RLC Transient Response (H.1)
1st Order Cicuit

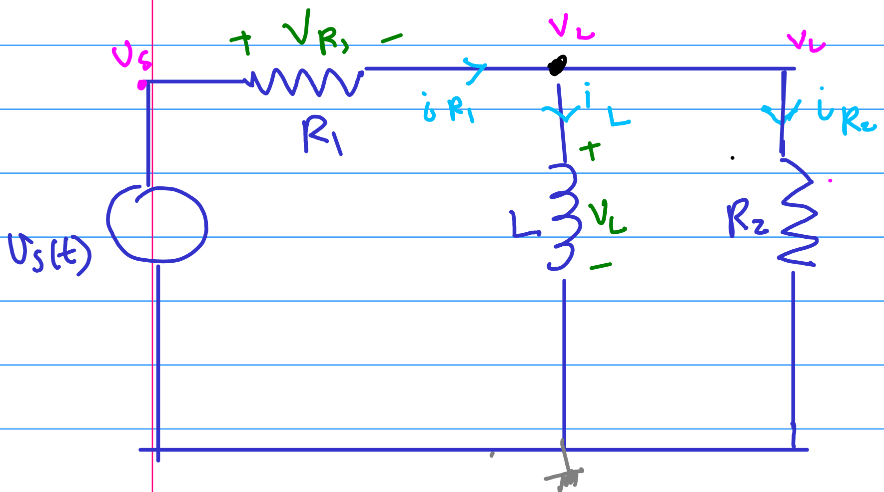
20150720

The necessities in Electric Circuit
wikiversity

$\boxed{\text{DC}}$ Steady State Solution

변하지 않는 input에 대한 Steady State 출력값

constant input



$$V_L = L\frac{di_L}{dt}$$

KCL

$$i_{R_1} - i_L - i_{R_2} = 0$$

$$\frac{V_S - V_L}{R_1} - i_L - \frac{V_L}{R_2} = 0$$

$$\frac{V_S}{R_1} - \frac{L}{R_1}\frac{di_L}{dt} - i_L - \frac{L}{R_2}\frac{di_L}{dt} = 0$$

$$-\left(\frac{L}{R_1} + \frac{L}{R_2}\right)\frac{di_L}{dt} - i_L + \frac{V_S}{R_1} = 0$$

$$\boxed{\boxed{\frac{L(R_1+R_2)}{R_1 R_2}}\frac{di_L}{dt} + i_L = \boxed{\frac{1}{R_1}} V_S}$$

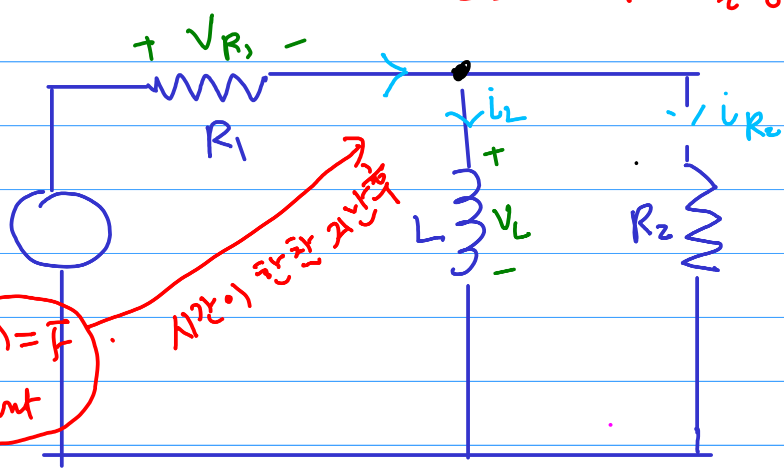
$$\boxed{\boxed{\tau}\frac{dx}{dt} + 1\,x(t) = \boxed{K_s}\ F}$$

$$\boxed{1\cdot\frac{di_L}{dt} + \frac{R_1 R_2}{L(R_1+R_2)} i_L = \frac{R_2}{L(R_1+R_2)} V_S}$$

$$\boxed{\frac{L(R_1+R_2)}{R_1R_2}}\frac{di_L}{dt}+i_L=\boxed{\frac{1}{R_1}}v_s$$

$$\boxed{\tau}\frac{dx}{dt}+1\,x(t)=\boxed{K_s}\,F$$

흐르는 전류가 일정 (변함 X) → $\boxed{\frac{di}{dt}=0}$



만약 $v_s(t)=F$ constant

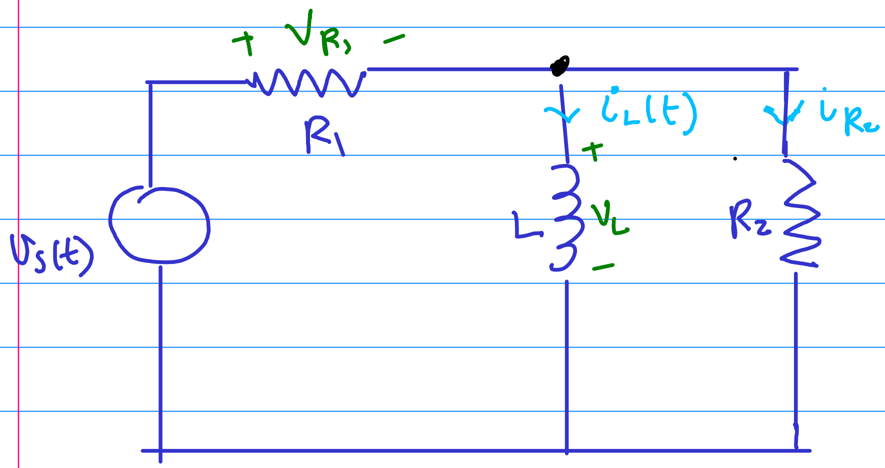
시간이 한참 지난후

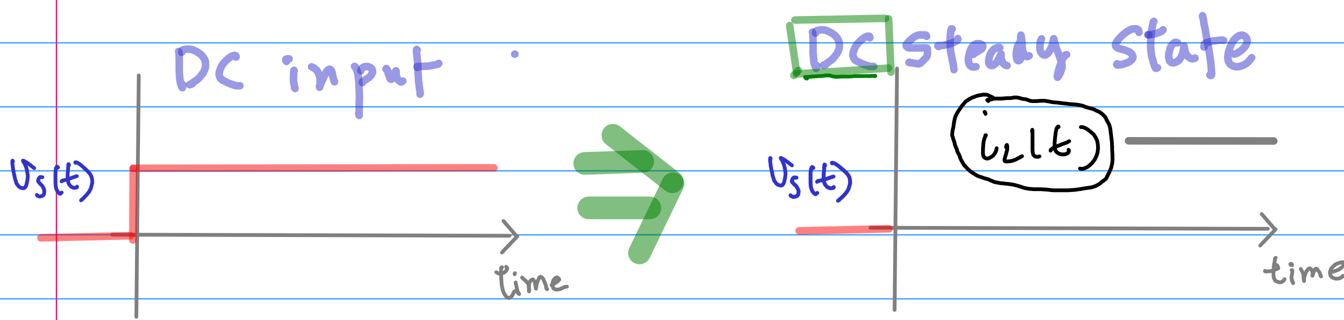
$$\boxed{\frac{L(R_1+R_2)}{R_1R_2}}\frac{di_L(\infty)}{dt}+i_L(\infty)=\boxed{\frac{1}{R_1}}F$$

