RLC Transient Response (H.1)
2nd Order Circuit

20150720

The necessities in Electric Circuit
wikiversity

Transient Response : 2^{nd} Order System

$$\boxed{\frac{1}{\omega_n^2}\frac{d^2x(t)}{dt} + 2\frac{\zeta}{\omega_n}\frac{dx(t)}{dt} + 1\,x(t) = Kf(t)}$$

$$x(t) \to \begin{cases} V_c(t) \\ i_L(t) \end{cases}$$

1. find $\boxed{x(0^-)}$ & $\boxed{x(\infty)}$ 2 steady state values
2. find $\boxed{x(0^+)}$ & $\boxed{\dot{x}(0^+)}$ initial conditions
3. find 2nd order Diff Eq.

$$\boxed{\frac{1}{\omega_n^2}\frac{d^2x(t)}{dt} + 2\frac{\zeta}{\omega_n}\frac{dx(t)}{dt} + 1\,x(t) = Kf(t)}$$

4. find $\boxed{\omega_n}$ & $\boxed{\zeta}$
5. find $\boxed{x(t) = x_h(t) + x_p(t)}$ $x(\infty)$
6. find $\boxed{\alpha_1}$ & $\boxed{\alpha_2}$

Constant input : DC

$$\boxed{\frac{1}{\omega_n^2}\frac{d^2x(t)}{dt^2} + 2\frac{\zeta}{\omega_n}\frac{dx(t)}{dt} + 1\,x(t) = Kf(t)}$$

(a) $\zeta > 1$	(b) $\zeta = 1$	(c) $\zeta < 1$
$\alpha_1 e^{(-\zeta\omega_n + \sqrt{\zeta^2-1}\,\omega_n)t}$ $+ \alpha_2 e^{(-\zeta\omega_n - \sqrt{\zeta^2-1}\,\omega_n)t}$ $+ x(\infty)$	$\alpha_1 e^{(-\zeta\omega_n)t}$ $+ \alpha_2 t e^{(-\zeta\omega_n)t}$ $+ x(\infty)$	$\alpha_1 e^{(-\zeta\omega_n + j\sqrt{1-\zeta^2}\,\omega_n)t}$ $+ \alpha_2 e^{(-\zeta\omega_n - j\sqrt{1-\zeta^2}\,\omega_n)t}$ $+ x(\infty)$

$x(\infty)$ .... particular

forced

Homogeneous sol → natural response → 0 vanishes

$$\alpha_1 e^{(-\zeta\omega_n + \sqrt{\zeta^2-1}\,\omega_n)t} + \alpha_2 e^{(-\zeta\omega_n - \sqrt{\zeta^2-1}\,\omega_n)t}$$

$$\alpha_1 e^{(-\zeta\omega_n)t} + \alpha_2 t e^{(-\zeta\omega_n)t}$$

$$\alpha_1 e^{(-\zeta\omega_n + j\sqrt{1-\zeta^2}\,\omega_n)t} + \alpha_2 e^{(-\zeta\omega_n - j\sqrt{1-\zeta^2}\,\omega_n)t}$$

$$e^{-\zeta\omega_n t}$$

$$e^{-\zeta\omega_n \infty} \to 0$$

$\boxed{\zeta > 0}$

$\omega_n > 0$

$t > 0$

$$\downarrow$$

$$c_1 e^{-\zeta\omega_n t}\cos\left(\sqrt{1-\zeta^2}\,\omega_n t\right) + c_2 e^{-\zeta\omega_n t}\sin\left(\sqrt{1-\zeta^2}\,\omega_n t\right)$$

