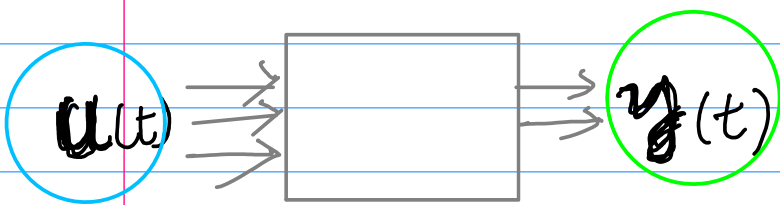
State Space (H.1)
Matrix Background

P 167. 5.5 State Space → Transfer fn



ZSR 의미

$\boxed{\text{초기 조건} = 0}$

inputs

state eq. $\dot{x}(t) = A\,x(t) + B\,u(t)$

inputs

output eq. $y(t) = C\,x(t) + D\,u(t)$

$x(0)$. initial conditions $\Rightarrow 0$

transfer fn 구할때는

state eq. $\dot{x}(t) = A\,x(t) + B\,u(t)$

$$sX(s) - x(0) = A\,X(s) + B\,U(s)$$

$$(sI - A)\,X(s) = B\,U(s) + x(0)$$

Output eq. $y(t) = C\,x(t) + D\,u(t)$

$$Y(s) = C\,X(s) + D\,U(s)$$

$$(sI - A)\,X(s) = \left(B\,U(s) + X(0)\right)$$

$$X(s) = (sI - A)^{-1}\left(B\,U(s) + X(0)\right)$$

$$X(s) = (sI - A)^{-1} B\,U(s) + (sI - A)^{-1} X(0)$$

input

초기치

Transfer fn ⇒ 0

$$Y(s) = C\,X(s) + D\,U(s)$$

$$Y(s) = C(sI - A)^{-1} B\,U(s) + C(sI - A)^{-1} X(0) + D\,U(s)$$

$$\phi(s)$$

$$Y(s) = \left[C(sI - A)^{-1} B + D\right] U(s) + C(sI - A)^{-1} X(0)$$

$$Y(s) = \left[C\,\phi(s)\,B + D\right] U(s) + C\,\phi(s)\,X(0)$$

$$(s\mathbf{I} - \mathbf{A})\,\mathbf{X}(s) = \mathbf{B}\,\mathbf{U}(s)$$

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B}\,\mathbf{U}(s)$$

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\,\underset{\uparrow \text{ input}}{\mathbf{U}(s)}$$

$$\mathbf{Y}(s) = \mathbf{C}\,\mathbf{X}(s) + \mathbf{D}\,\mathbf{U}(s)$$

$$\mathbf{Y}(s) = \mathbf{C}\,(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\,\mathbf{U}(s) + \mathbf{D}\,\mathbf{U}(s)$$

$$\phi(s)$$

$$\mathbf{Y}(s) = \left[\mathbf{C}\,(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}\right]\mathbf{U}(s)$$

$\Phi(s)$

전달함수 $$H(s) = C\,\underbrace{(sI - A)^{-1}}\,B \qquad \frac{1}{s+a}$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

임펄스응답 $$h(t) = C\,e^{At}\,B \qquad e^{-at}$$

$\Phi(t)$ ~~~ 상태천이행렬

State transition Matrix

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$At = t\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} a_1 t & a_2 t & a_3 t \\ b_1 t & b_2 t & b_3 t \\ c_1 t & c_2 t & c_3 t \end{bmatrix}$$

$$e^{At} = \xcancel{\begin{bmatrix} e^{a_1 t} & e^{a_2 t} & e^{a_3 t} \\ e^{b_1 t} & e^{b_2 t} & e^{b_3 t} \\ e^{c_1 t} & e^{c_2 t} & e^{c_3 t} \end{bmatrix}}$$

$$\Phi(s) = (s\mathbf{I} - \mathbf{A})^{-1} = \frac{\operatorname{adj}(s\mathbf{I} - \mathbf{A})}{|s\mathbf{I} - \mathbf{A}|}$$

$$\mathbf{Y}(s) = \left[\mathbf{C}\,\Phi(s)\,\mathbf{B} + \mathbf{D}\right]\mathbf{U}(s) + \mathbf{C}\,\Phi(s)\,\mathbf{X}(0)$$