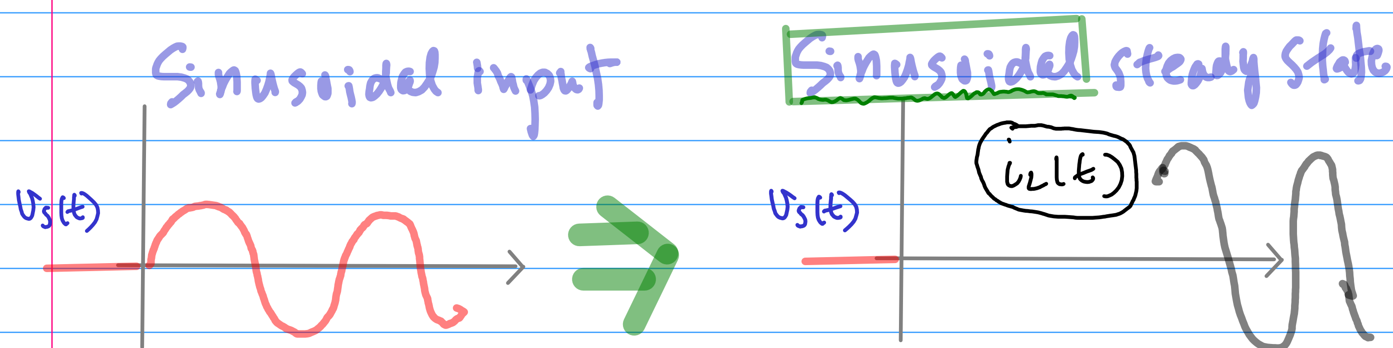
$$\boxed{\tau}\frac{dx(\infty)}{dt}+1\,x(\infty)=\boxed{K_s}\,F$$

$$i_L(\infty)=\boxed{\frac{1}{R_1}}F$$

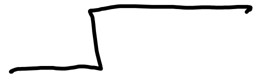
$\boxed{DC}$ steady state 값







phasor
+
Impedance

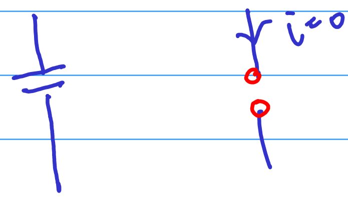


$\boxed{DC}$ Steady state

$\boxed{\text{at } t=\infty}$ $\quad \dfrac{d v_c(\infty)}{dt} \xrightarrow{} 0 = 0$

$\rightarrow i_c(\infty) = 0$

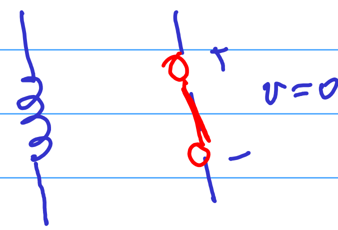
$\boxed{\text{at } t=\infty}$



$\boxed{\text{at } t=\infty}$ $\quad \dfrac{d i_L(\infty)}{dt} \xrightarrow{} 0 = 0$

$\rightarrow v_L(\infty) = 0$

$\boxed{\text{at } t=\infty}$



$$\tau \frac{dx}{dt} + 1\, x(t) = \boxed{K_s \quad F} \qquad \frac{dx}{dt} \rightarrow 0$$

$x(\infty) = K_s F$

$$\frac{1}{W_n^2} \frac{d^2 x(t)}{dt} + 2 \frac{\zeta}{W_n} \frac{dx(t)}{dt} + 1\, x(t) = \boxed{K f(t)}$$

$x(\infty) = K f(t)$

Constant input을 갖는

steady state 에서 $x(t)$ 는 변화가 없음

$t=\infty$

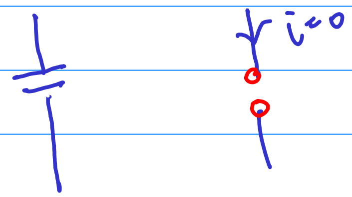
$$\boxed{\frac{d^2x}{dt} = \frac{dx}{dt} = 0}$$

Sinusoidal Steady state

$\boxed{\text{at } t=\infty}$ $\cancel{\dfrac{dv_c(\infty)}{dt}} = 0$

→ $i_c(\infty) = 0$

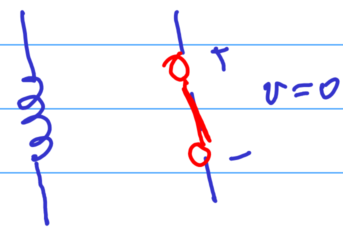
$\boxed{\text{at } t=\infty}$



$\boxed{\text{at } t=\infty}$ $\cancel{\dfrac{di_L(\infty)}{dt}} = 0$

→ $v_L(\infty) = 0$

$\boxed{\text{at } t=\infty}$



$i_c = C\dfrac{dv_c}{dt}$

$v_c(t) = A\cos\omega t$

$I = \dfrac{1}{j\omega C} V$

Transient Response : 1^{st} Order System

$$\boxed{\tau \frac{dx}{dt} + 1\, x(t) = K_s\, F}$$

$$x(t) \begin{cases} \rightarrow V_c(t) \\ \rightarrow i_L(t) \end{cases}$$

1. find $\boxed{x(0^-)}$ & $\boxed{x(\infty)}$ 2 steady state values
2. find $\boxed{x(0^+)}$ & $\boxed{\dot{x}(0^+)}$ initial conditions
3. find 1^{st} order Diff Eq.

$$\boxed{\tau \frac{dx}{dt} + 1\, x(t) = K_s\, F}$$

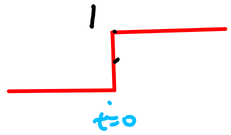
4. find $\boxed{\tau}$
5. find $\boxed{x(t) = x_h(t) + \overbrace{x_p(t)}^{x(\infty)}}$
6. find $\boxed{\alpha} = [x(0) - x(\infty)]$

@ this method works for DC input only

$$\boxed{\tau \frac{dx}{dt} + 1\, x(t) = K_s\, F}$$

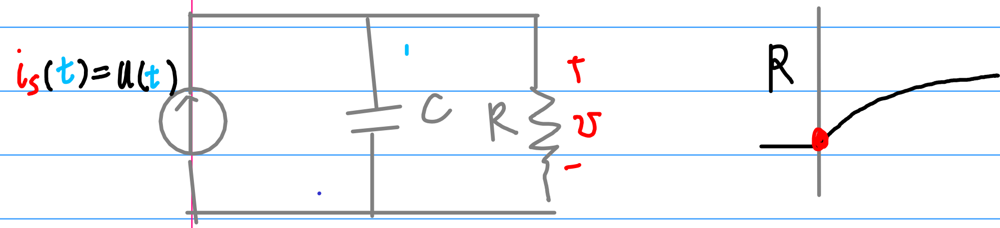
$$x(t) = [x(0) - x(\infty)]\, e^{-t/\tau} + x(\infty) \qquad t \geq 0$$

