Fourier Analysis Overview (0A)

- CTFS: Continuous Time Fourier Series
- CTFT: Continuous Time Fourier Transform
- DTFS: Discrete Time Fourier Series
- DTFT: Discrete Time Fourier Transform
- DFT: Discrete Fourier Transform

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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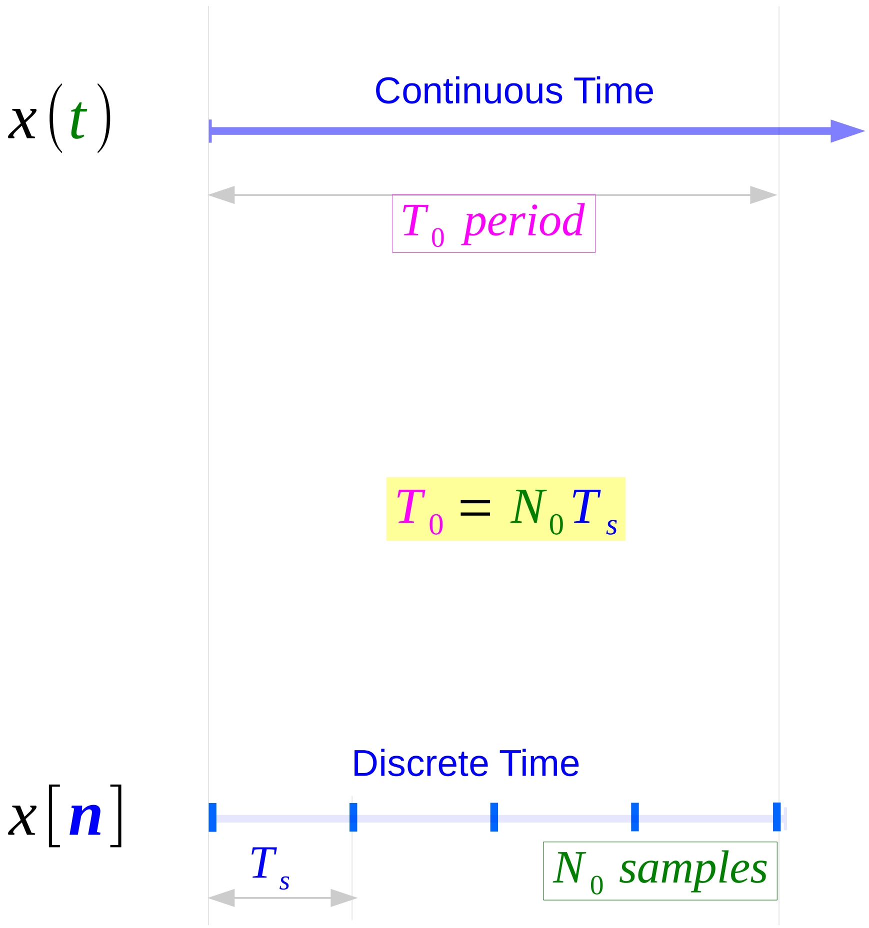
T_0 period and N_0 samples



frequency resolution

$$\omega_0 = \frac{2\pi}{T_0}$$

$$\omega_0 = \frac{2\pi}{N_0 T_s}$$

$\cdot T_s$

$$\frac{\omega_0}{1/T_s} = \frac{2\pi}{N_0 \cancel{T_s}} \cancel{T_s}$$

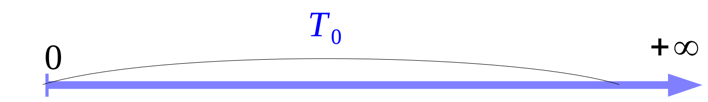
$\cdot f_s$

$$\hat{\omega}_0 = \frac{2\pi}{N_0}$$

normalized frequency resolution

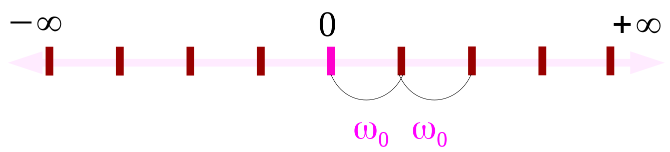
Periods and Resolutions $T_0\,,\ N_0$ & $\omega_0\,,\ \hat{\omega}_0$

CTFS



Continuous Time

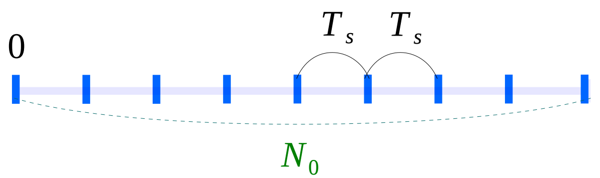
$\boxed{period:\ T_0\ seconds}$ $\quad T_0 = N_0 T_s$



