CT Impulse Function (4B)

- Continuous Time Impulse Function

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6/25/13

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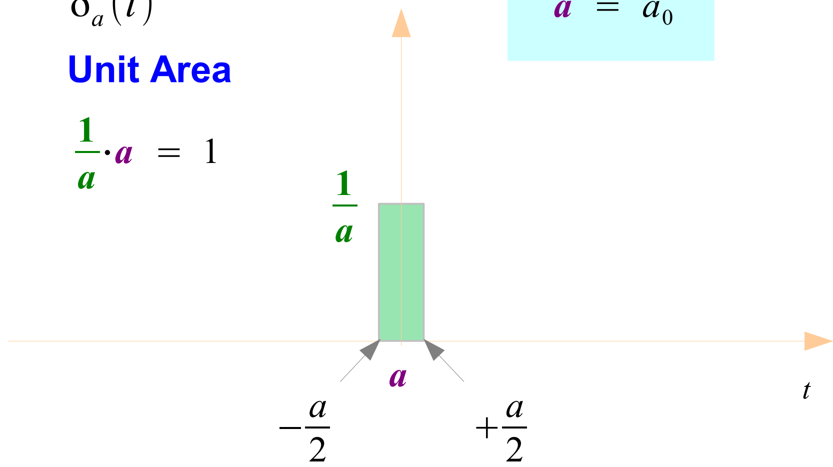
The Delta Function

$\delta_a(t)$

Unit Area

$\frac{1}{a} \cdot a = 1$

$a = a_0$

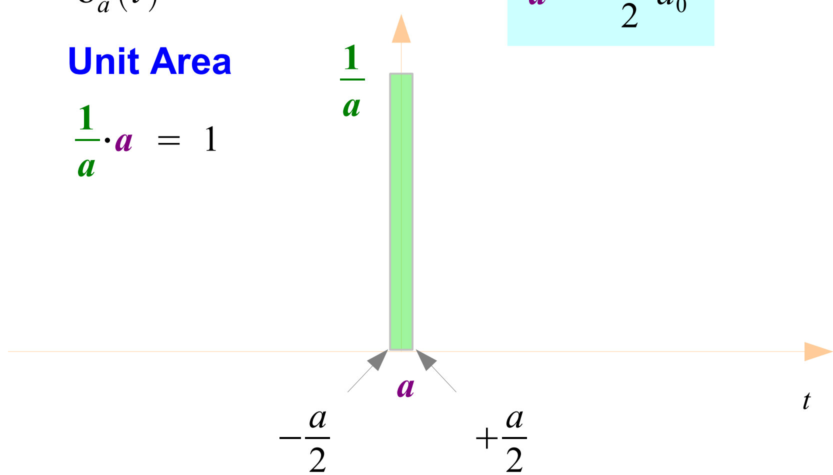


$\delta_a(t)$

Unit Area

$\frac{1}{a} \cdot a = 1$

$a = \frac{1}{2} \cdot a_0$



Height	Width	Area
$\frac{1}{a} = \frac{1}{a_0}$	$a = a_0$	$\frac{1}{a} \cdot a = 1$
$\frac{1}{a} = 2 \cdot \frac{1}{a_0}$	$a = \frac{1}{2} \cdot a_0$	$\frac{1}{a} \cdot a = 1$
$\frac{1}{a} \to \infty$	$a \to 0$	$\frac{1}{a} \cdot a = 1$