DC input

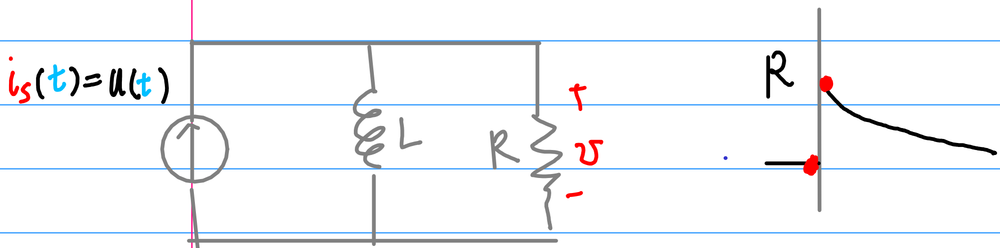


$$u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$

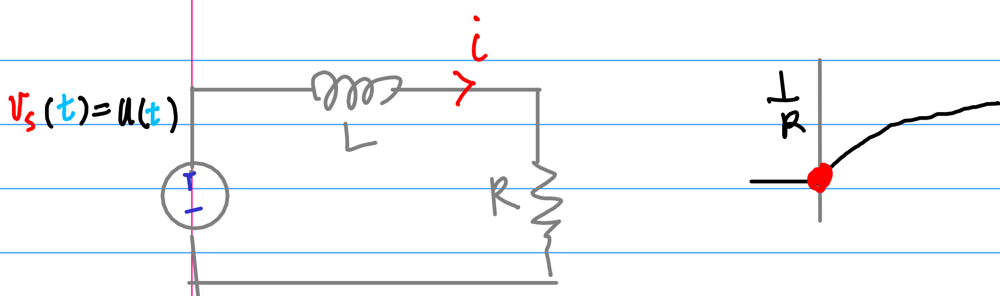
step function



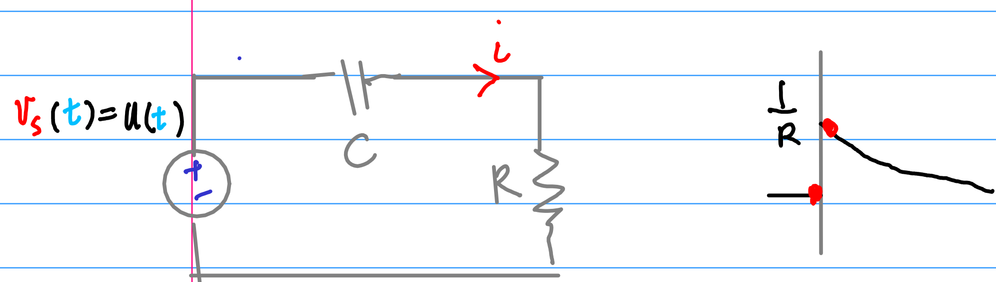
$$v = R(1 - e^{-t/RC})$$



$$v = R\, e^{-(R/L)t}$$

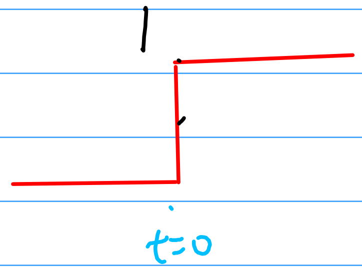


$$i = \frac{1}{R}(1 - e^{-(R/L)t})$$

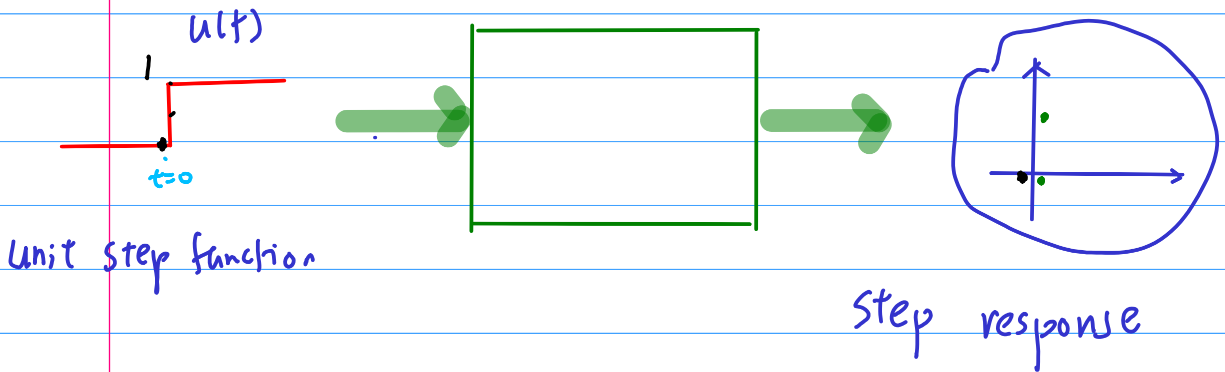


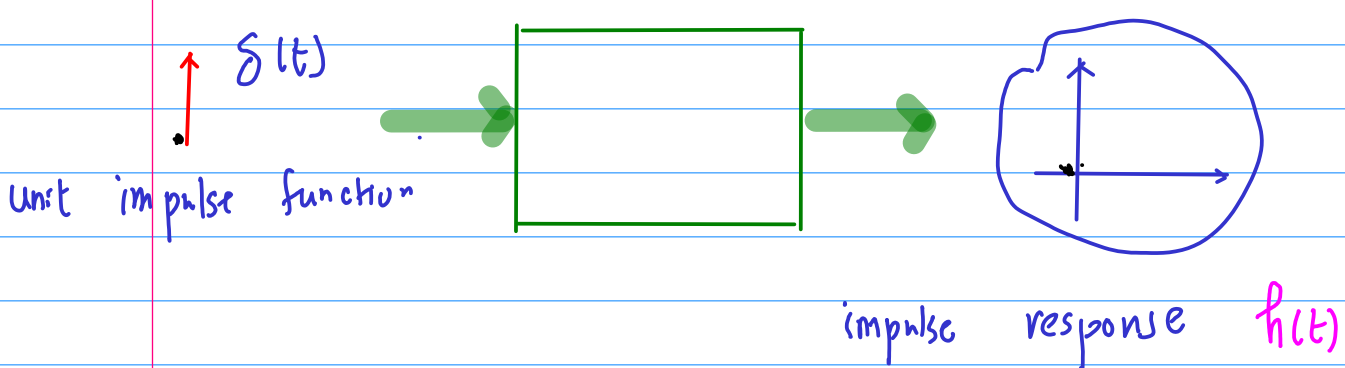
$$i = \frac{1}{R} e^{-t/RC}$$

Unit step function



$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

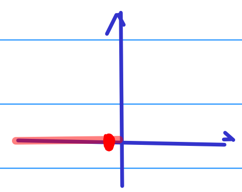


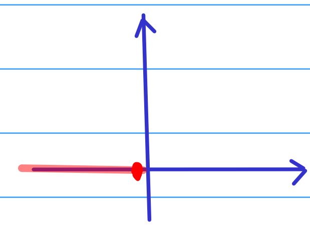


Zero State Response (ZSR)

$t<0$ input $x(t)=0$ no initial condition.

output $y(t)=0$





$t \geq 0$ 일때 non-zero input $x(t)$에 의한 output response

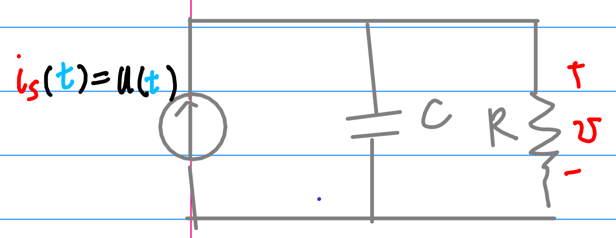
Zero Input Response (ZIR)

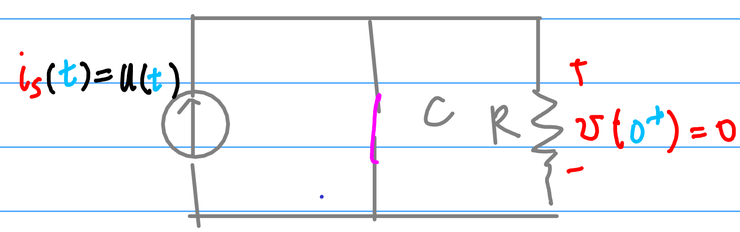
$t>0$ input $x(t)=0$ no input

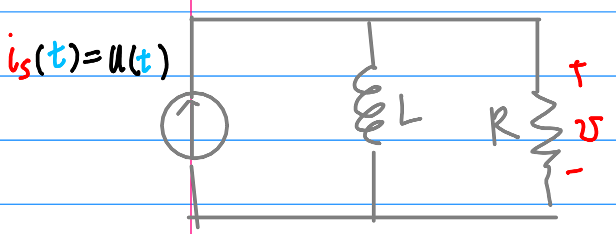
non-zero initial condition 으로 인한 output $t>0$ 나타남

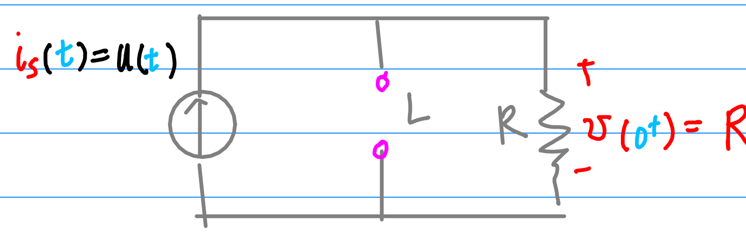
→ transient response 일부분.