Discrete Frequency

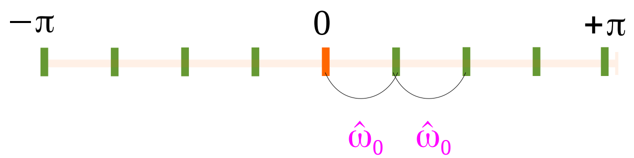
$$\omega_0 = \frac{2\pi}{T_0}$$

DTFS / DFT



Discrete Time

$\boxed{period:\ N_0\ samples}$



Normalized
Discrete Frequency

$$\hat{\omega}_0 = \frac{2\pi}{N_0}$$

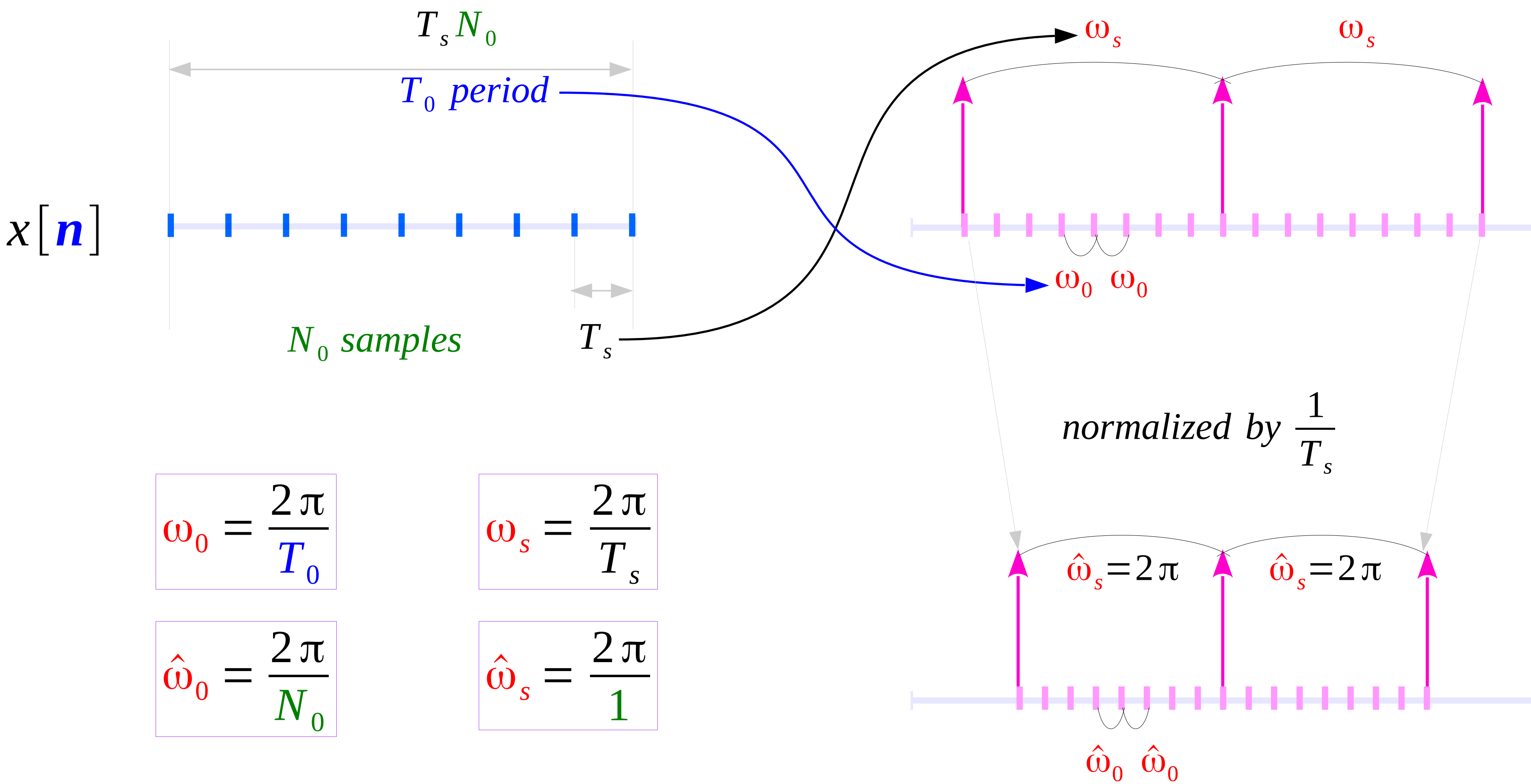
ω_s and ω_0

		$T_s = 1 \cdot T_s$	$T_0 = N_0 \cdot T_s$
		replication frequency	*frequency resolution*
$\omega = \dfrac{2\pi}{T}$	**Continuous Time**	$\omega_s = \dfrac{2\pi}{T_s}$	$\omega_0 = \dfrac{2\pi}{T_0}$
$\omega = \dfrac{\hat{\omega}}{T_s}$	**Discrete Time**	$\hat{\omega}_s = \dfrac{2\pi}{1}$	$\hat{\omega}_0 = \dfrac{2\pi}{N_0}$
		normalized *replication frequency*	***normalized*** *frequency resolution*

ω_s and ω_0

	$T_s = 1 \cdot T_s$	$T_0 = N_0 \cdot T_s$		
	(ω_s, T_s)	(ω_0, T_0)		
$\omega = \dfrac{2\pi}{T}$	$(\omega_s, 1 \cdot T_s)$	$(\omega_0, N_0 \cdot T_s)$	$\omega_s = \dfrac{2\pi}{T_s}$	$\omega_0 = \dfrac{2\pi}{T_0}$
$\omega = \dfrac{2\pi}{T}$	$(\hat{\omega}_s, 1)$	$(\hat{\omega}_0, N_0)$	$\hat{\omega}_s = \dfrac{2\pi}{1}$	$\hat{\omega}_0 = \dfrac{2\pi}{N_0}$
	$\omega = \dfrac{\hat{\omega}}{T_s}$	$\omega = \dfrac{\hat{\omega}}{T_s}$		