$\delta(t)$ **Dirac Delta**

Unit impulse function

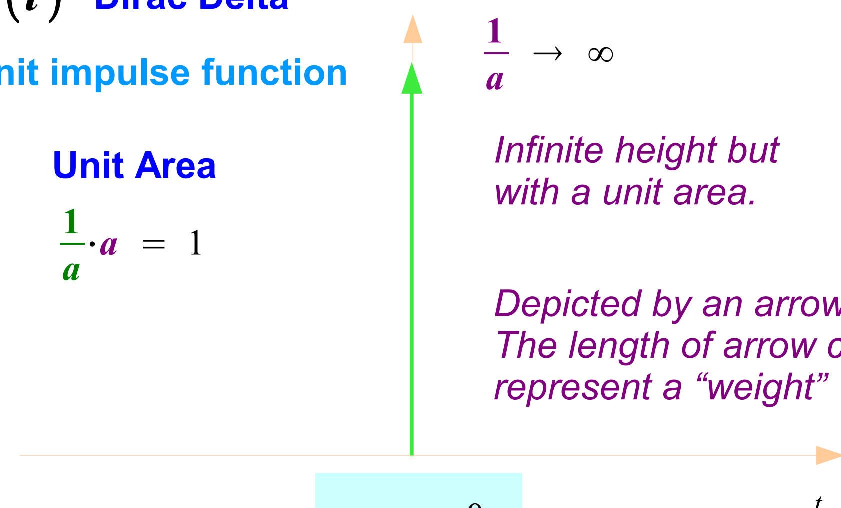
Unit Area

$\frac{1}{a} \cdot a = 1$

$\frac{1}{a} \to \infty$

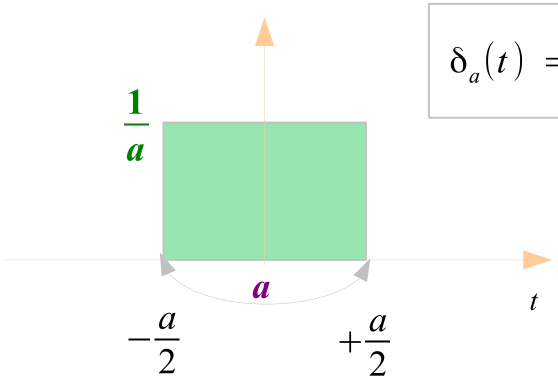
Infinite height but with a unit area.

Depicted by an arrow
The length of arrow can represent a “weight”

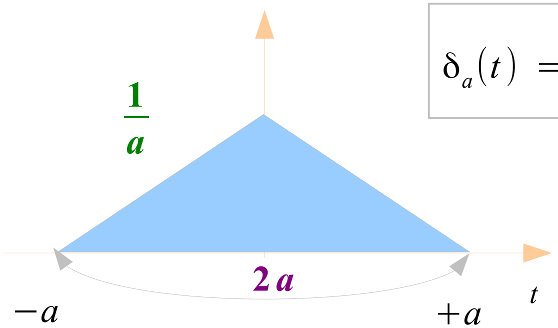


$a \to 0$

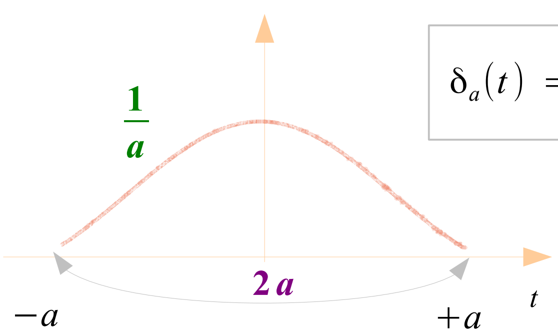
The Unit Impulse



$$\delta_a(t) = \frac{1}{a}\, \boldsymbol{rect}\left(\frac{1}{a} \cdot t\right)$$



$$\delta_a(t) = \frac{1}{a}\left(1 - \frac{1}{a}|t|\right)$$



$$\delta_a(t) = \frac{1}{a}\, \mathbf{exp}\left(\frac{\pi}{a^2} \cdot t^2\right)$$