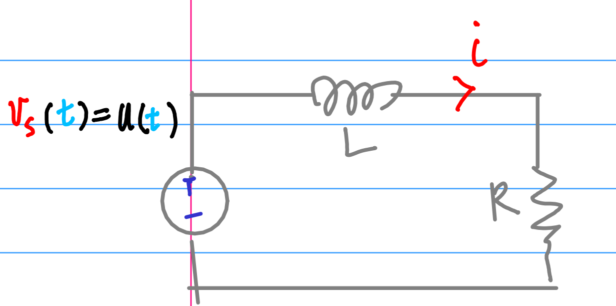
$t=0^+$

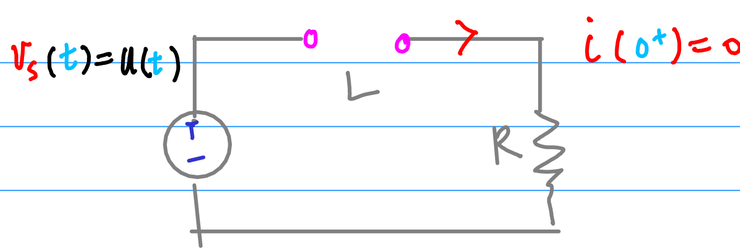

$i_s(t)=u(t)$
C
R
+
v
−

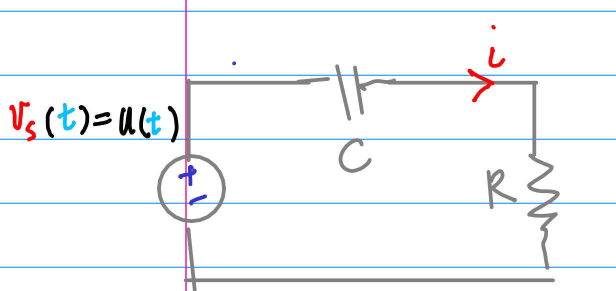

$i_s(t)=u(t)$
C
R
+
$v(0^+)=0$
−

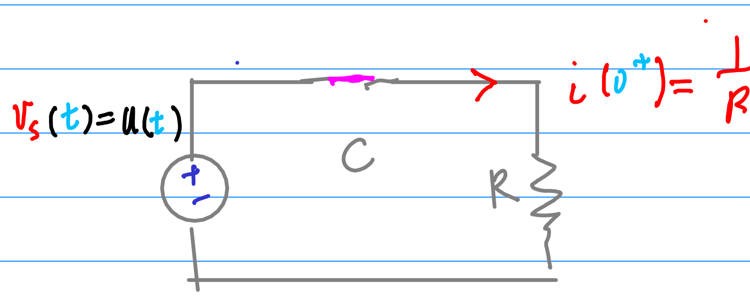

$i_s(t)=u(t)$
L
R
+
v
−


$i_s(t)=u(t)$
L
R
+
$v(0^+)=R$
−


i
$v_s(t)=u(t)$
L
R
+
−

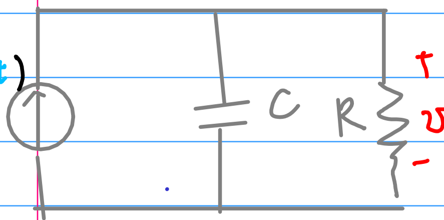

$v_s(t)=u(t)$
$i(0^+)=0$
L
R
+
−


i
$v_s(t)=u(t)$
C
R
+
−

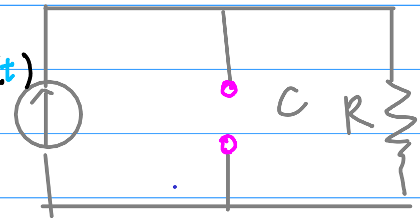

$v_s(t)=u(t)$
$i(0^+)=\frac{1}{R}$
C
R
+
−

$t=\infty$

$i_s(t)=u(t)$

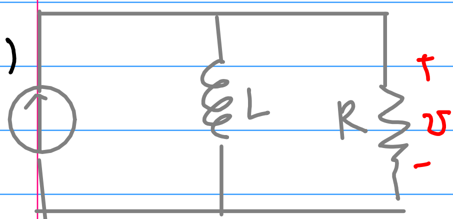


$i_s(t)=u(t)$

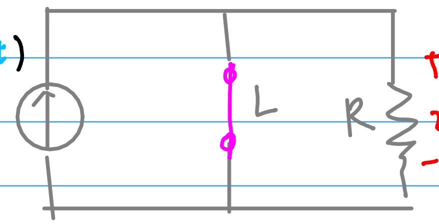


$v(\infty)=R$

$i_s(t)=u(t)$

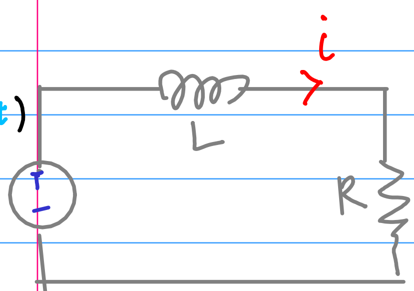


$i_s(t)=u(t)$

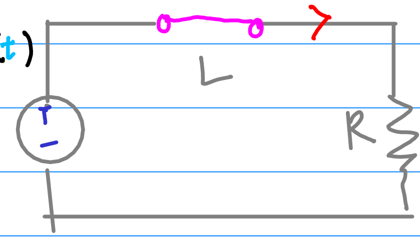


$v(\infty)=0$

$v_s(t)=u(t)$

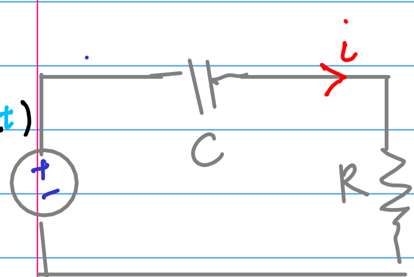


$v_s(t)=u(t)$

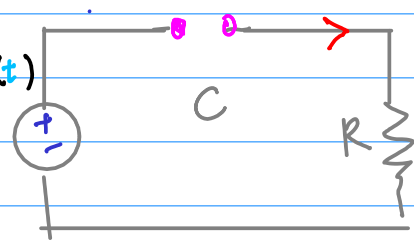


$i(\infty)=\frac{1}{R}$

$v_s(t)=u(t)$



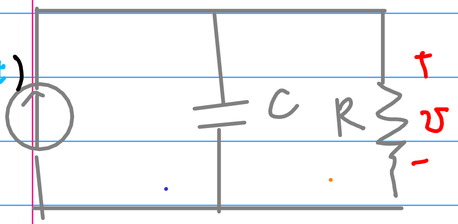
$v_s(t)=u(t)$



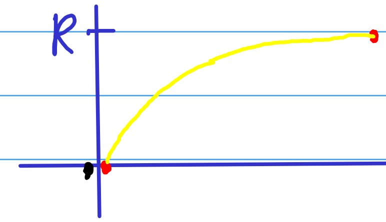
$i(\infty)=0$

$v(0^+)$ & $v(\infty)$

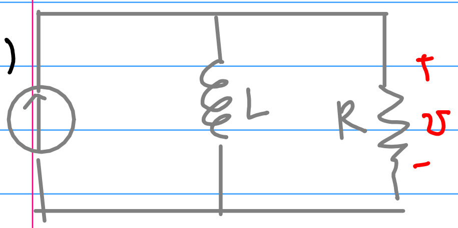
$i_s(t)=u(t)$



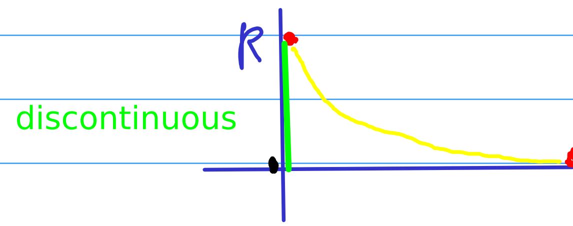
$v(0^+)=0$ $\quad$ $v(\infty)=R$



$i_s(t)=u(t)$

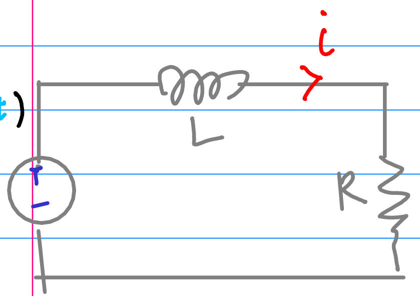


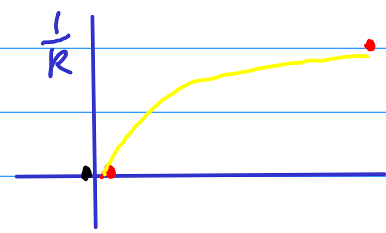
$v(0^+)=R$ $\quad$ $v(\infty)=0$



$i(0^+)=0$ $\quad$ $i(\infty)=\frac{1}{R}$

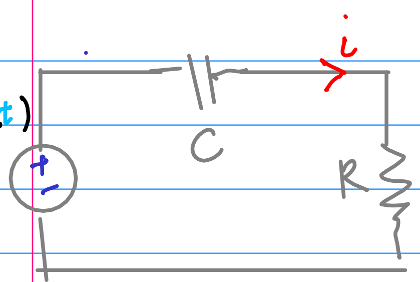
$v_s(t)=u(t)$

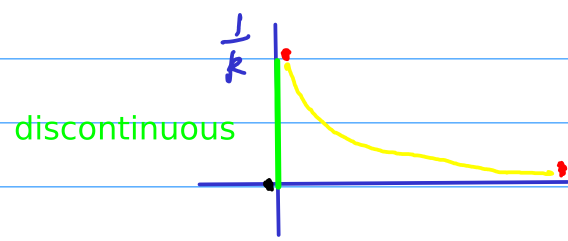