ω_s and ω_0



$$\omega_0 = \frac{2\pi}{T_0} \qquad \omega_s = \frac{2\pi}{T_s}$$

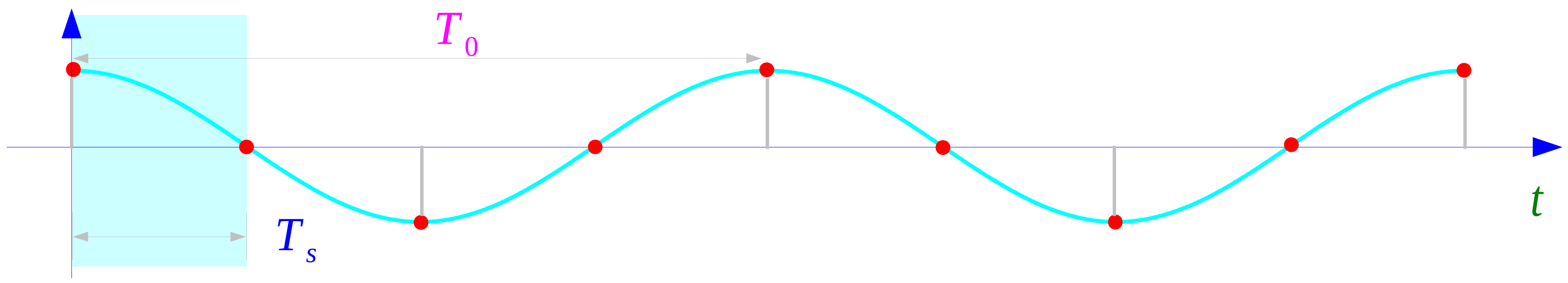
$$\hat{\omega}_0 = \frac{2\pi}{N_0} \qquad \hat{\omega}_s = \frac{2\pi}{1}$$

Frequency and Digital Frequency

Continuous Time

$$x(t) = \cos(\omega_0 t)$$

$$\boxed{\omega_0 = \frac{2\pi}{T_0}}$$



$$x[n] = x(nT_s)$$
$$= \cos(n\omega_0 T_s)$$
$$= \cos(n\hat{\omega}_0)$$

Discrete Time

$$\boxed{\hat{\omega}_0 = \frac{2\pi}{N_0}} \longleftarrow \boxed{\omega_0 = \frac{2\pi}{N_0 T_s}}$$

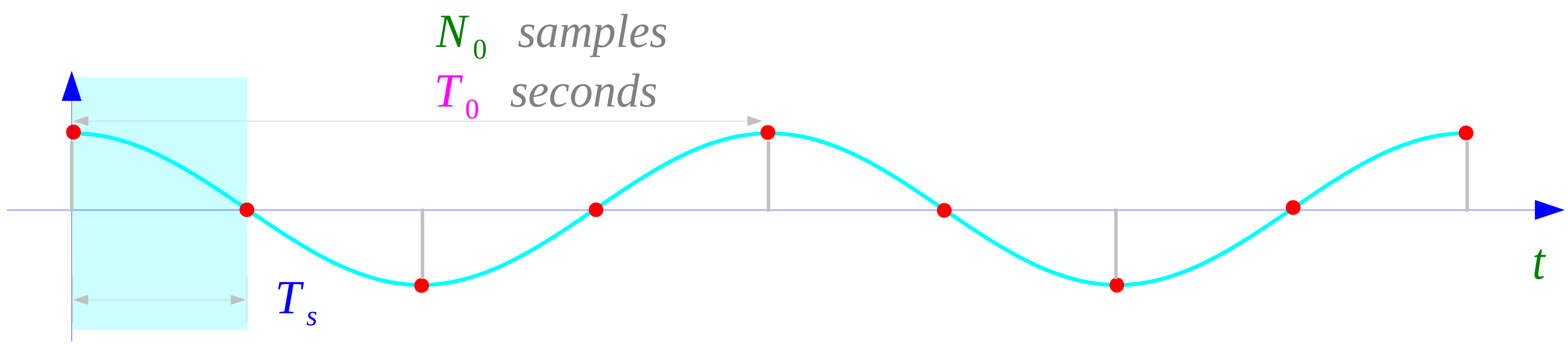
$$\hat{\omega} = \omega \cdot T_s = \frac{\omega}{f_s}$$

Frequency and Digital Frequency

Continuous Time

$$x(t) = \cos(\omega_0 t)$$

$$\boxed{\omega_0 = \frac{2\pi}{T_0}} \qquad rad/sec$$



$$x[n] = x(nT_s)$$
$$= \cos(n\omega_0 T_s)$$
$$= \cos(n\hat{\omega}_0)$$

Discrete Time

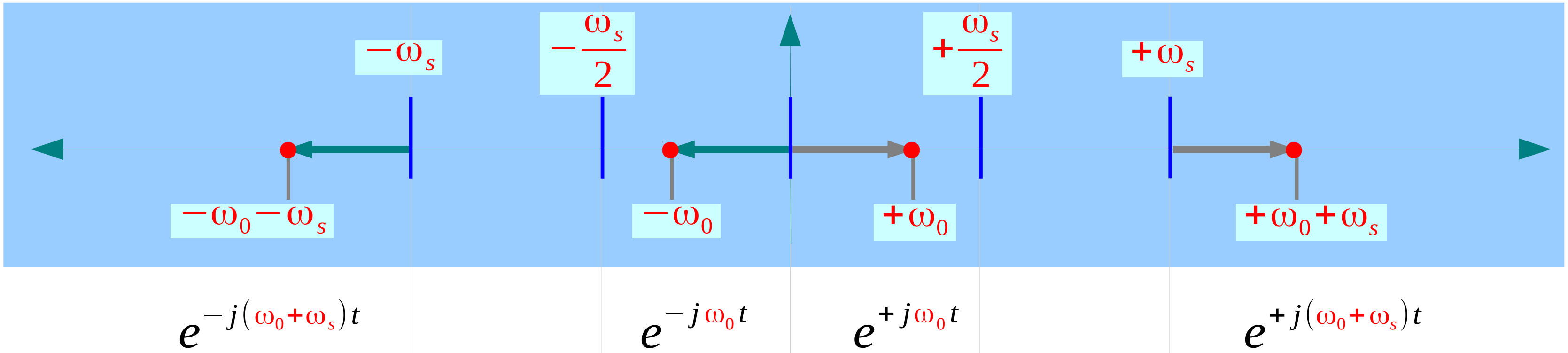
$$\boxed{\hat{\omega}_0 = \frac{2\pi}{N_0}} \qquad rad/sample$$