$$\lim_{a \to \infty} \delta_a(t) = \delta(t)$$

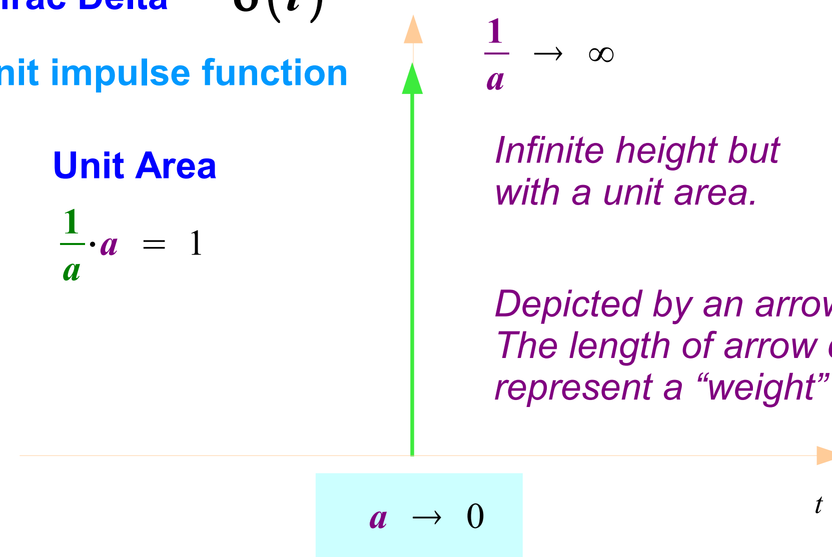
The shape does not matter in the limit
But the area matters : The Unit Area