$i(0^+)=\frac{1}{R}$ $\quad$ $i(\infty)=0$

$v_s(t)=u(t)$



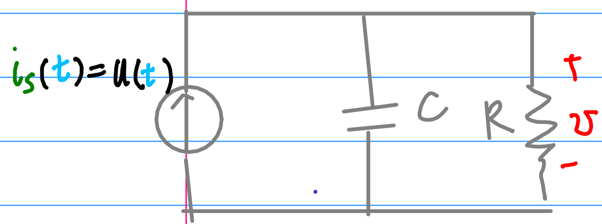


ODE

Zero State Response

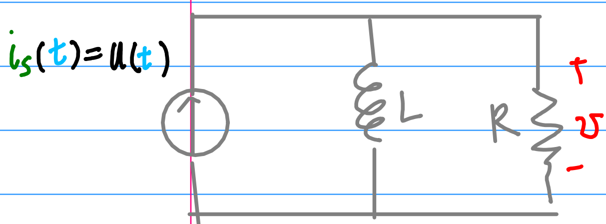
init cond$(0^+) = 0$

$\boxed{t > 0}$



$$\boxed{C\frac{dv}{dt} + \frac{v}{R} = i_s}$$

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{1}{C} i_s$$

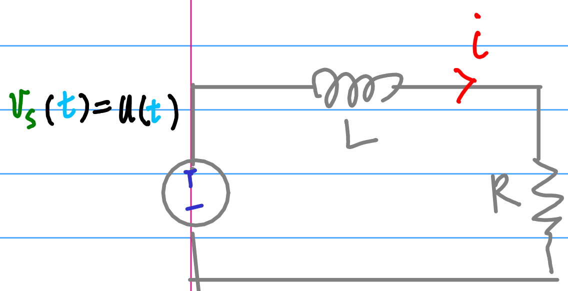


$$L\frac{di_L}{dt} = v_L \qquad i_L = \frac{1}{L}\int v_L(t)\,dt$$

$$\frac{1}{L}\int_0^t v\,dt + \frac{v}{R} = i_s$$

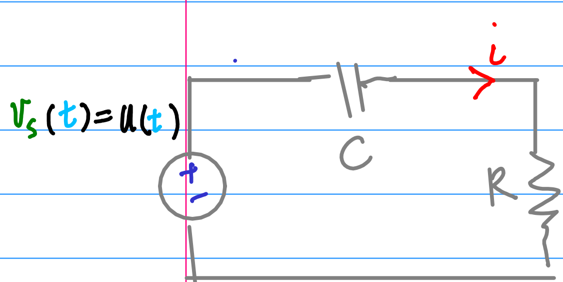
$$\boxed{\frac{1}{L} v + \frac{1}{R}\frac{dv}{dt} = \frac{di_s}{dt}}$$

$$\frac{dv}{dt} + \frac{R}{L} v = R\frac{di_s}{dt}$$



$$\boxed{L\frac{di}{dt} + Ri = v_s}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{1}{L} v_s$$



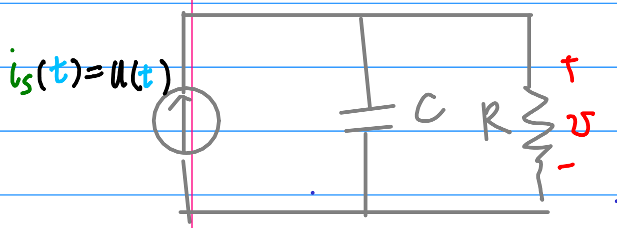
$$C\frac{dv_c}{dt} = i_c \qquad v_c = \frac{1}{C}\int_0^t i_c\,dt$$

$$\frac{1}{C}\int_0^t i\,dt + Ri = v_s$$

$$\boxed{\frac{1}{C} i + R\frac{di}{dt} = \frac{dv_s}{dt}}$$

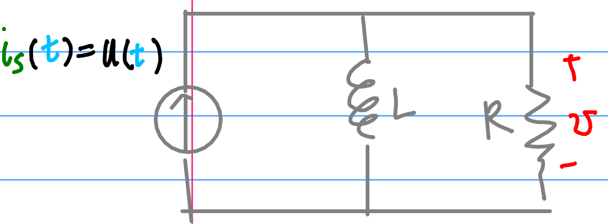
$$\frac{di}{dt} + \frac{1}{RC} i = \frac{1}{R}\frac{dv_s}{dt}$$

ODE for DC input



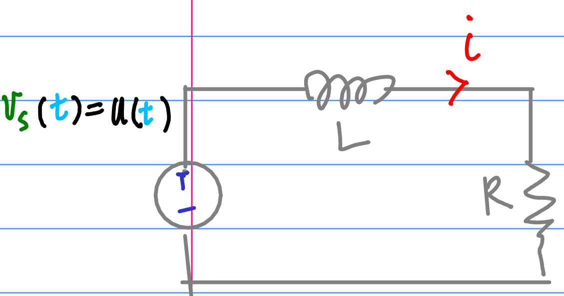
$$\boxed{\frac{dv}{dt} + \frac{v}{RC} = \frac{1}{C} i_s}$$

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{1}{C}$$



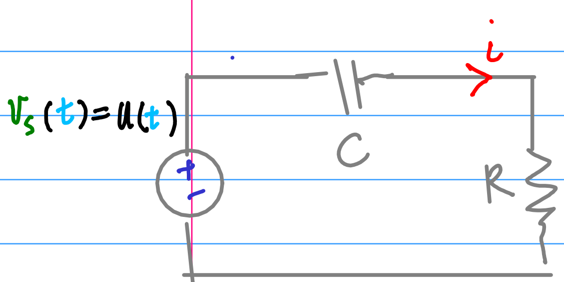
$$\boxed{\frac{dv}{dt} + \frac{R}{L} v = R \frac{di_s}{dt}}$$

$$\frac{dv}{dt} + \frac{R}{L} v = 0^{*}$$



$$\boxed{\frac{di}{dt} + \frac{R}{L} i = \frac{1}{L} v_s}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{1}{L}$$

