Logic (H.1)

20170403

Conjunction $\wedge$ AND

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

all T $\rightarrow$ T

any F $\rightarrow$ F

Disjunction $\lor$ OR

p	q	$p \lor q$
T	T	T
T	F	F
F	T	F
F	F	F

all F $\rightarrow$ F

any T $\rightarrow$ T

Exclusive OR $\oplus$ XOR

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

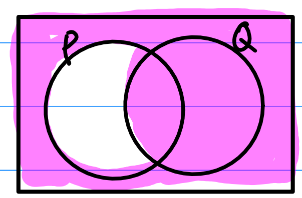
not both $\rightarrow$ T

Implication $\rightarrow$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$\sim p \vee q$

* material implication

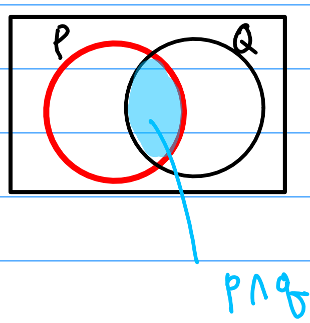


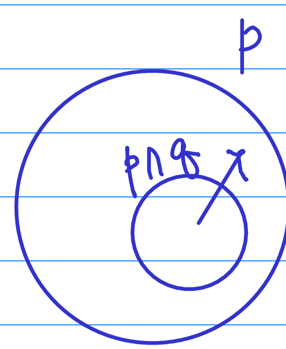
p	q	$p \wedge q$	p	$(p \wedge q) \rightarrow p$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	T
F	F	F	F	T

all true

Tautology

* logical implication





Tautology, Contradiction

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

any T → T
any T → T

tautology

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

any F → F
any F → F

Contradiction

neither tautology
nor contradiction → Contingency

Biconditionals $\longleftrightarrow$

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

De Morgan's Law

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

Distributive Law

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$$

$$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$$

$$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$$

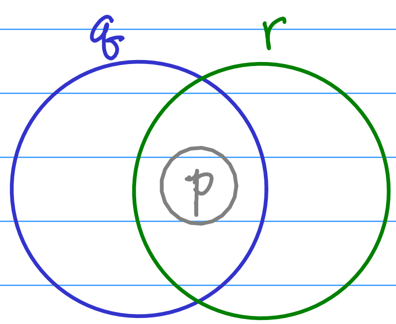
$$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$$

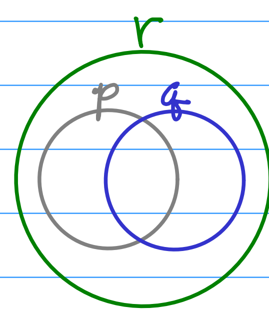
$$(p \to q) \land (p \to r) \equiv p \to (q \land r)$$

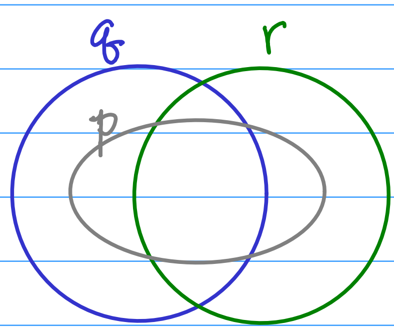
$$(p \to r) \land (q \to r) \equiv (p \lor q) \to r$$

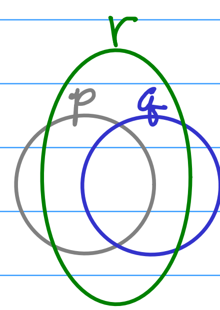
$$(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$$