전달함수 구하기 위해서 $\mathbf{X}(0) \Leftarrow 0$

$$\mathbf{G}(s) = \frac{\mathbf{Y}(s)}{\mathbf{U}(s)} = \left[\mathbf{C}\,\Phi(s)\,\mathbf{B} + \mathbf{D}\right]$$

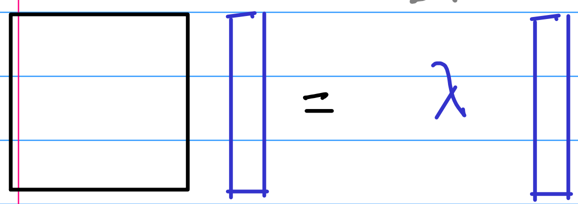
$$\Phi(s) = (s\mathbf{I} - \mathbf{A})^{-1} = \frac{\operatorname{adj}(s\mathbf{I} - \mathbf{A})}{|s\mathbf{I} - \mathbf{A}|}$$

pole

주어진 행렬 A

$$Ap = \lambda p$$

스라



어떤 λ ?

$$-Ap + \lambda p = 0$$

$$\lambda I p - Ap = 0$$

$$(\lambda I - A)p = 0$$

$$\phi(s) = \underbrace{(sI - A)}^{-1}$$

$$\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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$$(\lambda \mathbf{I} - \mathbf{A})\mathbf{p} = \mathbf{0}$$

① 역행열이 존재하면

$$\mathbf{p} = \underset{n \times n}{(\lambda \mathbf{I} - \mathbf{A})^{-1}} \ \underset{n \times 1}{\mathbf{0}} = \mathbf{0} \qquad \mathbf{p} = \mathbf{0}$$

유일한 한개의 해

② 역행열이 존재하지 않으면 $\therefore |\lambda \mathbf{I} - \mathbf{A}| = 0$

$\mathbf{p} \neq \mathbf{0}$ 해를 구할 수 있다

$\Downarrow$

non-zero $\mathbf{p}$를 구할 수 있다.

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

$$\lambda I - A = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda+2 & -1 \\ -1 & \lambda+2 \end{bmatrix}$$

$$\begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

$$\lambda = -1 \qquad \lambda = -3$$

$$|\lambda I - A| = 0 \Longrightarrow (\lambda+2)^2 - 1 = 0$$

$$\lambda^2 + 4\lambda + 3 = 0$$

$$(\lambda+3)(\lambda+1) = 0$$

$$\lambda = -1, \ -3$$

A의 eigenvalues

$$f(x) = x^2 + 4x + 3 = 0 \Rightarrow x = -1, \ x = -3$$

A의 eigen value를 구하기 위한 식.

$$f(A) = A^2 + 4A + 3I = 0$$

Caley-Hamilton Theorem

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \qquad (\lambda I - A) = \begin{bmatrix} \lambda+2 & -1 \\ -1 & \lambda+2 \end{bmatrix}$$

$$(\lambda I - A) = \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

$\lambda = -1$

$$(\lambda I - A) = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

$\lambda = -3$

$\boxed{\lambda = -1}$ 에 대한 eigenvector를 구하자

$$(\lambda I - A)\cdot p = 0$$

$$\begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\lambda = -1$

$a - b = 0$
$-a + b = 0.$

$a = b = \underline{1}$

$$\begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\lambda = -1$ 일 때 eigenvector

$\boxed{\lambda = -3}$ 에 대한 eigenvector를 구하자

$$(\lambda I - A)\cdot p = 0$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\lambda = -3$

$-a - b = 0$
$-a - b = 0.$

$a = -b = \underline{1}$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\lambda = -3$

$\lambda = -3$ 일 때 eigenvector

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

eigenvalue

$\lambda = -1$

eigenvector

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

eigenvalue

$\lambda = -3$

eigenvector

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

-1

$$\underset{A}{\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}} \underset{p}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

-3

$$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ +3 \end{bmatrix}$$

$$Ap = (-1)\,p$$

$$Ap = (3)\,p$$

p1n2

A ($n \times n$)

$\lambda_1, \lambda_2, \cdots, \lambda_n$: n개의 distinct eigenvalue

$$\lambda_1 \downarrow p_1, \quad \lambda_2 \downarrow p_2, \quad \cdots, \quad \lambda_n \downarrow p_n$$

eigenvector.

