State Space (H1) Canonical Forms

$$\boxed{AP = P\Lambda \qquad \begin{cases} A = P\Lambda P^{-1} \\ \Lambda = P^{-1}AP \end{cases}}$$

$$\underbrace{\begin{bmatrix} A \end{bmatrix}}_{n \times n}$$

$\lambda_1, \lambda_2, \cdots, \lambda_n$; n개의 distinct eigenvalue

$$\downarrow \qquad \downarrow \qquad\qquad \downarrow$$

$p_1, \quad p_2, \quad \cdots \quad p_n$ eigenvector.

$$\begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix} = \underbrace{\begin{bmatrix} P \end{bmatrix}}_{n \times n} \qquad \underbrace{\begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ & & \ddots \\ 0 & & & \lambda_n \end{bmatrix}}_{n \times n} \; \Lambda$$

$\Lambda \leftarrow A$	$A \leftarrow \Lambda$
$\Lambda = P^{-1}AP$	$A = P\Lambda P^{-1}$
$\Lambda^k = P^{-1}A^kP$	$A^k = P\Lambda^k P^{-1}$
$e^{\Lambda} = \sum_{k=0}^{\infty} \frac{\Lambda^k}{k!}$	$e^{A} = \sum_{k=0}^{\infty} \frac{A^k}{k!}$
$e^{\Lambda} \doteq P^{-1}e^{A}P$	$e^{A} \doteq Pe^{\Lambda}P^{-1}$

$\Lambda \leftarrow A$

$$(sI-\Lambda) = P^{-1}(sI-A)P$$

$$(sI-\Lambda)^{-1} = P^{-1}\underbrace{(sI-A)}^{-1}P$$

$$e^{\Lambda t} = P^{-1}e^{At}P$$

$$e^{\Lambda} = \sum_{k=0}^{\infty}\frac{\Lambda^k}{k!}$$

$$e^{\Lambda} \doteq P^{-1}e^{A}P$$

$A \leftarrow \Lambda$

$$(sI-A) = P(sI-\Lambda)P^{-1}$$

$$(sI-A)^{-1} = P\underbrace{(sI-\Lambda)}^{-1}P^{-1}$$

$$e^{At} = Pe^{\Lambda t}P^{-1}$$

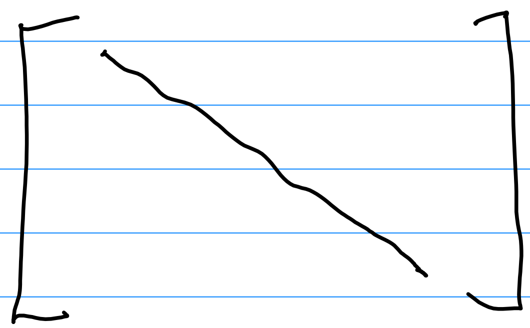
$$= \varphi(t)$$

$$e^{A} = \sum_{k=0}^{\infty}\frac{A^k}{k!}$$

$$e^{A} \doteq Pe^{\Lambda}P^{-1}$$

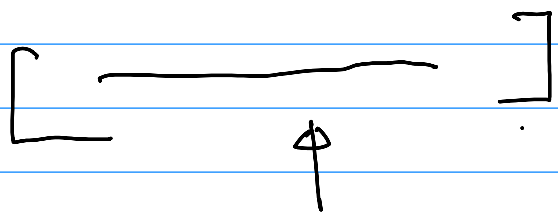
Diagonal Cannonical Form

st. eq



transfer functional
pole only

output eq



transfer functions
partial fraction
coefficients

transfer function

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1}s + b_n}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

$$= b_0 + \frac{c_1}{s+p_1} + \frac{c_2}{s+p_2} + \cdots + \frac{c_n}{s+p_n}.$$

T.F 의 pole [illegible]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -p_1 & & & & 0 \\ & -p_2 & & & \\ & & \ddots & & \\ & & & -p_{n-1} & \\ 0 & & & & -p_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} u$$

$$s \begin{bmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_n(s) \end{bmatrix} = \begin{bmatrix} -p_1 & & & \\ & -p_2 & & \\ & & \ddots & \\ & & & -p_n \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_n(s) \end{bmatrix} + \begin{bmatrix} U(s) \\ U(s) \\ \vdots \\ U(s) \end{bmatrix}$$

$$s X_1(s) = -p_1 X_1(s) + U(s)$$

$$(s+p_1) X_1(s) = U(s) \qquad \frac{X_1(s)}{U(s)} = \frac{1}{s+p_1}$$

$$X_1(s) = \frac{1}{(s+p_1)} U(s)$$

$$y = \begin{bmatrix} c_1 & c_2 & \dots & c_{n-1} & c_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + b_0 u$$