① Find $x(0^-)$ & $x(\infty)$

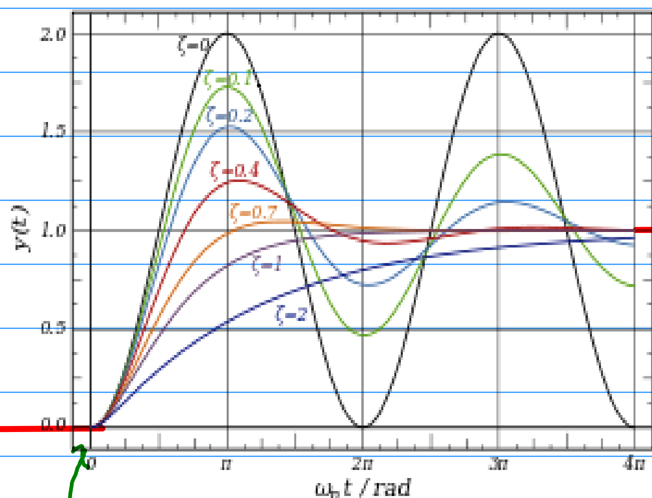
ON

$t = 0$ $\qquad\qquad\qquad\qquad$ $t = \infty$

0^- 0^+

$x(0^-)$ $\qquad\qquad\qquad\qquad$ $x(\infty)$

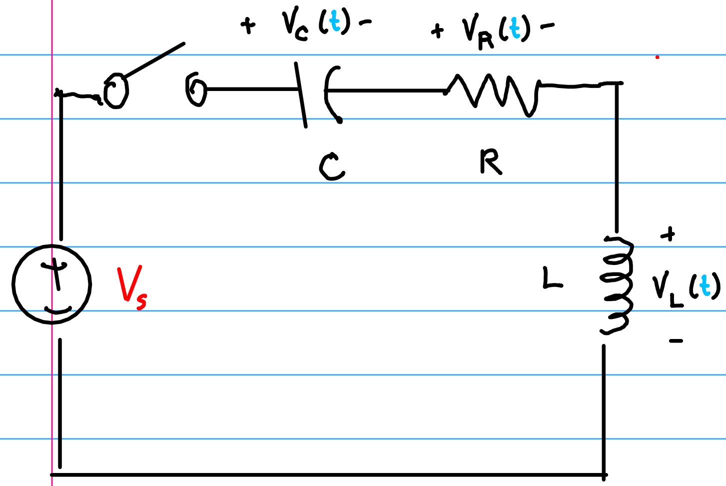
steady state Rsp $\qquad\qquad\qquad\qquad$ steady state Rsp



$t = \infty$

$t = 0^-$

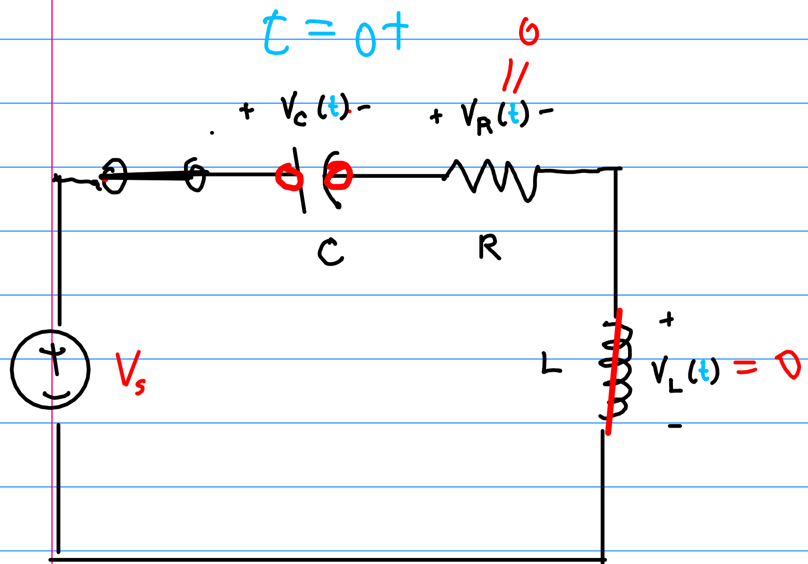
Fig 5.80 $t = 0^-$



문제에서

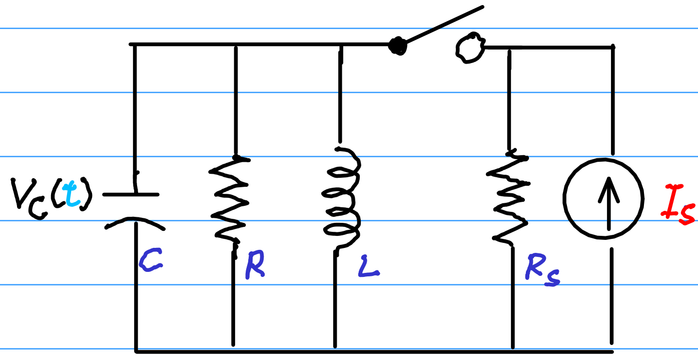
$$\begin{cases} v_C(0^-) = 5 \\ i_L(0^-) = 0 \end{cases}$$