$$(p \to r) \lor (q \to r) \equiv (p \land q) \to r$$



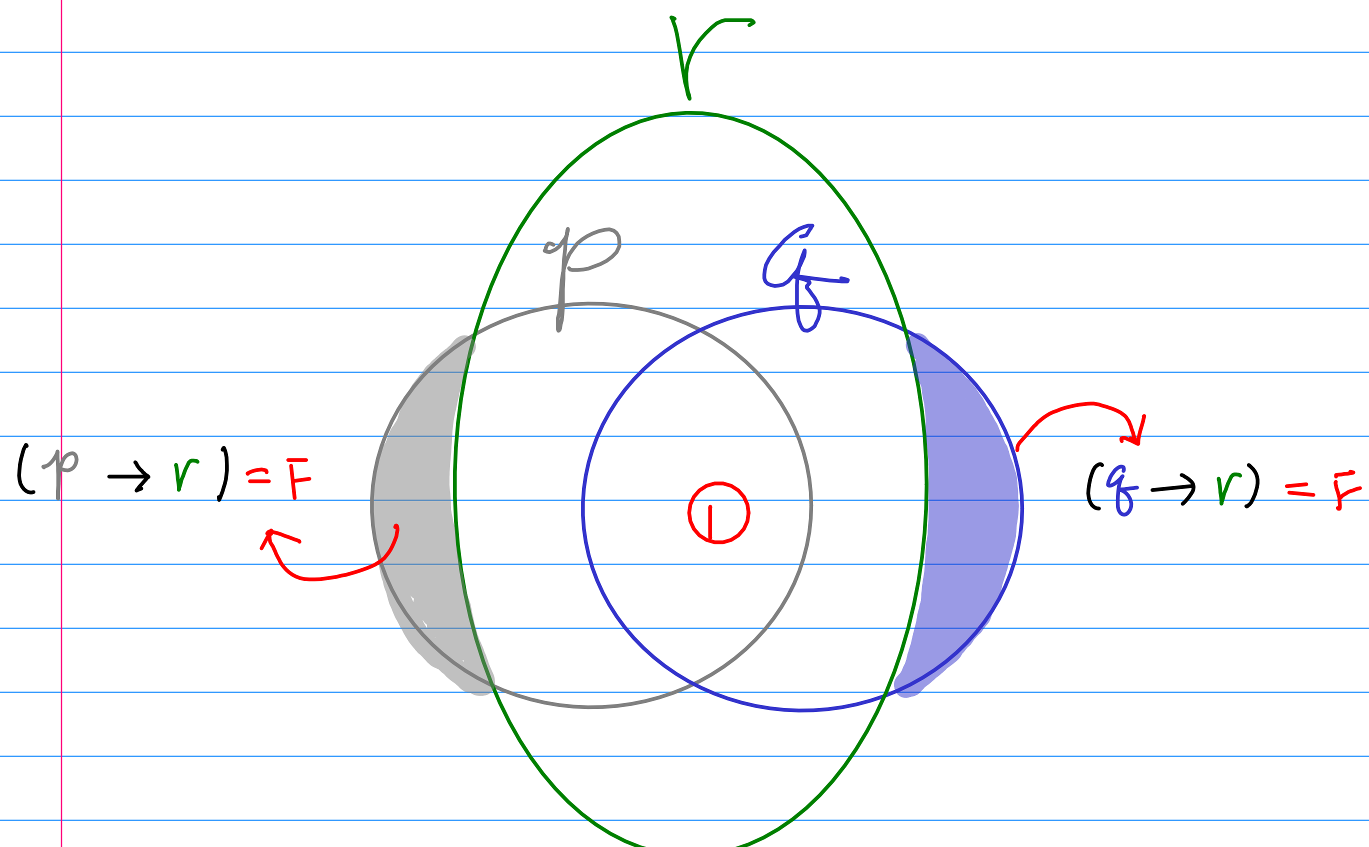






$$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$$

	$(p \to r)$	$(q \to r)$
1	T	T
2	T	F
3	F	T
	F	F



$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

when $p \wedge q$ = True $\Rightarrow$ when p = True & q = True $\Rightarrow$ $p = T$, $q = T$

when $\sim p \wedge \sim q$ = True $\Rightarrow$ when $\sim p$ = True & $\sim q$ = True $\Rightarrow$ $p = F$, $q = F$

	p	q	$p \leftrightarrow q$
when $p \wedge q$ = True $\Rightarrow$	T	T	T
	T	F	F
	F	T	F
when $\sim p \wedge \sim q$ = True $\Rightarrow$	F	F	T

$\Downarrow$

$p \leftrightarrow q$ becomes True

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

$p \leftrightarrow q$ becomes False when $p \wedge q$ = False

$p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$\sim p$	$\sim q$	$\sim p \leftrightarrow \sim q$
F	F	T
F	T	F
T	F	F
T	T	T

$\sim (p \leftrightarrow q) \equiv p \leftrightarrow \sim q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p	$\sim q$	$p \leftrightarrow \sim q$
T	F	F
T	T	T
F	F	T
F	T	F