$$\boxed{\omega_0 = \frac{2\pi}{N_0 T_s}}$$

$$\hat{\omega} = \omega \cdot T_s = \frac{\omega}{f_s}$$

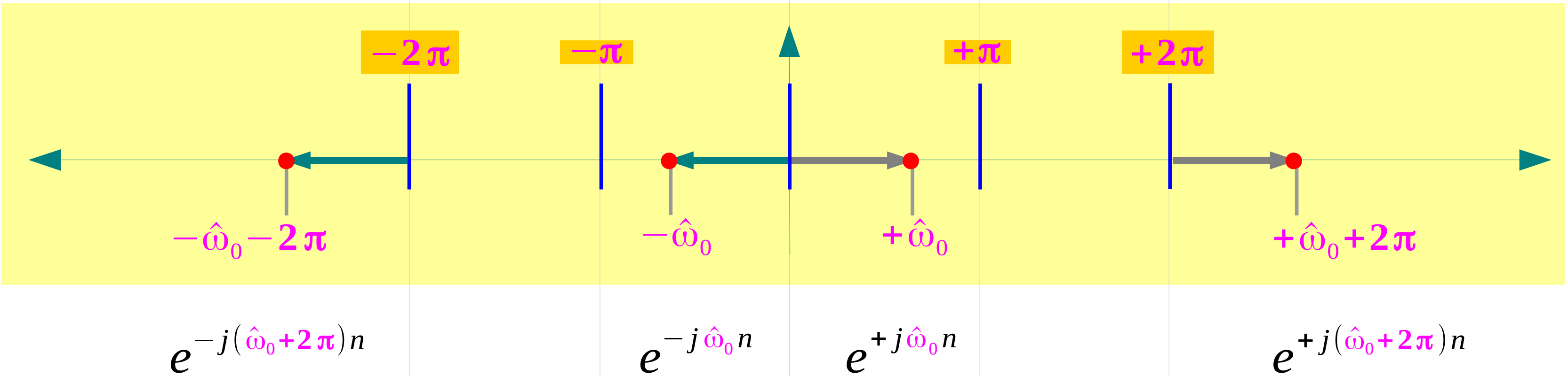
Frequency and Digital Frequency

Frequency $\omega \ (rad/sec)$



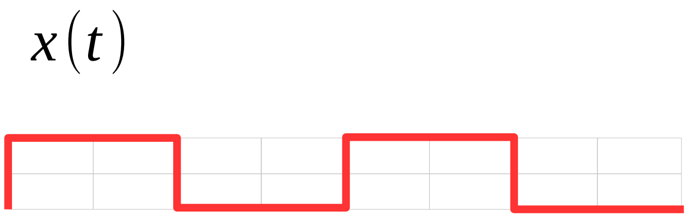
$$\omega = \frac{\hat{\omega}}{T_s}$$

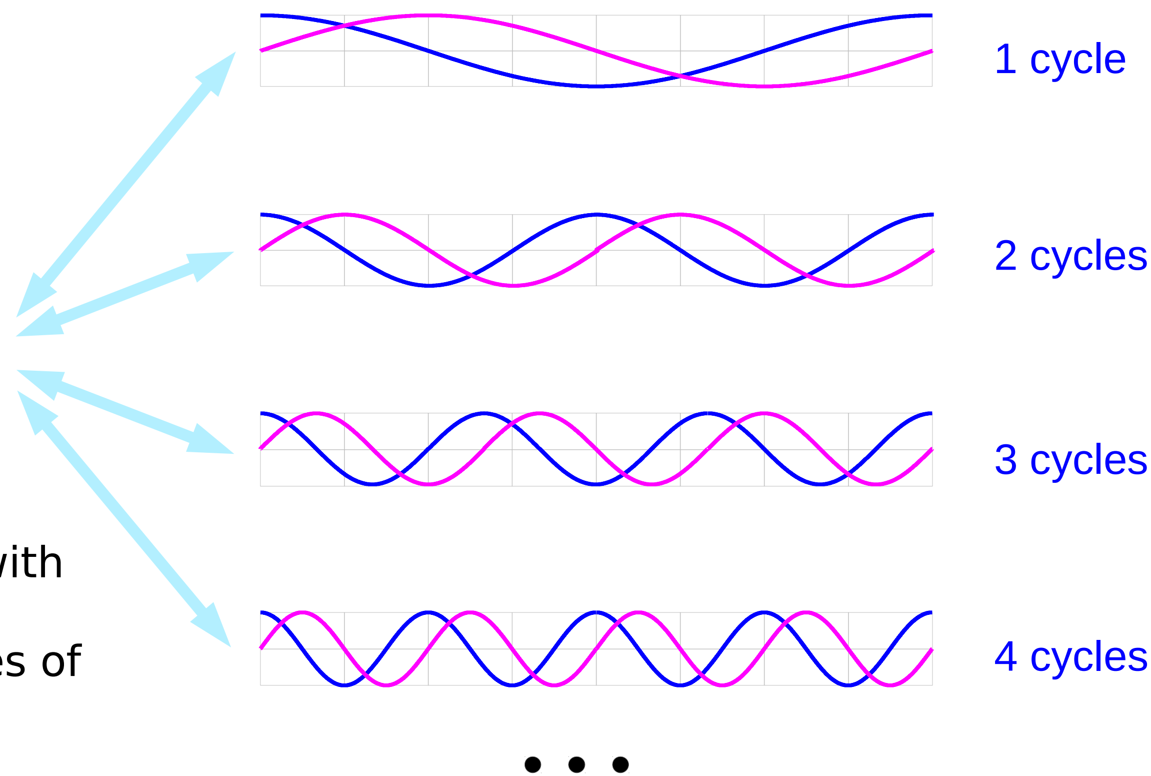
Digital Frequency $\hat{\omega} \ (rad/sample)$