$$\begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix} = P \quad (n \times n)$$

$$A \begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix} = AP$$

$$Ap_1 = \lambda_1 p_1, \quad Ap_2 = \lambda_2 p_2, \quad \cdots, \quad Ap_n = \lambda_n p_n$$

$$P \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \; \lambda_n \end{bmatrix} \rightarrow P\Lambda$$

$$AP = P\Lambda$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1x_1+a_2x_2+a_3x_3 & a_1y_1+a_2y_2+a_3y_3 & a_1z_1+a_2z_2+a_3z_3 \\ b_1x_1+b_2x_2+b_3x_3 & b_1y_1+b_2y_2+b_3y_3 & b_1z_1+b_2z_2+b_3z_3 \\ c_1x_1+c_2x_2+c_3x_3 & c_1y_1+c_2y_2+c_3y_3 & c_1z_1+c_2z_2+c_3z_3 \end{bmatrix}$$

$$= \left[\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \right]$$

$$A\,P_1 \qquad A\,P_2 \qquad \cdots \qquad A\,P_n$$

$$\begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 x_1 & a_1 y_1 & a_1 z_1 \\ b_2 x_2 & b_2 y_2 & b_2 z_2 \\ c_3 x_3 & c_3 y_3 & c_3 z_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 x_1 & b_2 y_1 & c_3 z_1 \\ a_1 \cdot x_2 & b_2 y_2 & c_3 z_2 \\ a_1 x_3 & b_2 y_3 & c_3 z_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} & b_2 \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} & c_3 \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \end{bmatrix}$$

$$\lambda_1 \mathbf{p}_1 \qquad \lambda_2 \mathbf{p}_2 \qquad \lambda_n \mathbf{p}_n$$

$$AP = P\Lambda$$

$$P^{-1}AP = \Lambda$$

$$A \rightarrow P \cdots \begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix}$$

eigenvector

$$P^{-1}AP = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

$$AP = P\Lambda$$

$$P^{-1}AP = \Lambda$$

$$A = P\Lambda P^{-1}$$

$$A^2 = AA = (P\Lambda \cancel{P^{-1}})(\cancel{P}\Lambda P^{-1}) = P\Lambda^2 P^{-1}$$

$$A^k = P\Lambda^k P^{-1}$$

$$= P \boxed{\begin{matrix} \lambda_1^k & & \\ & \lambda_2^k & \\ & & \ddots \\ & & & \lambda_n^k \end{matrix}} P^{-1}$$

$$\begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}\begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1^2 & 0 & 0 \\ 0 & b_2^2 & 0 \\ 0 & 0 & c_3^2 \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}^2$$

$$\begin{bmatrix} a_1^k & 0 & 0 \\ 0 & b_2^k & 0 \\ 0 & 0 & c_3^k \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}^k$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_n \end{bmatrix} \qquad \Lambda t = \begin{bmatrix} \lambda_1 t & & 0 \\ & \lambda_2 t & \\ 0 & & \lambda_n t \end{bmatrix}$$

$$e^{\Lambda t} \equiv \begin{bmatrix} e^{\lambda_1 t} & & 0 \\ & e^{\lambda_2 t} & \\ 0 & & e^{\lambda_n t} \end{bmatrix}$$

Taylor series

$$f(x) = e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$e^{\lambda t} = 1 + \frac{\lambda t}{1!} + \frac{\lambda^2 t^2}{2!} + \frac{\lambda^3 t^3}{3!} + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda^k$$

$$e^A = ?$$

$$f(x) = e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$f(A) = e^A = 1 + \frac{A}{1!} + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

$$e^{\lambda t} = \boxed{\sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda^k}$$

$$e^{\Lambda t} = \begin{bmatrix} e^{\lambda_1 t} & & \\ & e^{\lambda_2 t} & \\ & & e^{\lambda_n t} \end{bmatrix}$$

$$e^{\Lambda t} = \begin{bmatrix} \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_1^k & & \\ & \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_2^k & \\ & & \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_n^k \end{bmatrix}$$

$k=0$

$$\begin{bmatrix} \frac{t^0}{0!}\lambda_1^0 & & \\ & \frac{t^0}{0!}\lambda_2^0 & \\ & & \frac{t^0}{0!}\lambda_n^0 \end{bmatrix} = \frac{t^0}{0!}\Lambda^0$$

$k=1$

$$\begin{bmatrix} \frac{t^1}{1!}\lambda_1^1 & & \\ & \frac{t^1}{1!}\lambda_2^1 & \\ & & \frac{t^1}{1!}\lambda_n^1 \end{bmatrix} = \frac{t^1}{1!}\Lambda^1$$

