

Quantization (10B)

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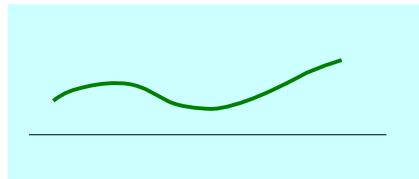
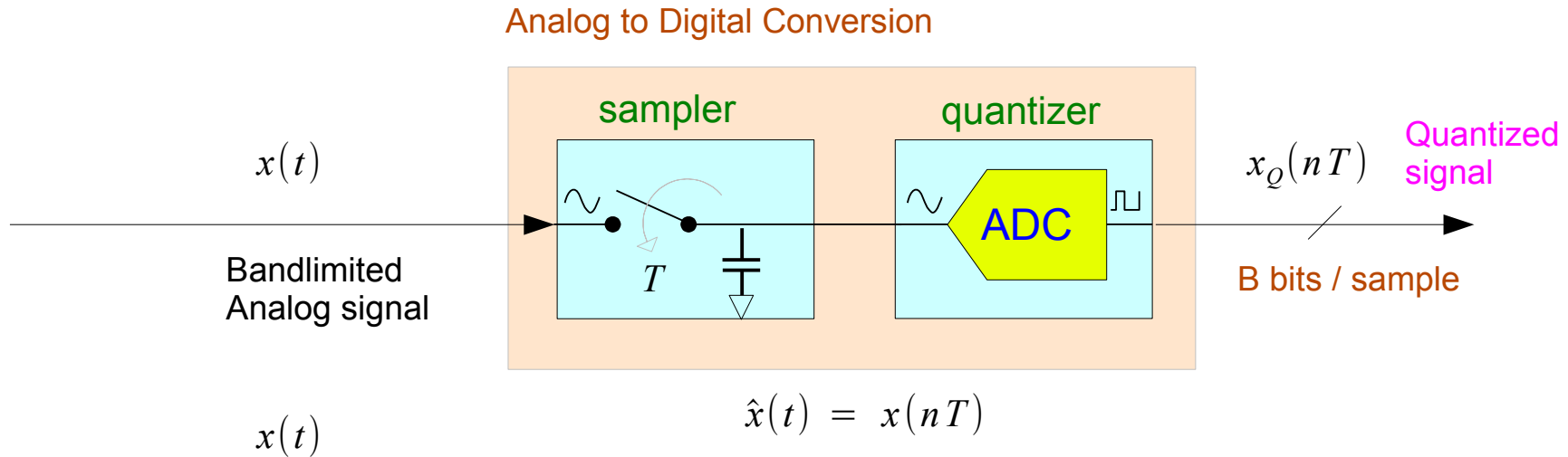
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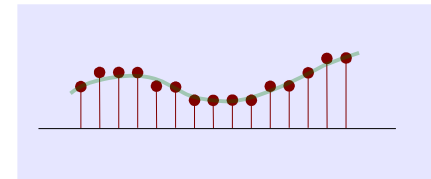
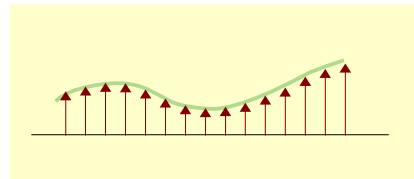
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Analog to Digital Conversion



Band-limited signal



Sampling Frequency

$$\frac{1}{T} = f_s$$



Periodicity in Freq Domain

spectrum replication f_s

Quantization Resolution

Quantization Resolution (Width)

$$Q = \frac{R}{2^B}$$

(range / level)

