Propositional Logic– Arguments (5B)

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11/30/16

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Based on

Contemporary Artificial Intelligence,
R.E. Neapolitan & X. Jiang

Logic and Its Applications,
Burkey & Foxley

Logical Consequences

$\mathbf{A} \vdash_{s} \mathbf{B}$	Syntactic Consequences

there is a derivation, in the proof-system **S**,
from the premise **A** to the conclusion **B**.
[If context fixes the relevant system S, we suppress the subscript.]

$\mathbf{A} \vDash_{L} \mathbf{B}$	Semantic Consequences

Logical Implication

on every possible interpretation of the non-logical vocabulary of language **L**, if **A** comes out true, so does **B**.
[If context fixes the relevant language L we suppress the subscript.]

A → B	Material Implication

on the truth-functional interpretation,
if the atomic wff **A** happens to be false and
the atomic wff **B** happens to the false too,
then **A→B** evaluates as true.
But we don't have **A ⊨ B**
(q isn't true on every valuation which makes p true).

A	B	A → B
True	True	True
True	False	False
False	True	True
False	False	True

http://math.stackexchange.com/questions/365569/whats-the-difference-between-syntactic-consequence-%E2%8A%A2-and-semantic-consequence-%E2%8A%A8

Double Turnstile ⊨

1. semantic consequence:
a set of sentences on the left
a single sentence on the right
to denote that if every sentence on the left is true,
the sentence on the right must be true, e.g. **Γ ⊨ φ**.
This usage is closely related to the single-barred turnstile
symbol which denotes syntactic consequence.

2. satisfaction:
a model (or truth-structure) on the left
a set of sentences on the right
to denote that the structure is a model
for (or satisfies) the set of sentences, e.g. **A ⊨ Γ** .

3. a tautology: **⊨ φ**
to say that the expression φ is
a semantic consequence of the empty set.

https://en.wikipedia.org/wiki/Double_turnstile

Syntactic Consequences

A formula **A** is a syntactic consequence
within some formal system **FS** of a set **Γ** of formulas
if there is a formal proof in **FS** of **A** from the set **Γ**.

$\mathbf{\Gamma} \vdash_{FS} \mathbf{A}$

- Sound argument
- Fallacy

Syntactic consequence does not depend on any
interpretation of the formal system.

A formal proof or derivation is
a finite sequence of sentences (called **wwf**),

each of which is an axiom, an assumption, or
which follows from the preceding sentences in the sequence
by a **rule of inference**.

The last sentence in the sequence is
a **theorem** of a formal system.

https://en.wikipedia.org/wiki/Double_turnstile

Semantic Consequences

A formula **A** is a semantic consequence within some formal system **FS** of a set of statements **Γ**

$$\Gamma \vDash_{FS} A$$

if and only if there is *no model **I***
in which all members of **Γ** are **true** and **A** is **false**.

the **set** of the interpretations that make all members of **Γ** **true**
is a **subset** of the **set** of the interpretations that make **A** **true**.

https://en.wikipedia.org/wiki/Double_turnstile

Summary

Syntactic consequence $\Gamma \vdash \varphi$

sentence **φ** is **provable** from the set of assumptions **Γ**.

Semantic consequence $\Gamma \models \varphi$

sentence **φ** is **true in all models of Γ**.

Soundness **If [$\Gamma \vdash \varphi$] then [$\Gamma \models \varphi$].**

Completeness **If [$\Gamma \models \varphi$] then [$\Gamma \vdash \varphi$].**

The propositional logic has

a proof system
(propositional calculus)
Syntactic consequences

a semantics
(truth-tables)
Semantic consequences

http://philosophy.stackexchange.com/questions/10785/semantic-vs-syntactic-consequence

Syntactic and Semantic Consequences (1)

Syntactic consequence Γ ⊢ φ

sentence **φ** is **provable** from the set of assumptions **Γ**.

$$\frac{A \qquad A \Rightarrow B}{B}$$

Semantic consequence Γ ⊨ φ

sentence **φ** is **true in all models of Γ**.

A	B	A⇒B	A∧(A⇒B)		A∧(A⇒B)⇒B
T	T	T	**T**	⇒	**T**
T	F	F	F		T
F	T	T	F		T
F	F	T	F		T

http://philosophy.stackexchange.com/questions/10785/semantic-vs-syntactic-consequence

Syntactic and Semantic Consequences (2)

A, A→B ⊢ **B** **Syntactic consequence Γ ⊢ φ**

if we take the assumptions A and A→B as given,
by **modus ponens** we can deduce B.

A, A→B ⊨ **B** **Semantic consequence Γ ⊨ φ**

in any model for which it is the case
that A is **true** and also A→B is **true**,
then, in that model, B is also **true**.

⊢ talks about the propostions themselves as syntactic objects,

⊨ talks about what the propositions mean i.e. semantics.

https://www.quora.com/What-are-the-differences-between-semantic-consequence-and-syntactic-consequence-in-logic

Soundness and Completeness (1)

Sound Deduction System :

if it derives only sound arguments

each of the inference rules is sound

Soundness. If [$\Gamma \vdash \varphi$] then [$\Gamma \vDash \varphi$].