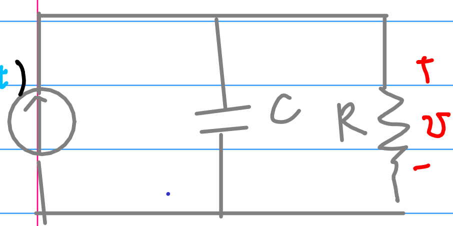


$$\boxed{\frac{di}{dt} + \frac{1}{RC} i = \frac{1}{R} \frac{dv_s}{dt}}$$

$$\frac{di}{dt} + \frac{1}{RC} i = 0^{*}$$

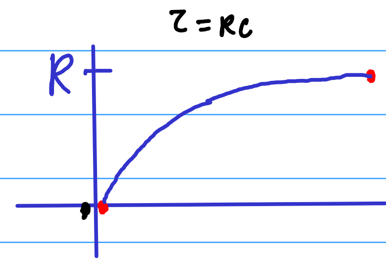
1. $\boxed{\tau \frac{dx}{dt} + 1\,x(t) = K_s\,F}$ $\qquad x(t) = \left[x(0) - x(\infty)\right] e^{-t/\tau} + x(\infty) \quad t \geq 0$

$i_s(t) = u(t)$

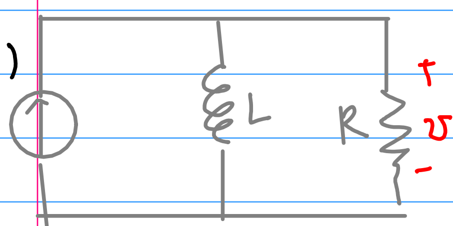


$$\frac{dv}{dt} + \frac{v}{RC} = \frac{1}{C}$$

$$v(t) = [0 - R]\,e^{-t/RC} + R$$

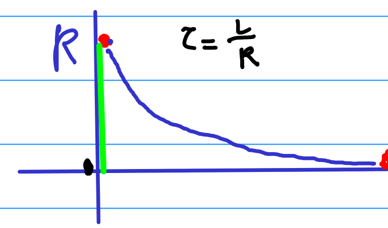


$i_s(t) = u(t)$

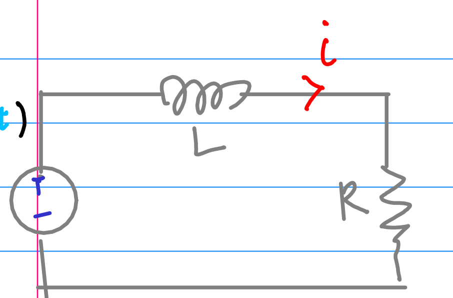


$$\frac{dv}{dt} + \frac{R}{L}\,v = 0^{*}$$

$$v(t) = [R - 0]\,e^{-t(L/R)} + 0$$

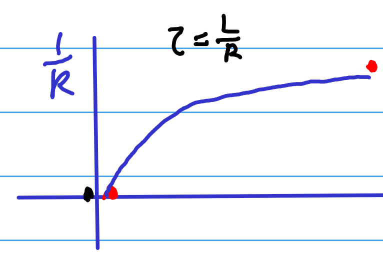


$V_s(t) = u(t)$

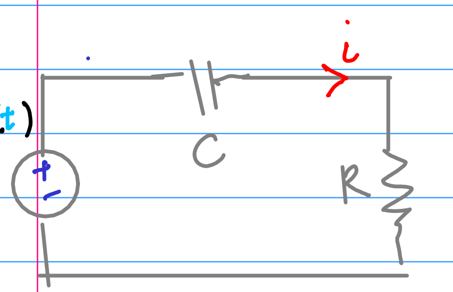


$$\frac{di}{dt} + \frac{R}{L}\,i = \frac{1}{L}$$

$$i(t) = [0 - R]\,e^{-t(L/R)} + R$$

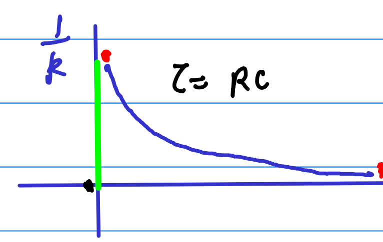


$V_s(t) = u(t)$



$$\frac{di}{dt} + \frac{1}{RC}\,i = 0^{*}$$

$$i(t) = [R - 0]\,e^{-t/RC} + 0$$



Classical Approach for solving ODE.

$y' + a y = g(t)$ 1st order Linear ODE

$m + a = 0$ aux. eq

$y_h(t) = C_1 e^{-a \cdot t}$

$t > 0$ step function input $g(t) \rightarrow$ Const ①

$y_p(t) = A$

$a \cdot A = 1$ $A = \frac{1}{a}$

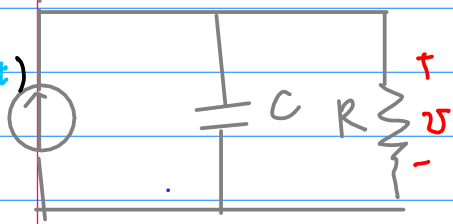
$y(t) = y_h(t) + y_p(t)$

$y(t) = C_1 e^{-a \cdot t} + \frac{1}{a}$

* $y(0^+) = C_1 e^{-a \cdot 0} + \frac{1}{a}$ ← determine C_1

2 $\boxed{y_h + y_p}$

$i_s(t) = u(t)$

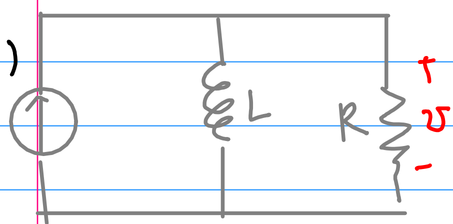


$$\frac{dv}{dt} + \frac{v}{RC} = \frac{1}{C} \qquad v_h(t) = c_1 e^{-t/RC}$$

$$\frac{v_p}{RC} = \frac{1}{C} \qquad v_p(t) = R$$

$$v(0^+) = 0 \qquad c_1 = -R \qquad \boxed{v = R(1 - e^{-t/RC})}$$

$i_s(t) = u(t)$

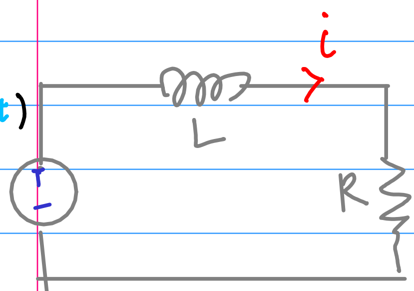


$$\frac{dv}{dt} + \frac{R}{L} v = 0 \; * \qquad v_h(t) = c_1 e^{-t(L/R)}$$

$$\frac{R}{L} v_p = 0 \qquad v_p(t) = 0$$

$$v(0^+) = R \qquad c_1 = R \qquad \boxed{v = R\, e^{-(R/L)t}}$$

$v_s(t) = u(t)$

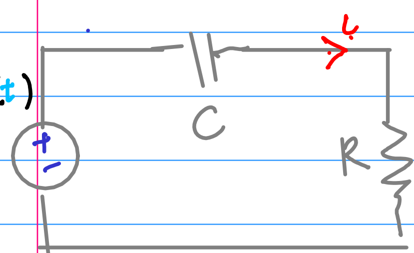


$$\frac{di}{dt} + \frac{R}{L} i = \frac{1}{L} \qquad i_h(t) = c_1 e^{-t(L/R)}$$

$$\frac{R}{L} i_p = \frac{1}{L} \qquad i_p(t) = \frac{1}{R}$$

$$i(0^+) = 0 \qquad c_1 = -\frac{1}{R} \qquad \boxed{i = \frac{1}{R}(1 - e^{-(R/L)t})}$$

$v_s(t) = u(t)$



$$\frac{di}{dt} + \frac{1}{RC} i = 0 \; * \qquad i_h(t) = c_1 e^{-t/RC}$$

$$\frac{1}{RC} i_p = 0 \qquad i_p(t) = 0$$

$$i(0^+) = \frac{1}{R} \qquad c_1 = \frac{1}{R} \qquad \boxed{i = \frac{1}{R} e^{-t/RC}}$$