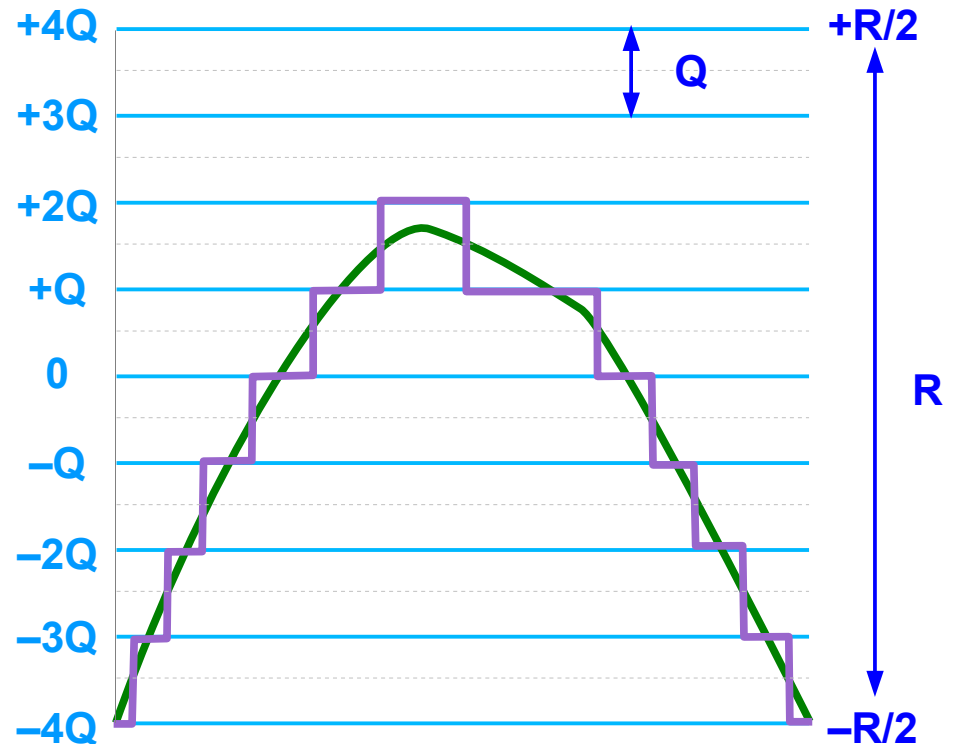
R full-scale Range

$$-\frac{R}{2} \leq X_Q(nT) < \frac{R}{2} \quad \text{Bipolar ADC}$$

$$0 \leq X_Q(nT) < R \quad \text{Unipolar ADC}$$

B bits $\rightarrow 2^B$ quantization levels

$$2^B = \frac{R}{Q} \quad \frac{\text{range}}{\text{(range / level)}}$$



Quantization Levels: SNR

B bits 2^B quantization levels

$$2^B = \frac{R}{Q}$$

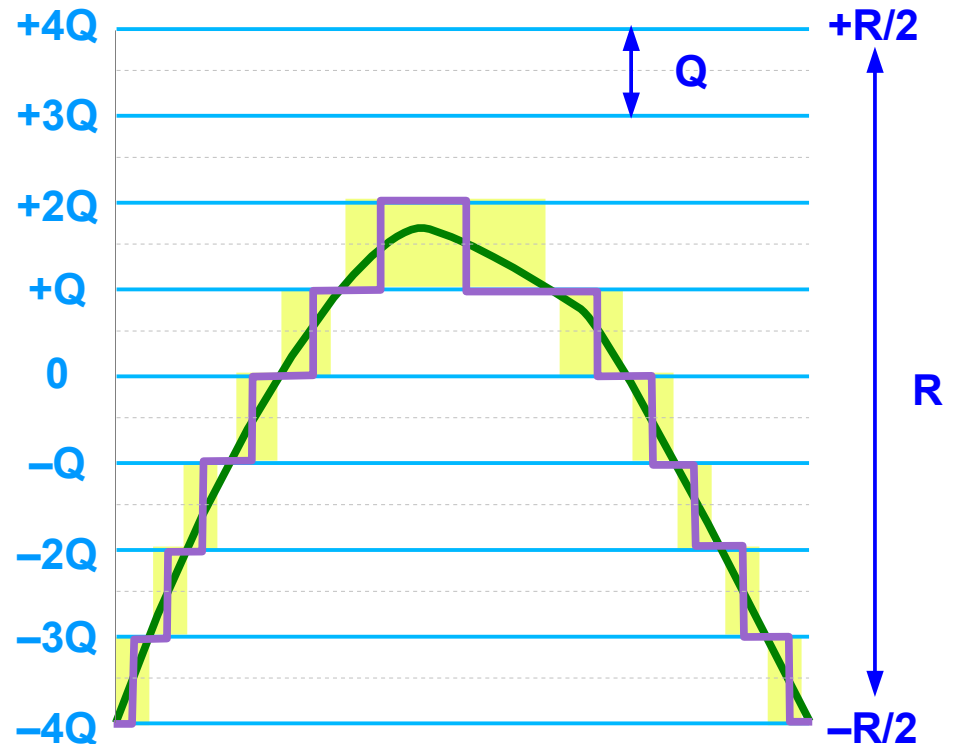
range
(range / level)