T.F partial frac ... 계수

$$y(t) = c_1 x_1(t) + c_2 x_2(t) + \cdots + c_n x_n(t) + b_0 u(t)$$

$$Y(s) = C_1 \underline{X_1(s)} + C_2 X_2(s) + \cdots + C_n X_n(s) + b_0 U(s)$$

$$= C_1 \frac{U(s)}{s+p_1} + C_2 \frac{U(s)}{s+p_2} + \cdots + C_n \frac{U(s)}{s+p_n} + b_0 U(s)$$

$$= \left(b_0 + \frac{C_1}{s+p_1} + \frac{C_2}{s+p_2} + \cdots + \frac{C_n}{s+p_n} \right) U(s)$$

Controllerable Cannonical Form

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}.$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} (b_n - a_n b_0) & (b_{n-1} - a_{n-1} b_0) & \cdots & (b_1 - a_1 b_0) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + b_0 u$$

$$y^{(n)} + a_1 \overset{x_n}{\overset{\parallel}{y^{(n-1)}}} + a_2 \overset{x_{n-1}}{\overset{\parallel}{y^{(n-2)}}} + \cdots + a_n \overset{x_1}{\overset{\parallel}{y}}$$

$$= b_0 u^{(n)} + b_1 u^{(n-1)} + \cdots + b_n u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} (b_n - a_n b_0) & (b_{n-1} - a_{n-1} b_0) & \dots & (b_1 - a_1 b_0) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + b_0 u$$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\vdots$

$\dot{x}_n = -a_n x_1 - a_{n-1} x_2 - \dots - a_1 x_n + u$

$x_1 = y$

$x_2 = y^{(1)}$

$x_3 = y^{(2)}$

$\vdots$

$x_n = y^{(n-1)}$

$\dot{x}_n = y^{(n)} =$

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix}
=
\begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
-a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u
$$

$$
y = \begin{bmatrix} (b_n - a_n b_0) & (b_{n-1} - a_{n-1} b_0) & \cdots & (b_1 - a_1 b_0) \end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + b_0 u
$$

$$
y^{(n)} + a_1 \overset{x_n \, = }{y^{(n-1)}} + a_2 \overset{x_{n-1} \, =}{y^{(n-2)}} + \cdots + a_n \overset{x_1 \, =}{y}
$$

$$
= b_0 u^{(n)} + b_1 u^{(n-1)} + \cdots + b_n u
$$

$$
\dot{x}_n = y^{(n)} = -a_1 y^{(n-1)} - a_2 y^{(n-2)} - \cdots - a_n y
$$

$$
b_0 u^{(n)} + b_1 u^{(n-1)} + \cdots + b_n u
$$

$$
= -a_1 x_n - a_2 x_{n-1} - \cdots - a_n x_1
$$

$$
b_0 u^{(n)} + b_1 u^{(n-1)} + \cdots + b_n u
$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} (b_n - a_n b_0) & (b_{n-1} - a_{n-1} b_0) & \dots & (b_1 - a_1 b_0) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + b_0 u$$

$$y = (b_n - a_n b_0) x_1 + (b_{n-1} - a_{n-1} b_0) x_2 +$$

$$+ (b_1 - a_1 b_0) x_n + b_0 u.$$

$$= b_n x_1 + b_{n-1} x_2 + \cdots + b_1 x_1 + b_0 u$$

$$- b_0 \left(\underline{a_n x_1 + a_{n-1} x_2 + \cdots + a_1 x_n} \right)$$

$$+ b_0 \dot{x}_n -$$

Observable Cannonical Form

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}.$$

$$y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \cdots + a_n y$$

$$= b_0 u^{(n)} + b_1 u^{(n-1)} + \cdots + b_n u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & \dots & 0 & -a_n \\ 1 & 0 & \dots & 0 & -a_{n-1} \\ 0 & 1 & \dots & 0 & -a_{n-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_n - a_n b_0 \\ b_{n-1} - a_{n-1} b_0 \\ b_{n-2} - a_{n-2} b_0 \\ \dots \\ b_1 - a_1 b_0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + b_0 u$$