Solving IVPs with Laplace Transform

$y' + a y = g(t)$ 1st order Linear ODE

$y(0^-)$ initial condition 0^-

Laplace Transform

$$sY(s) - y(0^-) + aY(s) = G(s)$$

$$(s+a)Y(s) = y(0^-) + G(s)$$

$$Y(s) = \underbrace{\frac{y(0^-)}{(s+a)}}_{ZIR} + \underbrace{\frac{G(s)}{(s+a)}}_{ZSR}$$

step function input $g(t) = u(t)$ $G(s) = \frac{1}{s}$

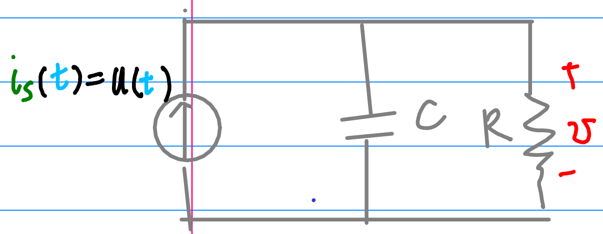
Zero State Response to the unit step input (ZSR)

$y(0^-) = 0$

$$Y(s) = \frac{1}{s(s+a)} = \frac{1}{a}\left(\frac{1}{s} - \frac{1}{s+a}\right)$$

$$y(t) = \frac{1}{a}\left(1 - e^{-at}\right)$$

③ ZSR using Laplace Transform



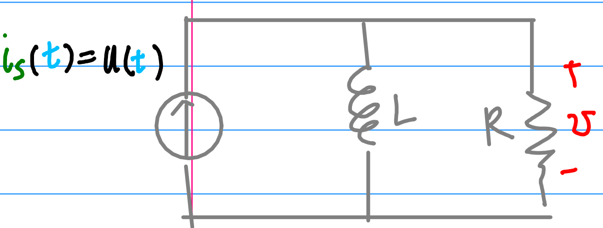
$$\boxed{\frac{dv}{dt} + \frac{v}{RC} = \frac{1}{C}u(t)}$$

$$v(0^-) = 0$$

$$V(s) = \frac{1}{C}\,\frac{1}{s\left(s+\frac{1}{RC}\right)}$$

$$= R\left(\frac{1}{s} - \frac{1}{s+\frac{1}{RC}}\right)$$

$$\boxed{v = R\left(1 - e^{-t/RC}\right)}$$

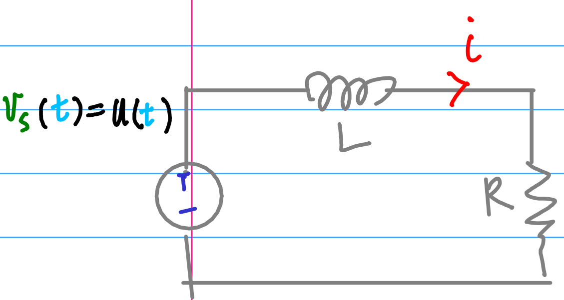


$$\boxed{\frac{dv}{dt} + \frac{R}{L}v = R\,\delta(t)}$$

$$v(0^-) = 0$$

$$V(s) = R\,\frac{1}{s+\frac{R}{L}}$$

$$\boxed{v = R\,e^{-(R/L)t}}$$



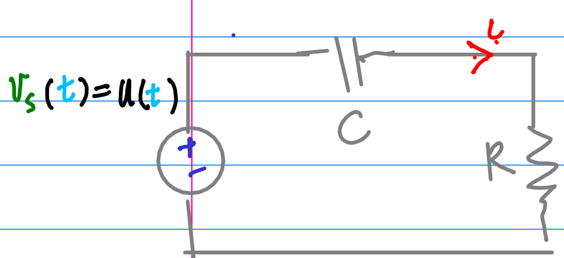
$$\boxed{\frac{di}{dt} + \frac{R}{L}i = \frac{1}{L}u(t)}$$

$$i(0^-) = 0$$

$$I(s) = \frac{1}{L}\,\frac{1}{s\left(s+\frac{R}{L}\right)}$$

$$= \frac{1}{R}\left(\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right)$$

$$\boxed{i = \frac{1}{R}\left(1 - e^{-(R/L)t}\right)}$$



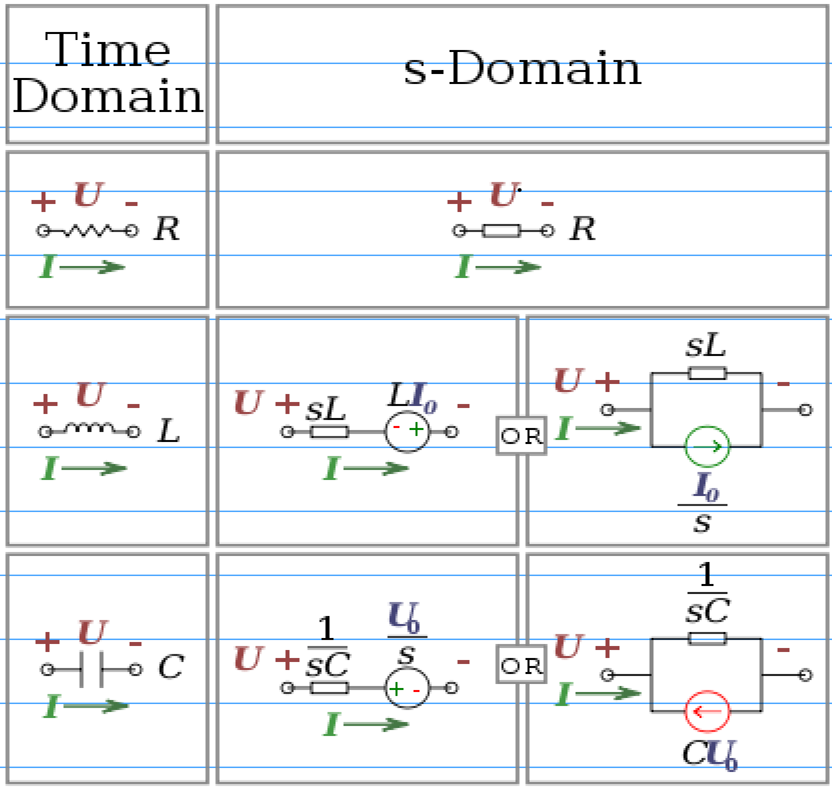
$$\boxed{\frac{di}{dt} + \frac{1}{RC}i = \frac{1}{R}\delta(t)}$$

$$i(0^-) = 0$$

$$I(s) = \frac{1}{R}\,\frac{1}{s+\frac{1}{RC}}$$

$$\boxed{i = \frac{1}{R}e^{-t/RC}}$$

S-domain Impedance



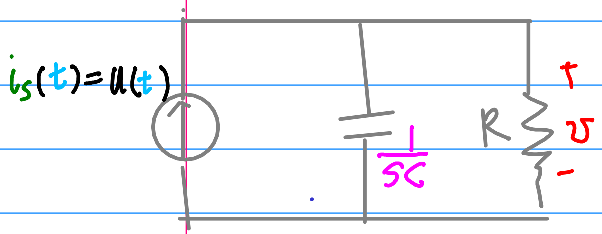
wikipedia.org

non-zero initial conditions

ZSR using s-domain impedance

Element	Impedance expression
Resistor	R
Inductor	sL
Capacitor	$\frac{1}{sC}$

④ ZSR using s-domain impedance

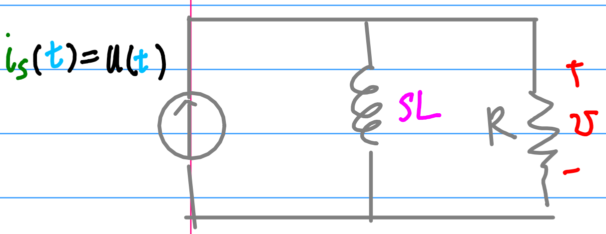


$$\frac{1}{sC} \| R = \frac{1}{sC + \frac{1}{R}}$$

$$\boxed{Z = \frac{1}{C}\frac{1}{(s+\frac{1}{RC})}}$$

$$V(s) = \frac{1}{C}\frac{1}{s(s+\frac{1}{RC})} = R\left(\frac{1}{s} - \frac{1}{s+\frac{1}{RC}}\right)$$