$t = 0^+$



$$\begin{cases} v_c(\infty) = 25 \\ i_L(\infty) = 0 \end{cases}$$

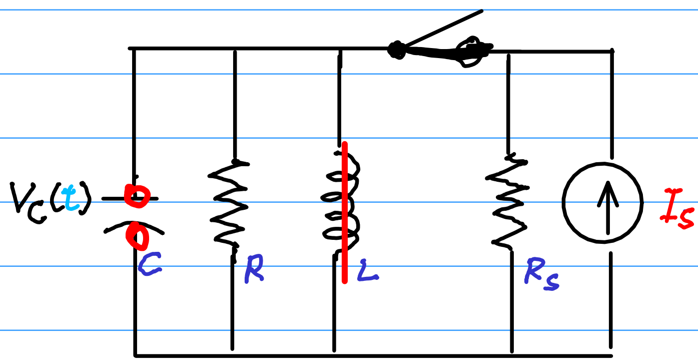
$t = 0^-$



$i_L(0^-) = 0$

$v_C(0^-) = 0$

$t = \infty$



$i_L(\infty) = I_s$

$v_C(\infty) = 0$

② find $x(0^+)$, $\dot{x}(0^+)$

$t=0$

continuous v_c · $v_c(0^-) = v_c(0^+)$

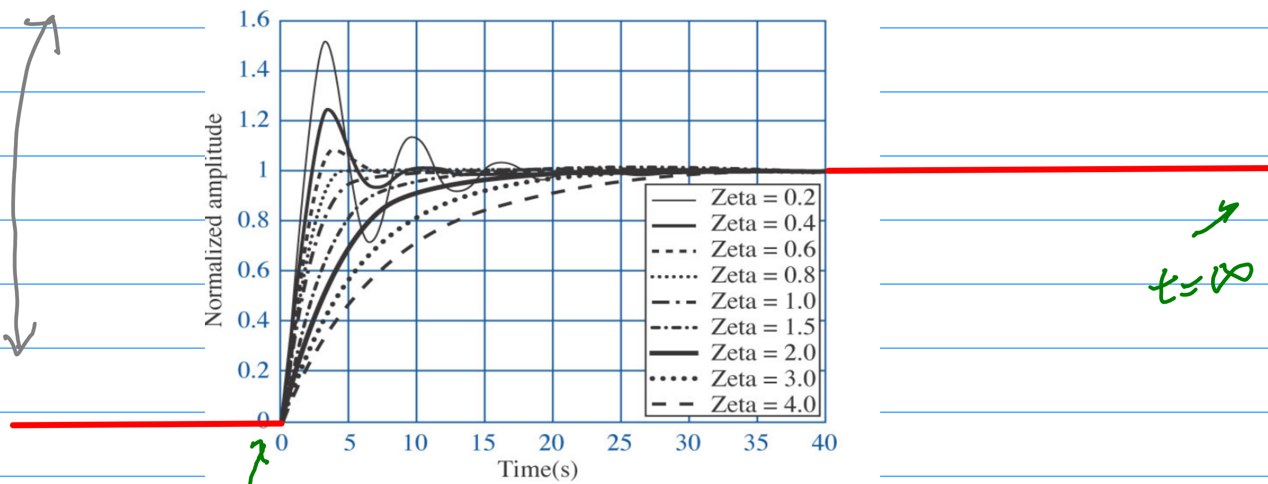
continuous i_L $i_L(0^-) = i_L(0^+)$

ON

$t=0$ $t=\infty$

0^- 0^+

$x(0^-) \neq x(0^+)$, $\dot{x}(0^+)$ may be continuous / discontinuous $x(\infty)$