R **→ Signal Range**

Q **→ Noise Range**

$$SNR = 20 \log_{10} \left(\frac{R}{Q} \right) = 6B \text{ dB}$$

$$20 \log_{10} 2^B = B \cdot 20 \log_{10} 2 = 6B \text{ dB}$$

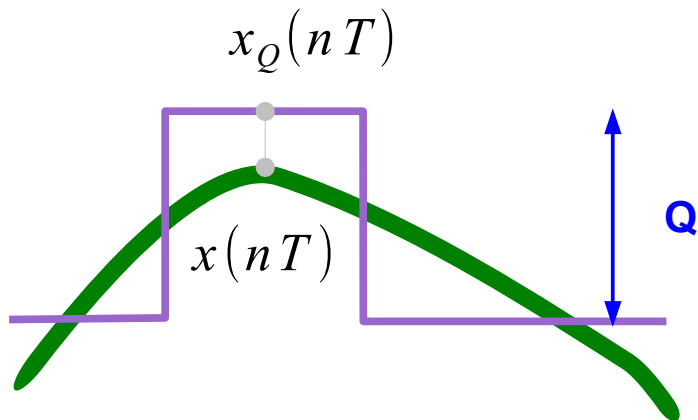


Quantization Error

Quantization Error

$$e(nT) = x_Q(nT) - x(nT)$$

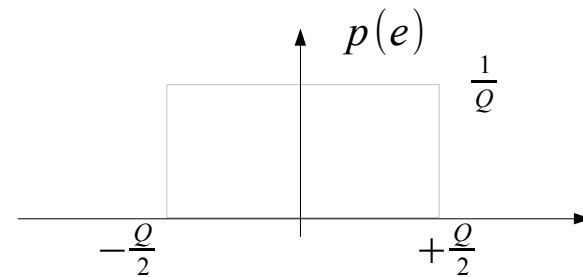
$$e_{rms} = \sqrt{\bar{e}^2} = \frac{Q}{\sqrt{12}}$$



$$e = x_Q - x \quad -\frac{Q}{2} \leq e \leq +\frac{Q}{2}$$

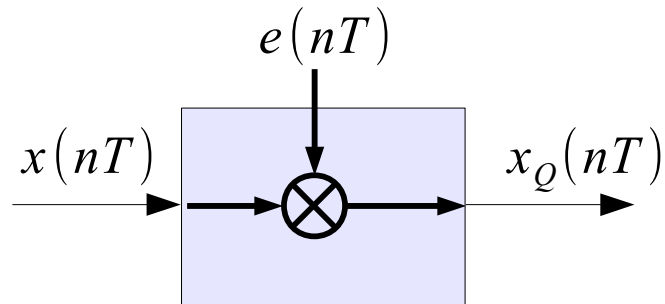
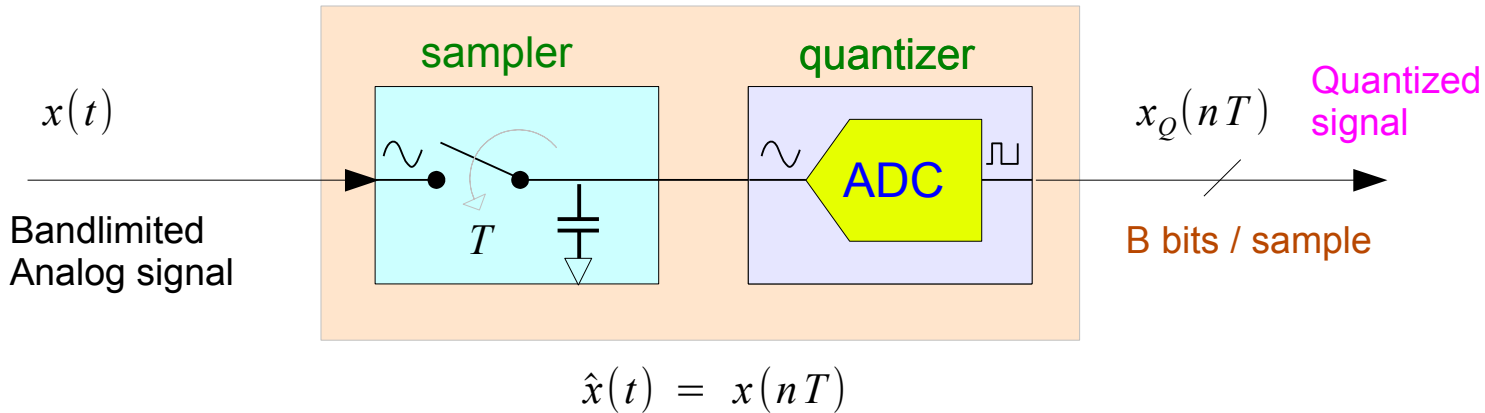
$$\bar{e} = \frac{1}{Q} \int_{-Q/2}^{+Q/2} e \, de = 0$$

$$\bar{e}^2 = \frac{1}{Q} \int_{-Q/2}^{+Q/2} e^2 \, de = \left[\frac{e^3}{3} \right]_{-Q/2}^{+Q/2} = \frac{Q^2}{12}$$



A Quantizer Model

Analog to Digital Conversion



$$\sigma_e^2 = E[e^2(n)] = \frac{Q^2}{12}$$

$$R_{ee}(k) = E[e(n+k)e(n)] = \sigma_e^2 \delta(k)$$

$$R_{ex}(k) = E[e(n+k)x(n)] = 0$$

References

- [1] <http://en.wikipedia.org/>
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- [4] R. G. Lyons, Understanding Digital Signal Processing, 1997
- [5] AVR121: Enhancing ADC resolution by oversampling
- [6] S.J. Orfanidis, Introduction to Signal Processing
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