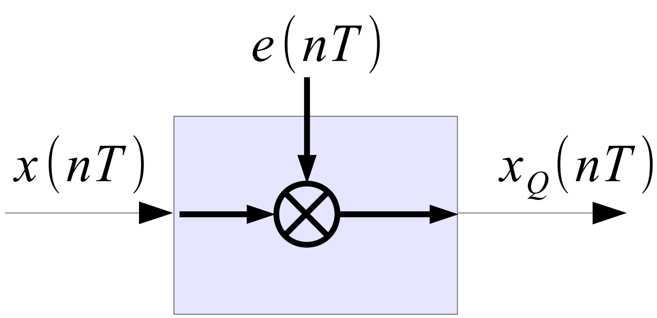
$$\overline{e^2} = \frac{1}{Q}\int_{-Q/2}^{+Q/2} e^2\, de = \left[\frac{e^3}{3}\right]_{-Q/2}^{+Q/2} = \frac{Q^2}{12}$$



A Quantizer Model



$$\hat{x}(t) = x(nT)$$



$$\sigma_e^2 = E[e^2(n)] = \frac{Q^2}{12}$$

$$R_{ee}(k) = E[e(n+k)e(n)] = \sigma_e^2\delta(k)$$

$$R_{ex}(k) = E[e(n+k)x(n)] = 0$$

References

[1] http://en.wikipedia.org/
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[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann
[4] R. G. Lyons, Understanding Digital Signal Processing, 1997
[5] AVR121: Enhancing ADC resolution by oversampling
[6] S.J. Orfanidis, Introduction to Signal Processing
www.ece.rutgers.edu/~orfanidi/intro2sp