Dirac Delta $\delta(t)$

Unit impulse function

Unit Area

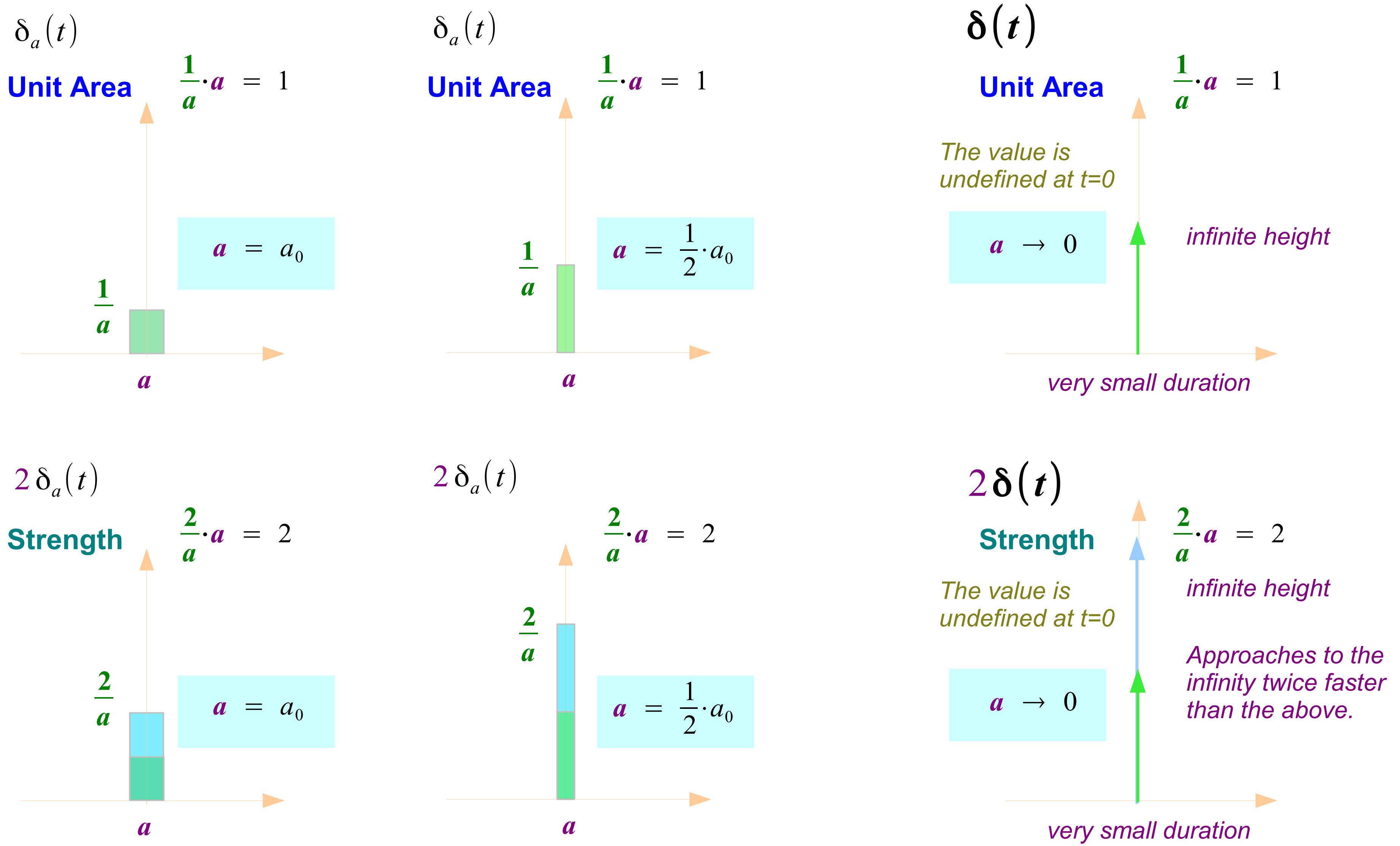
$$\frac{1}{a} \cdot a = 1$$



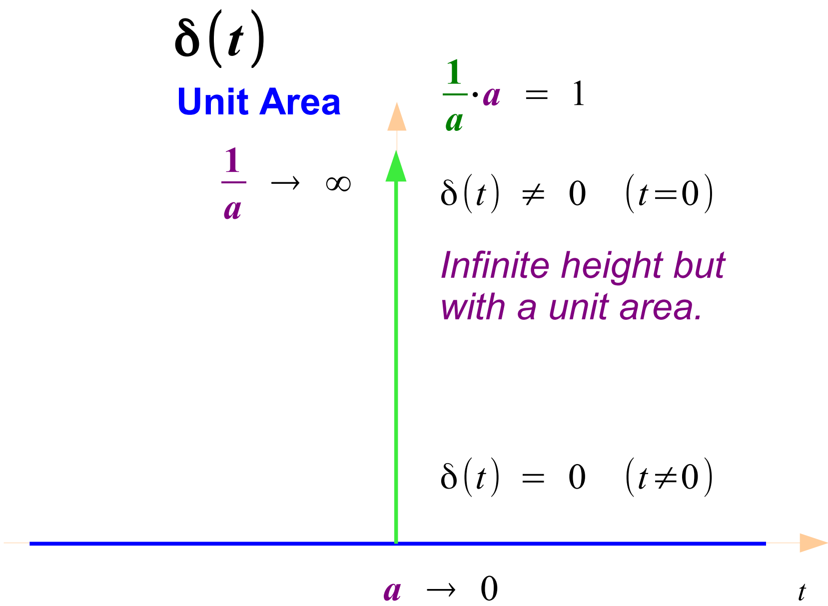
Infinite height but
with a unit area.

Depicted by an arrow
The length of arrow can
represent a "weight"