$k=2$

$$\begin{bmatrix} \frac{t^2}{2!}\lambda_1^2 & & \\ & \frac{t^2}{2!}\lambda_2^2 & \\ & & \frac{t^2}{2!}\lambda_n^2 \end{bmatrix} = \frac{t^2}{2!}\Lambda^2$$

$\vdots$

$k=n$

$$\begin{bmatrix} \frac{t^n}{n!}\lambda_1^n & & \\ & \frac{t^n}{n!}\lambda_2^n & \\ & & \frac{t^n}{n!}\lambda_n^n \end{bmatrix} = \frac{t^n}{n!}\Lambda^n$$

$\vdots$

$+)$

$$e^{\Lambda t} = \sum_{k=0}^{\infty} \frac{t^k}{k!}\Lambda^k$$

$$e^{\Lambda t} = \sum_{k=0}^{\infty} \frac{t^k}{k!} \Lambda^k$$

$$= \begin{bmatrix} \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_1^k & & 0 \\ & \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_2^k & \\ 0 & & \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_n^k \end{bmatrix}$$

$$P e^{\Lambda t} P^{-1} = P \left(\sum_{k=0}^{\infty} \frac{t^k}{k!} \Lambda^k \right) P^{-1}$$

$$= \sum_{k=0}^{\infty} \frac{t^k}{k!} \boxed{P \Lambda^k P^{-1}}$$

$$\overbrace{\underbrace{\boxed{P\Lambda P^{-1}}}_{A}\ \underbrace{\boxed{P\Lambda P^{-1}}}_{A}\ \cdots\ \underbrace{\boxed{P\Lambda P^{-1}}}_{A}}^{k} = A^k$$

$$P e^{\Lambda t} P^{-1} = P\left(\sum_{k=0}^{\infty} \frac{t^k}{k!} \Lambda^k\right) P^{-1}$$

$$= \sum_{k=0}^{\infty} \frac{t^k}{k!} \boxed{P \Lambda^k P^{-1}}$$

$$= \sum_{k=0}^{\infty} \frac{t^k}{k!} A^k$$

$$e^{at} = \sum_{k=0}^{\infty} \frac{t^k}{k!} a^k$$

$$e^{At} = \sum_{k=0}^{\infty} \frac{t^k}{k!} A^k$$

$$\boxed{\begin{aligned} e^{a(t_1+t_2)} &= e^{at_1} \cdot e^{at_2} \\ e^{at} \cdot e^{-at} &= 1 \\ \left(e^{at}\right)^{-1} &= e^{-at} \\ \frac{d}{dt}\left(e^{at}\right) &= a e^{at} \end{aligned}}$$

$$\boxed{\begin{aligned} e^{A(t_1+t_2)} &= e^{At_1} \cdot e^{At_2} \\ e^{At} \cdot e^{-At} &= I \\ \left(e^{At}\right)^{-1} &= e^{-At} \\ \frac{d}{dt}\left(e^{At}\right) &= A e^{At} \end{aligned}}$$

$$\Phi(s)$$

$$Y(s) = \left[C \underbrace{(sI - A)^{-1}}_{\Phi(s)} B + D \right] U(s)$$

$$+ C (sI - A)^{-1} X(0)$$

$$Y(s) = \left[C \, \Phi(s) B + D \right] U(s) + C \Phi(s) X(0)$$

$$\Phi(s) = (sI - A)^{-1} \Longleftrightarrow \varphi(t)$$

$$(AB)^{-1} = B^{-1} A^{-1}$$

$$(sI - A) = sI - P \Lambda P^{-1}$$

$$= s P I P^{-1} - P \Lambda P^{-1}$$

$$= P (sI - \Lambda) P^{-1}$$

$$(sI - A) = P (sI - \Lambda) P^{-1}$$

$$(sI - A)^{-1} = \left[(sI - \Lambda) P^{-1} \right]^{-1}$$

$$(P^{-1})^{-1} (sI - \Lambda)^{-1} P^{-1}$$

$$\Phi(s) = (sI - A)^{-1} = P(sI-\Lambda)^{-1}P^{-1}$$

$$\Updownarrow$$

$$\varphi(t) = \mathcal{L}^{-1}\left\{P(sI-\Lambda)^{-1}P^{-1}\right\}$$

$$= P\,\boxed{\mathcal{L}^{-1}\left\{(sI-\Lambda)^{-1}\right\}}\,P^{-1}$$

$$(sI-\Lambda)^{-1} = \begin{bmatrix} \frac{1}{(s-\lambda_1)} & & 0 \\ & \frac{1}{(s-\lambda_2)} & \\ 0 & & \frac{1}{(s-\lambda_n)} \end{bmatrix}$$