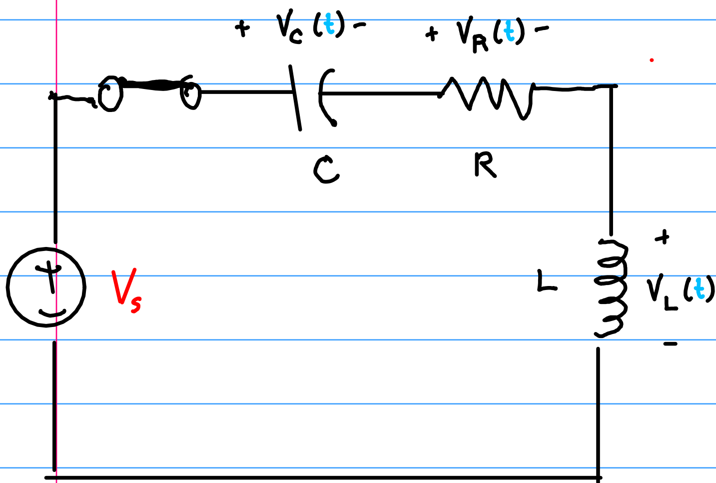


$t=\infty$

$t=0^-$

step2

$t = 0^+$



문제에서

$$\begin{cases} v_C(0^-) = 5 = v_C(0^+) \\ i_L(0^-) = 0 = i_L(0^+) \end{cases}$$

continuous

$i_L'(0^+) = ?$

$t = 0^+$ KVL

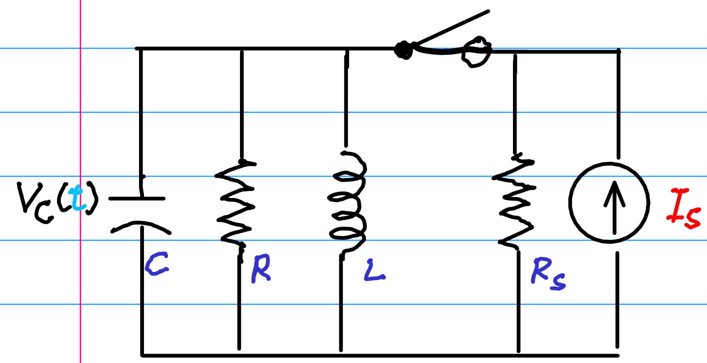
$$V_s - v_C(0^+) - R\,i_L(0^+) - v_L(0^+) = 0$$

$$V_s - v_C(0^+) - R\,i_L(0^+) - L\frac{di_L(0^+)}{dt} = 0$$

$$\frac{di_L(0^+)}{dt} = \frac{1}{L}\Big(\underset{2V}{V_s} - \underset{5}{v_C(0^+)} - \underbrace{R\,i_L(0^+)}_{0}\Big)$$

step2

$t=0^+$



$$i_L(0^-)=0=i_L(0^+)$$

$$v_c(0^-)=0=v_c(0^+)$$

$$v_c'(0^+)=?$$

$$I_s-\frac{v_c(t)}{R_s}-i_L(t)-\frac{v_c(t)}{R}-C\frac{dv_c(t)}{dt}=0$$

$$I_s-\cancelto{0}{\frac{v_c(0^+)}{R_s}}-\cancelto{0}{i_L(0^+)}-\cancelto{0}{\frac{v_c(0^+)}{R}}-C\frac{dv_c(0^+)}{dt}=0$$

$$\frac{dv_c(0^+)}{dt}=\frac{1}{C}I_s$$

③ find 2nd order Diff Eq.

$$\boxed{\frac{1}{W_n^2}\frac{d^2x(t)}{dt} + 2\frac{\zeta}{W_n}\frac{dx(t)}{dt} + 1\, x(t) = K f(t)}$$

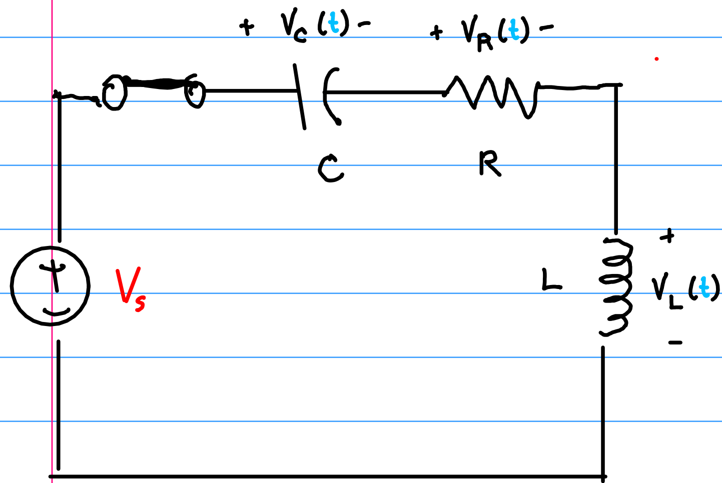
t>0 ⇒ switch "ON"