Impulse Strength



The Properties of the Delta Function



The Equivalence Property

$$g(t)\,\delta(t) = g(0)\,\delta(t)$$

$$g(t)\,\delta(t-t_0) = g(t_0)\,\delta(t-t_0)$$

The Sampling Property

$$\int_{-\infty}^{+\infty} g(t)\,\delta(t)\,dt = g(0)$$

$$\int_{-\infty}^{+\infty} g(t)\,\delta(t-t_0)\,dt = g(t_0)$$

An Even Function

$$\delta(-t) = \delta(t)$$

The Replication Property

$$\int_{-\infty}^{+\infty} g(\tau)\,\delta(t-\tau)\,d\tau = g(t)$$

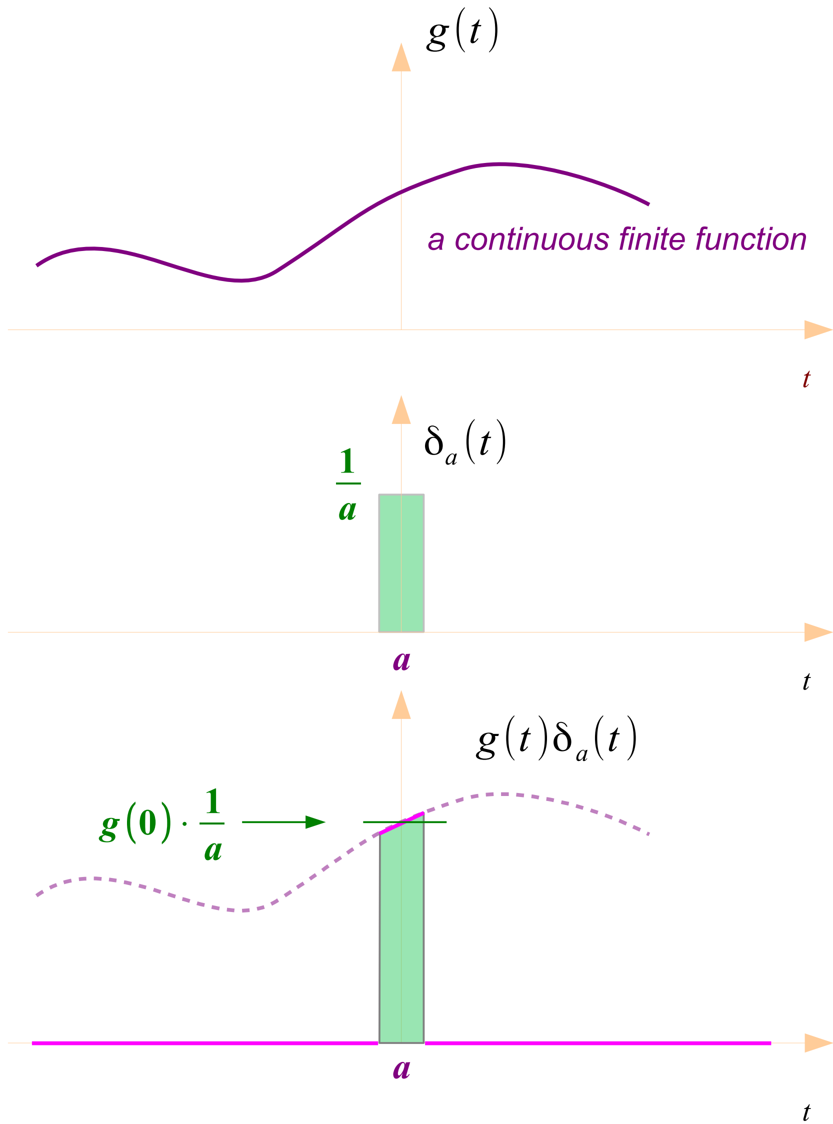
The Scaling Property

$$\delta(a(t-t_0)) = \frac{1}{|a|}\delta(t-t_0)$$

The Fourier Transform Property