$$\hat{\omega} = \omega \cdot T_s$$

CTFS Correlation Process

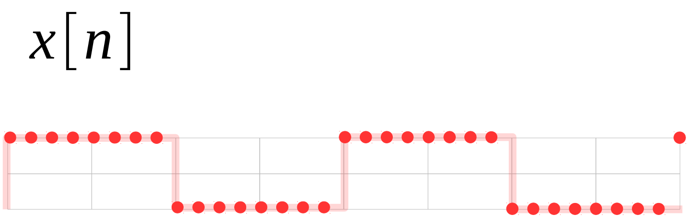


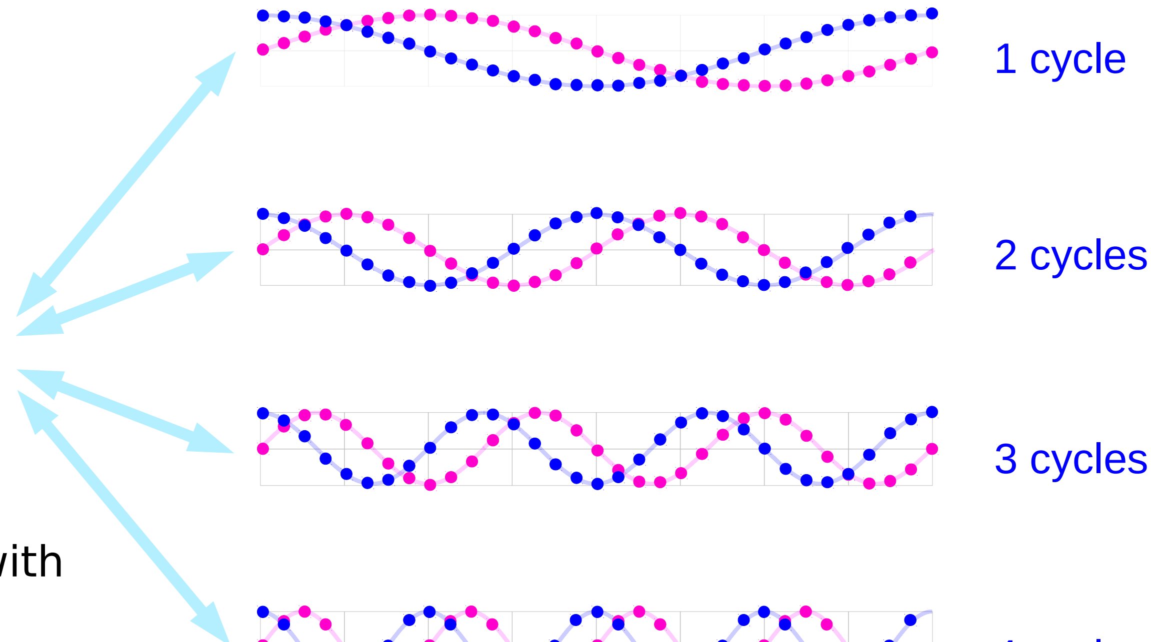


Measure the degree of correlation with these cosine and sine waves whose frequencies are the integer multiples of the fundamental frequency

$$C_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt \quad \Longleftrightarrow \quad x(t) = \sum_{n=-\infty}^{+\infty} C_n\, e^{+jn\omega_0 t}$$