$$x(t) = \begin{cases} V_c(t) & \leftarrow \text{parallel same volt.} \\ i_L(t) & \leftarrow \text{series same current} \end{cases}$$

$$LC\frac{d^2 i_L}{dt} + R_1\frac{di_L}{dt} + 1\, i_L(t) = 0$$

$$LC\frac{d^2 V_c}{dt^2} + L\left(\frac{R_1 + R}{R_1 R}\right)\frac{dV_c}{dt} + 1\, V_c(t) = 0$$

$\boxed{t>0}$



$$V_s - v_c(t) - R\cdot i_L(t) - L\frac{di_L}{dt} = 0$$

$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_L(t)\,dt$$

$$\frac{d}{dt}\left\{ V_s - \frac{1}{C}\int_{-\infty}^{t} i_L(t)\,dt - R\cdot i_L(t) - L\frac{di_L}{dt} = 0 \right\}$$

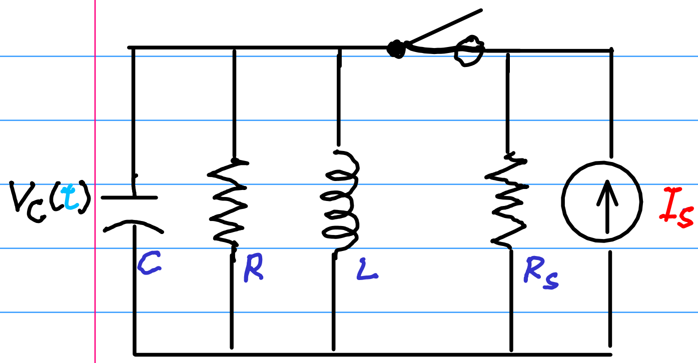
상수

$$-\frac{1}{C}\, i_L(t) - R\cdot\frac{di_L}{dt} - L\frac{d^2 i_L}{dt^2} = 0$$

$$i_L(t) + RC\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt} = 0$$

$$LC\frac{d^2 i_L}{dt} + RC\frac{di_L}{dt} + 1\, i_L(t) = 0$$

$t > 0$



$$L\frac{di_L}{dt} = v_L$$

$$i_L = \frac{1}{L}\int v_L\,dt$$

KCL

$$I_s - \frac{V_C(t)}{R_s} - i_L(t) - \frac{V_C(t)}{R} - C\frac{dV_C(t)}{dt} = 0$$

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{\infty} v_L(t)\,dt$$

$$\frac{d}{dt}\left\{ I_s - \frac{V_C(t)}{R_s} - \frac{1}{L}\int_{-\infty}^{\infty} v_L(t)\,dt - \frac{V_C(t)}{R} - C\frac{dV_C(t)}{dt} = 0 \right\}$$

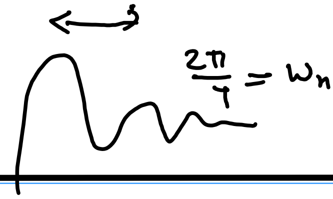
상수

0

$$\left(\frac{1}{R_s} + \frac{1}{R}\right)\frac{dV_C}{dt} + \frac{1}{L}v_L(t) + C\frac{d^2V_C}{dt^2} = 0$$

$$LC\frac{d^2V_C}{dt^2} + L\left(\frac{R_s + R}{R_s R}\right)\frac{dV_C}{dt} + 1\,v_L(t) = 0$$