$$\int_{-\infty}^{+\infty} \delta(t)\,e^{-j2\pi f t}\,dt = 1$$

The Equivalence Property



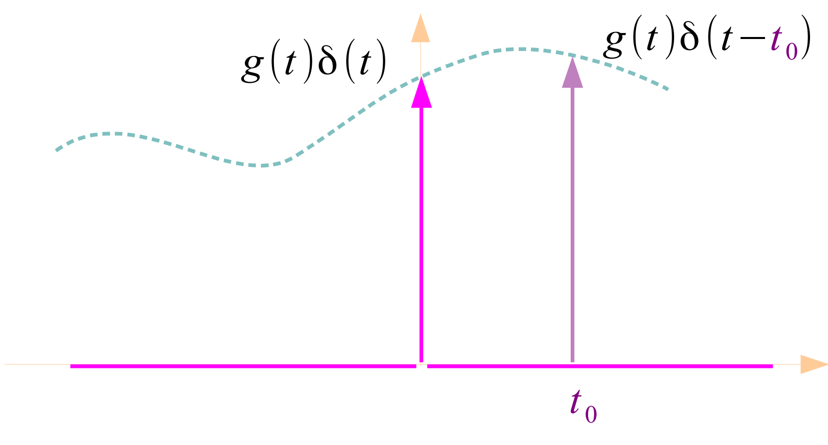
$$\lim_{a \to \infty} \delta_a(t) = \delta(t)$$

$$\lim_{a \to \infty} g(t)\delta_a(t) = g(0)\delta(t)$$

$$g(t)\delta(t) = 0 \qquad (t \neq 0)$$

$$g(0)\delta(t) = g(t)\delta(t)$$

$$g(t_0)\delta(t-t_0) = g(t)\delta(t-t_0)$$



The Sampling Property

$$g(0)\delta(t) = g(t)\delta(t)$$

$$\int_{-\infty}^{+\infty} g(0)\delta(t)\,dt = \int_{-\infty}^{+\infty} g(t)\delta(t)\,dt$$

$$g(0)\int_{-\infty}^{+\infty} \delta(t)\,dt = g(0)$$

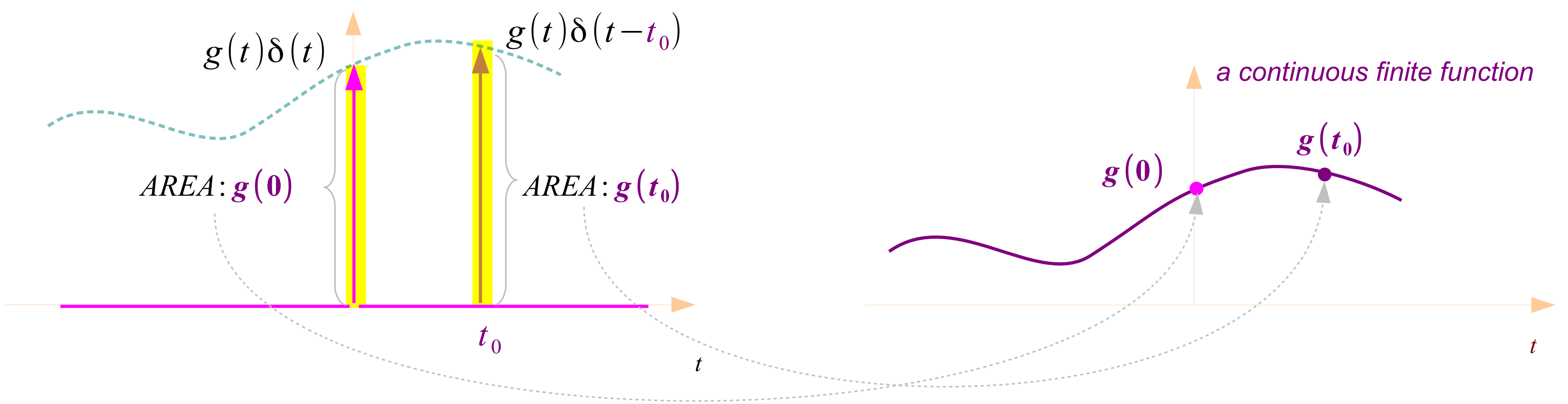
$$g(0) = \int_{-\infty}^{+\infty} g(t)\delta(t)\,dt$$