DTFS Correlation Process

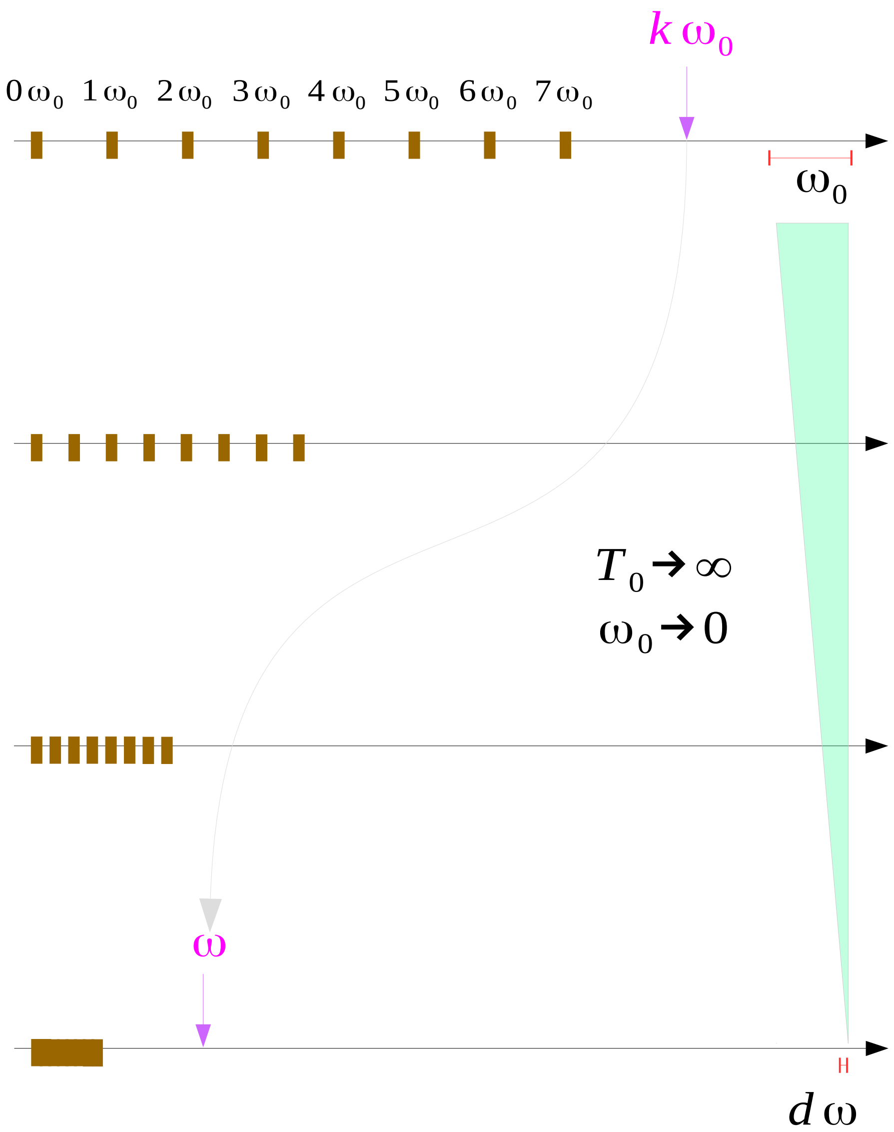




Measure the degree of correlation with these cosine and sine waves whose frequencies are the integer multiples of the fundamental frequency

$$\gamma_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\left(\frac{2\pi}{N}\right)kn} \quad \Longleftrightarrow \quad x[n] = \sum_{k=-M}^{+M} \gamma_k\, e^{+j\left(\frac{2\pi}{N}\right)kn}$$

CTFS → CTFT



$$x_{T_0}(t) = \sum_{k=-\infty}^{+\infty} C_k\, e^{+j\omega_0 k t} \cdot 1$$

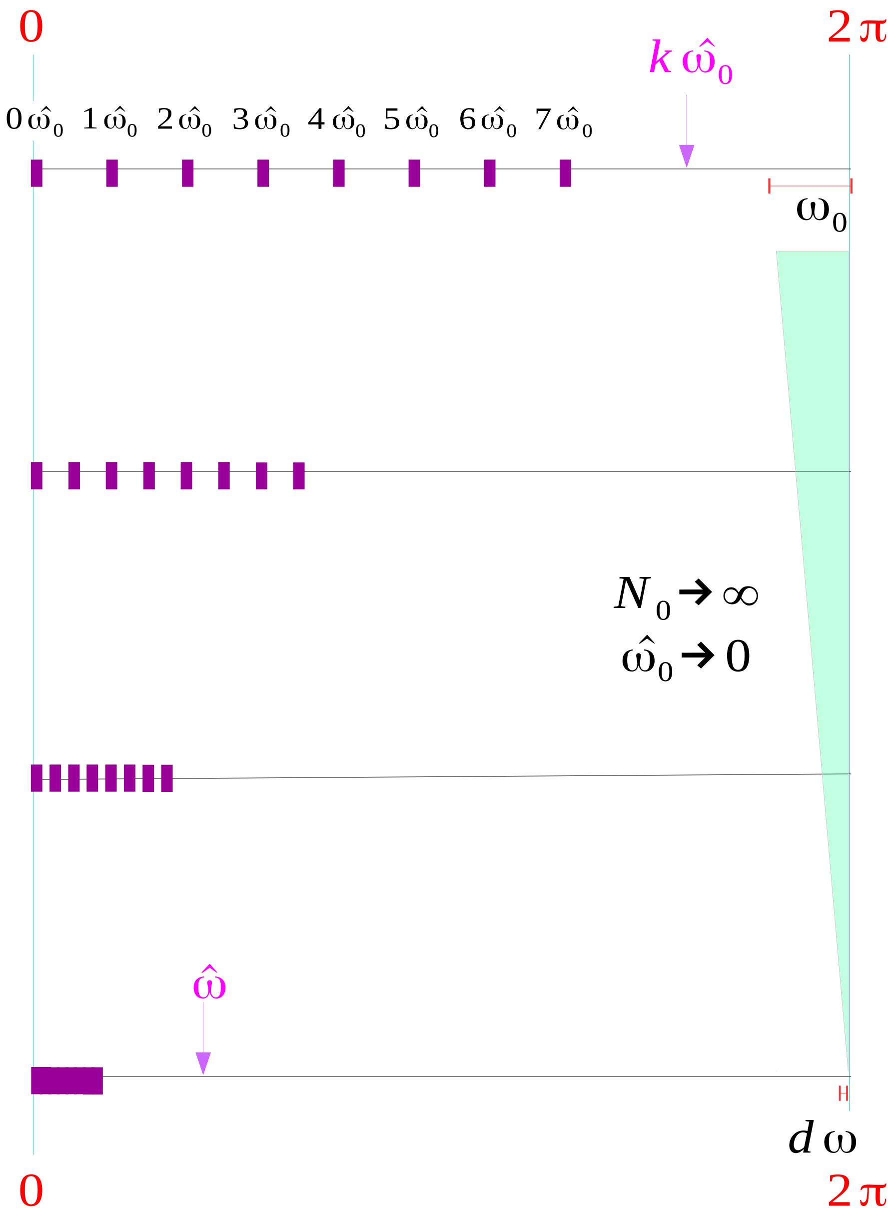
$$= \sum_{k=-\infty}^{+\infty} C_k\, e^{+j\omega_0 k t} \cdot \left(\frac{T_0}{2\pi}\right) \cdot \left(\frac{2\pi}{T_0}\right)$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} C_k T_0\, e^{+j\omega_0 k t} \cdot \left(\frac{2\pi}{T_0}\right)$$

$$x_{T_0}(t) = \frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} C_k T_0\, e^{+j\omega_0 k t} \cdot \omega_0$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