$\sim (p \leftrightarrow q) \equiv \sim p \leftrightarrow q$

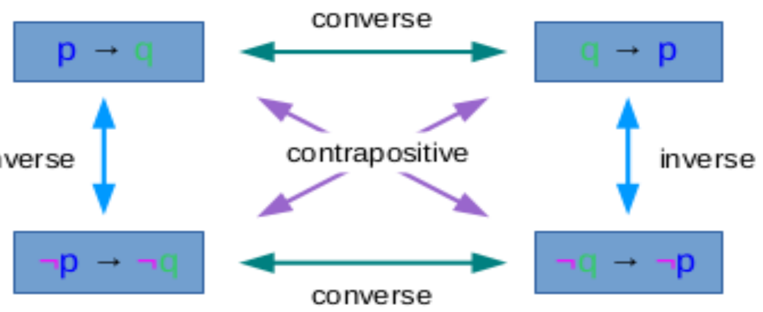
p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$\sim p \leftrightarrow q$
F	T	F
F	F	T
T	T	T
T	F	F

Comparison

from en.wikipedia.org

implication	if P then Q	first statement implies truth of second
inverse	if ~P then ~Q	negation of both statements
converse	if Q then P	reversal of both statements
contrapositive	if ~Q then ~P	reversal and negation of both
statements		
negation	P and ~Q	contradicts the implication



Contraposition

In logic, contraposition is a law, which says that a conditional statement is logically equivalent to its contrapositive.

The contrapositive of the statement has its antecedent and consequent **inverted** and **flipped**: the contrapositive of $P \rightarrow Q$ is thus $\neg Q \rightarrow \neg P$.

For instance, the **proposition** "*All bats are mammals*"

can be restated as the **conditional** "*If something is a bat, then it is a mammal*".

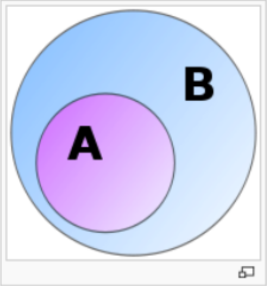
Now, the law says that statement is identical to the **contrapositive** "*If something is not a mammal, then it is not a bat.*"

Note that if $P \rightarrow Q$ is true and we are given that Q is false, $\neg Q$, it can logically be concluded that P must be false, $\neg P$.

This is often called the **law of contrapositive**, or the **modus tollens** rule of inference.

Contraposition

Consider the Euler diagram shown. According to this diagram, if something is in A, it must be in B as well. So we can interpret "all of A is in B" as:



$$A \to B$$

It is also clear that anything that is **not** within B (the white region) **cannot** be within A, either. This statement,

$$\neg B \to \neg A$$

is the contrapositive. Therefore we can say that

$$(A \to B) \to (\neg B \to \neg A).$$

Practically speaking, this may make life much easier when trying to prove something. For example, if we want to prove that every girl in the United States (A) is blonde (B), we can either try to directly prove $A \to B$ by checking all girls in the United States to see if they are all blonde. Alternatively, we can try to prove $\neg B \to \neg A$ by checking all non-blonde girls to see if they are all outside the US. This means that if we find at least one non-blonde girl within the US, we will have disproved $\neg B \to \neg A$, and equivalently $A \to B$.

To conclude, for any statement where A implies B, then *not B* always implies *not A*. Proving or disproving either one of these statements automatically proves or disproves the other. They are fully equivalent.

http://en.wikipedia.org/wiki/Derivative

Contraposition

$$\frac{\square \rightarrow \bigcirc}{\neg \bigcirc \rightarrow \neg \square}$$

Hypothetical Syllogism

$$\frac{\begin{array}{c}\square \rightarrow \bigcirc \\ \bigcirc \rightarrow \triangle\end{array}}{\square \rightarrow \triangle}$$

$$\left.\begin{array}{c} p \to q \\ \neg p \to r \\ r \to s \end{array}\right\} \text{premises}$$

$$\overline{}$$

$$\neg q \to s \quad \} \text{ conclusion}$$

$$\begin{array}{c} p \to q \\ \hline \neg q \to \neg p \end{array} \qquad \begin{array}{c} \boxed{p} \longrightarrow q \\ \hline \neg\, q \longrightarrow \neg \boxed{p} \end{array}$$

$$\boxed{\neg q} \longrightarrow \neg p$$

$$\neg p \to r \qquad \neg p \longrightarrow r$$

$$\neg q \to r \qquad \begin{array}{c} \hline \boxed{\neg q} \longrightarrow r \end{array}$$

$$\boxed{\neg q} \longrightarrow r$$

$$r \to s \qquad r \longrightarrow s$$

$$\neg q \to s \qquad \begin{array}{c} \hline \boxed{\neg q} \longrightarrow s \end{array}$$