$$\boxed{v = R(1-e^{-t/RC})}$$

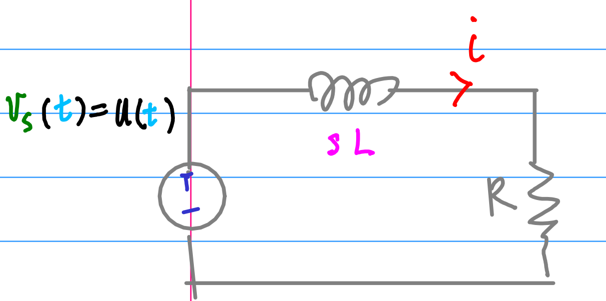


$$sL \| R = \frac{1}{\frac{1}{sL}+\frac{1}{R}} = \frac{sLR}{sL+R}$$

$$\boxed{Z = R\frac{s}{s+\frac{R}{L}}}$$

$$V(s) = R\frac{1}{s+\frac{R}{L}}$$

$$\boxed{v = R e^{-(R/L)t}}$$

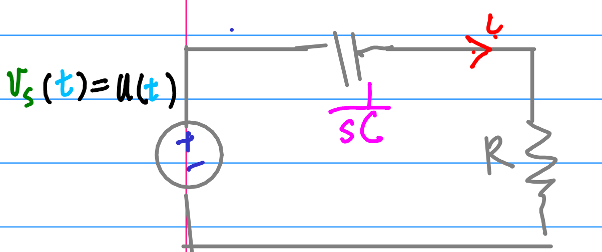


$$sL + R = L\left(s+\frac{R}{L}\right)$$

$$\boxed{Y = \frac{1}{L}\frac{1}{(s+\frac{R}{L})}}$$

$$I(s) = \frac{1}{L}\frac{1}{s(s+\frac{R}{L})} = \frac{1}{R}\left(\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right)$$

$$\boxed{i = \frac{1}{R}(1-e^{-(R/L)t})}$$



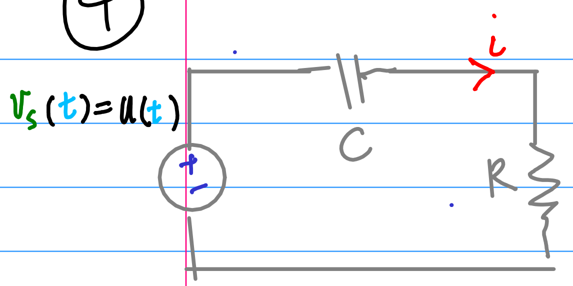
$$\frac{1}{sC} + R = \frac{1}{sC}(1+sRC) = \frac{R}{s}\left(\frac{1}{RC}+s\right)$$

$$\boxed{Y = \frac{1}{R}\frac{s}{s+\frac{1}{RC}}}$$

$$I(s) = \frac{1}{R}\frac{1}{s+\frac{1}{RC}}$$

$$\boxed{i = \frac{1}{R}e^{-t/RC}}$$

(4)

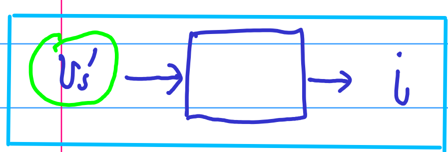


$$C\frac{dv_c}{dt} = i_c \qquad v_c = \frac{1}{C}\int_0^t i_c\,dt$$

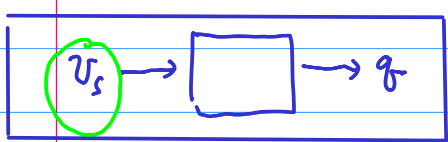
$$\frac{1}{C}\int_0^t i\,dt + Ri = v_s$$

$$\frac{1}{C}i + R\frac{di}{dt} = \frac{dv_s}{dt}$$

$$\boxed{\frac{di}{dt} + \frac{1}{RC}i = \frac{1}{R}\frac{dv_s}{dt}}$$



$$\boxed{\frac{dq}{dt} + \frac{1}{RC}q = \frac{1}{R}v_s}$$



$$q_h(t) = c_1 e^{-\frac{t}{RC}}$$

$$q_p(t) = A \qquad 0 + \frac{1}{RC}A = \frac{1}{R}$$

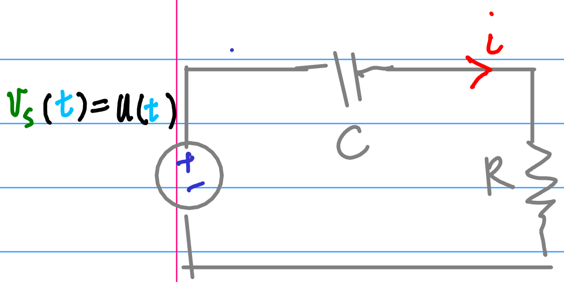
$$= C$$

$$q(t) = c_1 e^{-\frac{t}{RC}} + C$$

$$q(0^+) = c_1 e^{-\frac{0^+}{RC}} + C = 0 \qquad c_1 = -C$$

$$q(t) = C\left(1 - e^{-\frac{t}{RC}}\right)$$

$$i(t) = q'(t) = \frac{1}{R}e^{-\frac{t}{RC}}$$



$$C\frac{dv_c}{dt}=i_c \qquad v_c=\frac{1}{C}\int_0^t i_c\,dt$$

$$\frac{1}{C}\int_0^t i\,dt + R\,i = v_s$$

$$\frac{1}{C}\,i + R\frac{di}{dt} = \frac{dv_s}{dt}$$

$$\frac{di}{dt} + \frac{1}{RC}\,i = \frac{1}{R}\frac{dv_s}{dt}$$

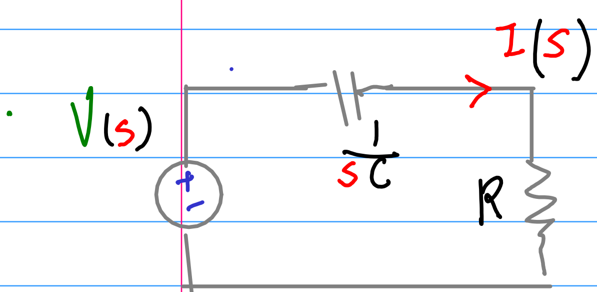
$$s\,I(s) - i(0^-) + \frac{1}{RC}I(s) = \frac{1}{R}\left(s\,V(s) - v(0^-)\right)$$

$$\left(s+\frac{1}{RC}\right)I(s) = \frac{1}{R}\,s\,\frac{1}{s}$$

$$I(s) = \frac{1}{R}\,\frac{1}{\left(s+\frac{1}{RC}\right)}$$

$$i(t) = \frac{1}{R}\,e^{-\frac{t}{RC}}$$

$$\frac{1}{sC} \qquad sL$$



$$I(s) = \frac{V(s)}{\left(\frac{1}{sC}+R\right)} = \frac{\frac{1}{s}}{\left(\frac{1}{sC}+R\right)}$$

$$\frac{1/R}{\frac{1}{RC}+s}$$

$$\frac{1}{R}e^{-\frac{t}{RC}}$$

output

input

$$y^{(N)} + a_1 y^{(N-1)} + \cdots + a_{N-1} y^{(1)} + a_N y = x$$

base system

$$x(t) \longrightarrow \boxed{h_0(t)} \longrightarrow y(t)$$

notation: $y^{(N)} = \dfrac{d^N y}{dt^N} = \dfrac{d^N}{dt^N} y(t)$

$$x'(t) \longrightarrow \boxed{h_0(t)} \longrightarrow y'(t)$$

$$x''(t) \longrightarrow \boxed{h_0(t)} \longrightarrow y''(t)$$

$$x^{(3)}(t) \longrightarrow \boxed{h_0(t)} \longrightarrow y^{(3)}(t)$$

$$y'' + a y' + b y(t) = x(t)$$

$$y''' + a y'' + b y'(t) = x'(t)$$

$$\frac{d^2}{dt^2}\left(\frac{dy}{dt}\right) + a \frac{d}{dt}\left(\frac{dy}{dt}\right) + b\left(\frac{dy}{dt}\right) = \left(\frac{dx}{dt}\right)$$

$$y'' + a y' + b y(t) = x(t)$$