DTFS → DTFT



$$x_{N_0}[n] = \sum_{k=0}^{N_0} \gamma_k \, e^{+j\left(\frac{2\pi}{N_0}\right)kn} \cdot 1$$

$$= \sum_{k=0}^{N_0} \gamma_k \, e^{+j\left(\frac{2\pi}{N_0}\right)kn} \cdot \left(\frac{N_0}{2\pi}\right) \cdot \left(\frac{2\pi}{N_0}\right)$$

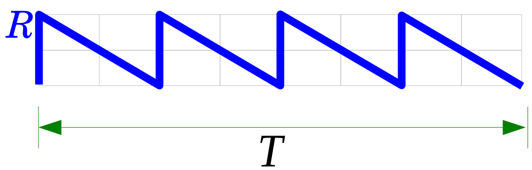
$$= \frac{1}{2\pi} \sum_{k=0}^{N_0} \gamma_k N_0 \, e^{+j\left(\frac{2\pi}{N_0}\right)kn} \cdot \left(\frac{2\pi}{N_0}\right)$$

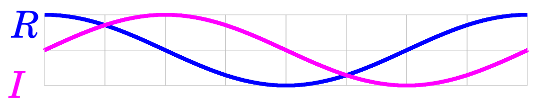
$$\boxed{x_{N_0}[n] = \frac{1}{2\pi} \sum_{k=0}^{N_0} \gamma_k N_0 \, e^{+j\left(\frac{2\pi}{N_0}\right)kn} \cdot \left(\frac{2\pi}{N_0}\right)}$$

$$\boxed{x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X\left(e^{j\hat{\omega}}\right) e^{+j\hat{\omega}n} \, d\hat{\omega}}$$

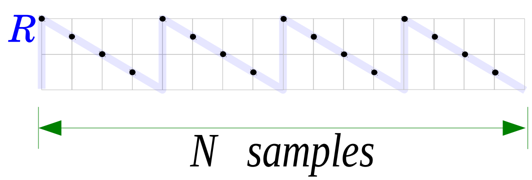
CTFS & DTFS Correlation Processes

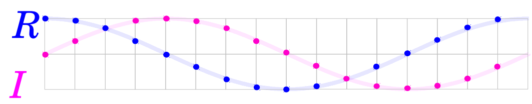
$x(t)$ $\qquad e^{+j\left(\frac{2\pi}{T}\right)k\,t}$ $\qquad \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt \; = \; C_k$





$x[n]$ $\qquad e^{+j\left(\frac{2\pi}{N}\right)k\,n}$ $\qquad \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\left(\frac{2\pi}{N}\right)n\,k} \; = \; \gamma_k$





DTFS and DFT – position of 1/N

Discrete Time Fourier <u>Series</u> **DTFS**

$$\gamma[k] = \boxed{\frac{1}{N}} \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \quad \Longleftrightarrow \quad x[n] = \sum_{k=0}^{N-1} \gamma[k]\, e^{+j(2\pi/N)kn}$$

Discrete Fourier <u>Transform</u> **DFT**

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \quad \Longleftrightarrow \quad x[n] = \boxed{\frac{1}{N}} \sum_{k=0}^{N-1} X[k]\, e^{+j(2\pi/N)kn}$$

$$DTFS(x[n]) = \frac{1}{N} DFT(x[n])$$

$$\gamma[n] = \frac{1}{N} X[n]$$

$$\hat{\omega}_0 = \left(\frac{2\pi}{N}\right)$$

DTFS and DFT coefficients relationship

Discrete Time Fourier Series DTFS

$$\gamma[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \quad \Leftrightarrow \quad x[n] = \sum_{k=0}^{N-1} \gamma[k]\, e^{+j(2\pi/N)kn}$$

$$X[k] = N\cdot\frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \quad \Leftrightarrow \quad x[n] = \sum_{k=0}^{N-1} \frac{1}{N} X[k]\, e^{+j(2\pi/N)kn}$$

Discrete Fourier Transform DFT

$$X[k] = N\,\gamma[k]$$

$$\gamma[k] = \frac{1}{N} X[k]$$

Converting DTFS and DFT Coefficients

$$DFT(x[n]) = N\, DTFS(x[n])$$

$$X[n] = N\, \gamma[n]$$

$$\underset{\text{DTFS}}{\boxed{\gamma[k]}} \underset{\cdot \frac{1}{N}}{\overset{\cdot N}{\rightleftarrows}} \underset{\text{DFT}}{\boxed{X[k]}}$$

$$DTFS(x[n]) = \frac{1}{N} DFT(x[n])$$

$$\gamma[n] = \frac{1}{N} X[n]$$

Fourier Transform Types

Continuous Time Fourier **Series**

$$C_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt \quad \Leftrightarrow \quad x(t) = \sum_{k=-\infty}^{+\infty} C_k\, e^{+jk\omega_0 t}$$

Discrete Time Fourier **Series**

$$\gamma[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\hat{\omega}_0 n} \quad \Leftrightarrow \quad x[n] = \sum_{k=0}^{N-1} \gamma[k]\, e^{+jk\hat{\omega}_0 n}$$

Continuous Time Fourier **Transform**

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

Discrete Time Fourier **Transform**

$$X(j\hat{\omega}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\hat{\omega} n} \quad \Leftrightarrow \quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(j\hat{\omega})\, e^{+j\hat{\omega} n}\, d\hat{\omega}$$

Frequency Resolution

Continuous Time Fourier **Series**

$$C_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt \quad \Longleftrightarrow \quad x(t) = \sum_{k=-\infty}^{+\infty} C_k\, e^{+jk\omega_0 t}$$