A sound **argument:**

If the **premises** **entails** the **conclusion**

A **fallacy:**

If the **premises** does **not** **entail** the **conclusion**

Complete Deduction System :

It can drive every sound argument

must contain deduction theorem rule

Completeness. If [$\Gamma \vDash \varphi$] then [$\Gamma \vdash \varphi$].

Soundness and Completeness (2)

Soundness is the property of only being able **to prove "true" things**.

Completeness is the property of being able **to prove all true things**.

So a given logical system is **sound** if and only if
the inference rules of the system admit only valid formulas.
Or another way, if we start with valid premises,
the inference rules do not allow an invalid conclusion to be drawn.

A system is **complete** if and only if

all valid formula can be derived from the axioms and the inference rules.
So there are no valid formula that we can't prove.

http://philosophy.stackexchange.com/questions/6992/the-difference-between-soundness-and-completeness

Invalid Argument Examples

P1 Grass is green
P2 Paris is the capital of France
C A poodle is a dog

P1, P2 and C all true, but argument not deductively valid.

If you object that this doesn't count as an argument because there is *no connection* between the Ps or between the Ps and the C, try

P1 All atoms are tiny
P2 The smallest particle of hydrogen gas is tiny
C The smallest particle of hydrogen gas is an atom

All true, but not deductively valid.
To see this, substitute 'oxygen' for 'hydrogen'
(the smallest part of oxygen gas is a molecule not an atom, so C false)
or 'pollen' for 'hydrogen gas' (the smallest particle of pollen is a grain, C false)

https://askaphilosopher.wordpress.com/2012/02/08/invalid-argument-with-true-premisses-and-true-conclusion/

Unsound Argument Examples

P1 Craig is a Scot
P2 All Scots are drunks
C Craig is a drunk

Here, P1 can be true, C follows from the Ps (validity),
C can be true, but the argument is unsound because P2 is false.
So, although the C is true we can't rely on the argument to establish it.
It is an unsound argument.

P1,	P2		C
T,	T	|=	T
T,	F	|=	T

https://askaphilosopher.wordpress.com/2012/02/08/invalid-argument-with-true-premisses-and-true-conclusion/

Invalid Argument Examples

- Either Elizabeth owns a Honda or she owns a Saturn. (True / False)
- Elizabeth does not own a Honda.
- Therefore, Elizabeth owns a Saturn.

A valid argument
even if one of the premises is actually false,
that *if they had been true the conclusion would have been true as well*

- All toasters are items made of gold. (False)
- All items made of gold are time-travel devices.
- Therefore, all toasters are time-travel devices.

A valid and unsound argument
even if one of the premises is actually false,
that if they had been true the conclusion would have been true as well

- No felons are eligible voters.(True)
- Some professional athletes are felons. (True)
- Therefore, some professional athletes are not eligible voters. (True)

A valid and sound argument

http://www.iep.utm.edu/val-snd/

Valid Argument Examples

- All tigers are mammals.
- No mammals are creatures with scales.
- Therefore, no tigers are creatures with scales.

A valid and sound argument **[P1,P2 ⊢ C], [P1, P2 ⊨ C]**

- All spider monkeys are elephants. (False)
- No elephants are animals. (False)
- Therefore, no spider monkeys are animals. (False)

A valid but unsound argument **[P1,P2 ⊢ C], [P1, P2 ⊭ C]**

These arguments share the same form: **A valid arguments**, **Syntactic Consequences**

- All A are B;
- No B are C;
- Therefore, No A are C.

http://www.iep.utm.edu/val-snd/

Invalid Argument Examples

- All basketballs are round.
- The Earth is round.
- Therefore, the Earth is a basketball.

An invalid and unsound argument **[P1,P2 $\nvdash$ C], [P1, P2 $\nvDash$ C]**

- All popes reside at the Vatican. (True)
- John Paul II resides at the Vatican. (True)
- Therefore, John Paul II is a pope. (True)

An invalid and unsound argument **[P1,P2 $\nvdash$ C], [P1, P2 $\nvDash$ C]**

These arguments also have the same form: **an invalid arguments**

- All A's are F;
- X is F;
- Therefore, X is an A.

http://www.iep.utm.edu/val-snd/

Syntactic and Material Consequences

The most widely prevailing view on how to best account for logical consequence is to appeal to formality. This is to say that whether statements follow from one another logically depends on the structure or logical form of the statements without regard to the contents of that form.

Syntactic accounts of logical consequence rely on schemes using inference rules. For instance, we can express the logical form of a valid argument as:

All A are B.
All C are A.
Therefore, all C are B.

This argument is formally valid, because every instance of arguments constructed using this scheme are valid.

This is in contrast to an argument like "Fred is Mike's brother's son. Therefore Fred is Mike's nephew." Since this argument depends on the meanings of the words "brother", "son", and "nephew", the statement "Fred is Mike's nephew" is a so-called material consequence of "Fred is Mike's brother's son," not a formal consequence. A formal consequence must be true *in all cases*, however this is an incomplete definition of formal consequence, since even the argument "P is Q's brother's son, therefore P is Q's nephew" is valid in all cases, but is not a *formal* argument.[1]

https://en.wikipedia.org/wiki/Logical_consequence

Logical Equivalences

¬, ∧ ,
∨

¬, ∧ ,
∨

∧∨⌐¬
⇒⇔≡⇛
⊨

∧∨⌐¬
⇒⇔≡⇛
⊨

⇒
⇔
≡

⇒
⇔
≡

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