④ Find ω_n & ζ



$\uparrow$ natural freq $\uparrow$ damping ratio

$$\boxed{\frac{1}{\omega_n^2}\frac{d^2x(t)}{dt} + \frac{2\zeta}{\omega_n}\frac{dx(t)}{dt} + 1\,x(t) = Kf(t)}$$

$$LC\frac{d^2 i_L}{dt} + RC\frac{di_L}{dt} + 1\,i_L(t) = 0$$

$$\omega_n = \sqrt{\frac{1}{LC}} = 1000$$

$$\zeta = RC\frac{\omega_n}{2} = 2.5 \quad \underline{\text{Overdamping}}$$

$$LC\frac{d^2 v_c}{dt^2} + L\left(\frac{R_s + R}{R_s R}\right)\frac{dv_c}{dt} + 1\,v_c(t) = 0$$

$$\omega_n = \sqrt{\frac{1}{LC}} = 560$$

$$\zeta = L(\ )\frac{\omega_n}{2} = 1 \quad \underline{\text{Critical damping}}$$

⑤ find $x(t) = x_h(t) + x_p(t)$ $x(\infty)$

ζ 에 따라서

$\zeta > 1$ $D > 0$ 2 distinct real roots m_1, m_2

$x_h(t) \Leftarrow$ $\boxed{x_h(t) = C_1 e^{m_1 t} + C_2 e^{m_2 t}}$

$\zeta = 1$ $D = 0$ repeated real root m_1

$x_h(t) \Longleftarrow$ $\boxed{x_h(t) = C_1 e^{m_1 t} + C_2 t \cdot e^{m_2 t}}$

$\zeta < 1$ $D < 0$ 2 complex conjugate root $m_1 = \alpha + j\beta$, $m_2 = \alpha - j\beta$

$x_h(t) \Longleftarrow$

$$\boxed{\begin{aligned} x_h(t) &= C_1 e^{(\alpha + j\beta)t} + C_2 e^{(\alpha - j\beta)t} \\ &= e^{\alpha t}\left(C_3 \cos(\beta t) + C_4 \sin(\beta t)\right) \end{aligned}}$$

ⓐ $\zeta > 1$

$$x(t) = x_N(t) + x_F(t)$$

$$\boxed{\dot{=} \alpha_1 e^{(-\zeta\omega_n + \sqrt{\zeta^2-1}\,\omega_n)t} + \alpha_2 e^{(-\zeta\omega_n - \sqrt{\zeta^2-1}\,\omega_n)t} + x(\infty)}$$

$t \geq 0$

ⓑ $\zeta = 1$

$$x(t) = x_N(t) + x_F(t)$$

$$\boxed{\dot{=} \alpha_1 e^{(-\zeta\omega_n)t} + \alpha_2 t e^{(-\zeta\omega_n)t} + x(\infty)}$$

ⓒ $\zeta < 1$

$$x(t) = x_N(t) + x_F(t)$$

$$\boxed{\dot{=} \alpha_1 e^{(-\zeta\omega_n + j\sqrt{1-\zeta^2}\,\omega_n)t} + \alpha_2 e^{(-\zeta\omega_n - j\sqrt{1-\zeta^2}\,\omega_n)t} + x(\infty)}$$

⑥ find α_1 & α_2

initial condition

$$\begin{cases} i_L(0^+) \\ \dot{i}_L(0^+) \end{cases} \qquad \begin{cases} v_c(0^+) \\ \dot{v}_c(0^+) \end{cases}$$

Step 2

ⓐ $\zeta > 1$

$$\left.\begin{aligned} x(t) &= \alpha_1 e^{(-\zeta\omega_n + \sqrt{\zeta^2-1}\,\omega_n)t} \\ &+ \alpha_2 e^{(-\zeta\omega_n - \sqrt{\zeta^2-1}\,\omega_n)t} \\ &+ x(\infty) \end{aligned}\right\} \quad x'(t)\Big|_{t=0^+}$$

$t \geq 0$

ⓑ $\zeta = 1$

$$\begin{aligned} x(t) &= \alpha_1 e^{(-\zeta\omega_n)t} \quad \Rightarrow \quad x'(t)\Big|_{t=0^+} \\ &+ \alpha_2 t e^{(-\zeta\omega_n)t} \\ &+ x(\infty) \end{aligned}$$

ⓒ $\zeta < 1$

$$\begin{aligned} x(t) &= \alpha_1 e^{(-\zeta\omega_n + j\sqrt{1-\zeta^2}\,\omega_n)t} \\ &+ \alpha_2 e^{(-\zeta\omega_n - j\sqrt{1-\zeta^2}\,\omega_n)t} \\ &+ x(\infty) \end{aligned}$$