$$g(t_0)\delta(t-t_0) = g(t)\delta(t-t_0)$$

$$\int_{-\infty}^{+\infty} g(t_0)\delta(t-t_0)\,dt = \int_{-\infty}^{+\infty} g(t)\delta(t-t_0)\,dt$$

$$g(t_0)\int_{-\infty}^{+\infty} \delta(t)\,dt = g(t_0)$$

$$g(t_0) = \int_{-\infty}^{+\infty} g(t)\delta(t-t_0)\,dt$$



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The Replication Property

$$g(t_0) = \int_{-\infty}^{+\infty} g(t)\,\delta(t-t_0)\,dt$$

$t \leftarrow t_0 \qquad \tau \leftarrow t$

$$g(t) = \int_{-\infty}^{+\infty} g(\tau)\,\delta(\tau-t)\,d\tau$$

$\delta(-t) = \delta(t)$

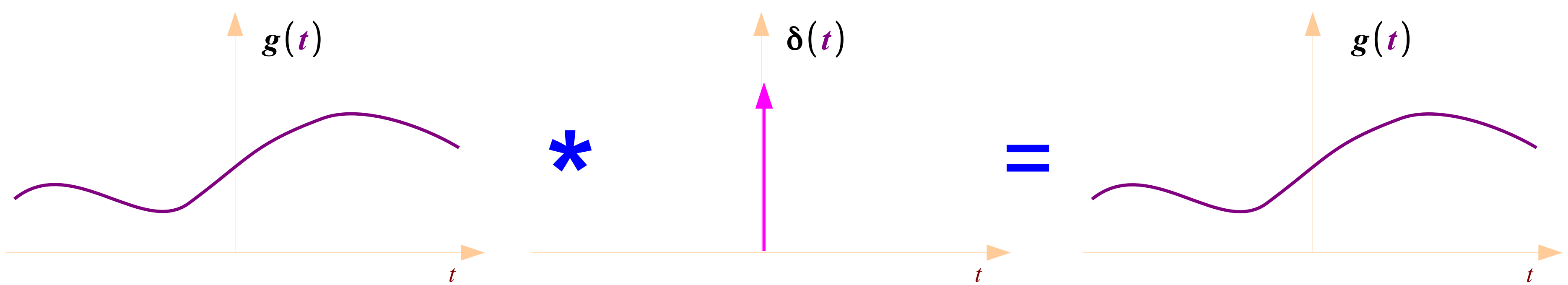
$$g(t) = \int_{-\infty}^{+\infty} g(\tau)\,\delta(t-\tau)\,d\tau$$

Impulse response

$\delta(t) \rightarrow$ [$g(t)$] $\rightarrow g(t)$

Replication

$g(t) \rightarrow$ [$\delta(t)$] $\rightarrow g(t)$



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The Fourier Transform Property

$$1 = \int_{-\infty}^{+\infty} \delta(t)\, e^{-j2\pi f t}\, dt$$

$\delta(t) \Longleftrightarrow 1$

$1 \Longleftrightarrow \delta(f)$

$e^{+j2\pi f_c t} \Longleftrightarrow \delta(f - f_c)$

$e^{-j2\pi f_c t} \Longleftrightarrow \delta(f + f_c)$

$$\cos(2\pi f_c t) = \frac{1}{2}\left[e^{+j2\pi f_c t} + e^{-j2\pi f_c t}\right]$$

$$\sin(2\pi f_c t) = \frac{1}{2j}\left[e^{+j2\pi f_c t} - e^{-j2\pi f_c t}\right]$$

$\cos(2\pi f_c t) \Longleftrightarrow \frac{1}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$

$\sin(2\pi f_c t) \Longleftrightarrow \frac{1}{2j}\left[\delta(f - f_c) - \delta(f + f_c)\right]$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] M. J. Roberts, Fundamentals of Signals and Systems
[4] S. Haykin, An Introduction to Analog & Digital Communications