$$\begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \begin{bmatrix} & ? & \\ & & \\ & & \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \begin{bmatrix} a_1^{-1} & 0 & 0 \\ 0 & b_2^{-1} & 0 \\ 0 & 0 & c_3^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Phi(s) = (sI - \Lambda)^{-1} = \begin{bmatrix} \frac{1}{(s-\lambda_1)} & & 0 \\ & \frac{1}{(s-\lambda_2)} & \\ 0 & & \ddots \\ & & & \frac{1}{(s-\lambda_n)} \end{bmatrix}$$

$$\boxed{\mathcal{L}^{-1}\left\{(sI-\Lambda)^{-1}\right\}} = \begin{bmatrix} e^{\lambda_1 t} & & 0 \\ & e^{\lambda_2 t} & \\ 0 & & \ddots \\ & & & e^{\lambda_n t} \end{bmatrix}$$

$$= \boxed{e^{\Lambda t}\, u(t)}$$

$$\Phi(s) = (s\mathbf{I} - \mathbf{A})^{-1} = \mathbf{P}(s\mathbf{I} - \Lambda)^{-1}\mathbf{P}^{-1}$$

$$\Updownarrow$$

$$\varphi(t) = \mathcal{L}^{-1}\left\{\mathbf{P}(s\mathbf{I} - \Lambda)^{-1}\mathbf{P}^{-1}\right\}$$

$$= \mathbf{P}\,\boxed{\mathcal{L}^{-1}\left\{(s\mathbf{I} - \Lambda)^{-1}\right\}}\,\mathbf{P}^{-1}$$

$$= \mathbf{P}\,\boxed{e^{\Lambda t}\,u(t)}\,\mathbf{P}^{-1}$$

$$= \boxed{\mathbf{P}\,e^{\Lambda t}\,\mathbf{P}^{-1}}\,u(t)$$

$$\varphi(t) = e^{\mathbf{A}t}\,u(t)$$

$$\Phi(s) = (s\mathbf{I} - \mathbf{A})^{-1} \qquad \phi(s) = (s-a)^{-1}$$

$$\Updownarrow \qquad \Updownarrow$$

$$\varphi(t) = e^{\mathbf{A}t}\,u(t) \qquad \varphi(t) = e^{at}\,u(t)$$

state transition matrix

$$X(s) = \Phi(s) B U(s) + \Phi(s) x(0)$$

$$Y(s) = C X(s) + D U(s)$$

$$X(s) = \underbrace{(sI - A)^{-1}}_{\phi(s)} B\, U(s) + \underbrace{(sI - A)^{-1}}_{\phi(s)} X(0)$$

$$Y(s) = \left[C\, \phi(s) B + D \right] U(s) + C\, \phi(s) X(0)$$

$$\boxed{\begin{aligned} X(s) &= \phi(s) B\, U(s) + \phi(s) X(0) \\ Y(s) &= C\, \phi(s) B\, U(s) + D\, U(s) + C\, \phi(s) X(0) \end{aligned}}$$

$$f(t) \longleftrightarrow F(s)$$

$$g(t) \longleftrightarrow G(s)$$

$$f(t) * g(t) \longleftrightarrow F(s) \cdot G(s)$$

$$\int f(t-\tau) \cdot g(\tau)\, d\tau$$

$$X(s) = \boxed{\Phi(s)}\,\boxed{B\,U(s)} + \boxed{\Phi(s)}\,X(0)$$

$$x(t) = \varphi(t) * B\,u(t) + \varphi(t)\,x(0)$$

$$= \int_0^t \varphi(t-\tau) B\,u(\tau)\, d\tau + \varphi(t)\,x(0)$$

$$Y(s) = \boxed{C\,\Phi(s)}\,\boxed{B\,U(s)} + \boxed{D\,U(s)} + \boxed{C\Phi(s)}\,X(0)$$

$$y(t) = C\,\varphi(t) * B\,u(t) + D\,u(t) + C\varphi(t)\,x(0)$$

$$= C\int^t \varphi(t-\tau) B\,u(\tau)\, d\tau + D\,u(t) + C\varphi(t)\,x(0)$$