Frequency Resolution $\quad \omega_0 = \left(\frac{2\pi}{T_0}\right) \quad$ *Signal Period* $\quad T_0$

Discrete Time Fourier **Series**

$$\gamma[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\hat{\omega}_0 n} \quad \Longleftrightarrow \quad x[n] = \sum_{k=0}^{N-1} \gamma[k]\, e^{+jk\hat{\omega}_0 n}$$

Frequency Resolution $\quad \hat{\omega}_0 = \left(\frac{2\pi}{N_0}\right) \quad$ *Sample Counts* $\quad N_0$

Frequency Variable Notations

Continuous Time Fourier **Transform**

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

$$j\omega \rightarrow e^{j\omega} \rightarrow X(e^{j\omega}) \qquad j\omega \;\; \textit{always appears as} \;\; e^{j\omega}$$

Discrete Time Fourier **Transform**

$$X(j\hat{\omega}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\hat{\omega} n} \quad \Leftrightarrow \quad x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(j\hat{\omega})\, e^{+j\hat{\omega} n}\, d\hat{\omega}$$

$$j\hat{\omega} \rightarrow e^{j\hat{\omega}} \rightarrow X(e^{j\hat{\omega}}) \qquad j\hat{\omega} \;\; \textit{always appears as} \;\; e^{j\hat{\omega}}$$

CT	$x(t)$	**FS**	$\frac{1}{T}\int_0^T \; dt$	$\boxed{e^{-jk\omega_0 t}}$	**DF**	C_k	**AF** $\sum_{k=-\infty}^{+\infty}$
DT	$x[n]$	**FS**	$\frac{1}{N}\sum_{n=0}^{N-1}$	$\boxed{e^{-jk\hat{\omega}_0 n}}$	**DF**	$\gamma[k]$	**PF** $\sum_{k=0}^{N-1}$
CT	$x(t)$	**FT**	$\int_{-\infty}^{+\infty} \; dt$	$\boxed{e^{-j\omega t}}$	**CF**	$X(j\omega)$	**AF** $\frac{1}{2\pi}\int_{-\infty}^{+\infty} \; d\omega$
DT	$x[n]$	**FT**	$\sum_{n=-\infty}^{+\infty}$	$\boxed{e^{-j\hat{\omega} n}}$	**CF**	$X(j\hat{\omega})$	**PF** $\frac{1}{2\pi}\int_{-\pi}^{+\pi} \; d\hat{\omega}$

Continuous **T**ime **F**ourier **S**eries **C**ontinuous **F**req **A**periodic **F**req

Discrete **T**ime **F**ourier **T**ransform **D**iscrete **F**req **P**eriodic **F**req

CT	$x(t)$	**PT**	$\frac{1}{T}\int_{0}^{T} \; dt$	$\boxed{e^{-jk\omega_0 t}}$	**DF**	C_k	**AF**	$\sum_{k=-\infty}^{+\infty}$
DT	$x[n]$	**PT**	$\frac{1}{N}\sum_{n=0}^{N-1}$	$\boxed{e^{-jk\hat{\omega}_0 n}}$	**DF**	$\gamma[k]$	**PF**	$\sum_{k=0}^{N-1}$
CT	$x(t)$	**AT**	$\int_{-\infty}^{+\infty} \; dt$	$\boxed{e^{-j\omega t}}$	**CF**	$X(j\omega)$	**AF**	$\frac{1}{2\pi}\int_{-\infty}^{+\infty} \; d\omega$
DT	$x[n]$	**AT**	$\sum_{n=-\infty}^{+\infty}$	$\boxed{e^{-j\hat{\omega} n}}$	**CF**	$X(j\hat{\omega})$	**PF**	$\frac{1}{2\pi}\int_{-\pi}^{+\pi} \; d\hat{\omega}$

Continuous **T**ime **F**ourier **S**eries **C**ontinuous **F**req **A**periodic **F**req

Discrete **T**ime **F**ourier **T**ransform **D**iscrete **F**req **P**eriodic **F**req

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PT	$\frac{1}{T}\int_0^T \quad dt$	**DF**	C_k
PT	$\frac{1}{N}\sum_{n=0}^{N-1}$	**DF**	$\gamma[k]$
AT	$\int_{-\infty}^{+\infty} \quad dt$	**CF**	$X(j\omega)$
AT	$\sum_{n=-\infty}^{+\infty}$	**CF**	$X(j\hat{\omega})$

Continuous Time **F**ourier **S**eries **C**ontinuous **F**req **A**periodic **F**req
Discrete Time **F**ourier **T**ransform **D**iscrete **F**req **P**eriodic **F**req

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CT $x(t)$ **AF** $\sum_{k=-\infty}^{+\infty}$

DT $x[n]$ **PF** $\sum_{k=0}^{N-1}$

CT $x(t)$ **AF** $\frac{1}{2\pi}\int_{-\infty}^{+\infty} \quad d\omega$

DT $x[n]$ **PF** $\frac{1}{2\pi}\int_{-\pi}^{+\pi} \quad d\hat{\omega}$

Continuous Time **F**ourier **S**eries **C**ontinuous **F**req **A**periodic **F**req
Discrete Time **F**ourier **T**ransform **D**iscrete **F**req **P**eriodic **F**req

CT $x(t)$ **PT** $\frac{1}{T}\int_0^T \quad dt$

DT $x[n]$ **PT** $\frac{1}{N}\sum_{n=0}^{N-1}$

PT $\frac{1}{T}\int_0^T 1\,dt = \frac{T}{T}$

PT $\frac{1}{N}\sum_{n=0}^{N-1} 1 = \frac{N}{N}$

$X(j\omega) \approx T \cdot C_k$ **CF** $\left(\frac{1}{2\pi}\right) \cdot T \cdot \left(\frac{2\pi}{T}\right)$

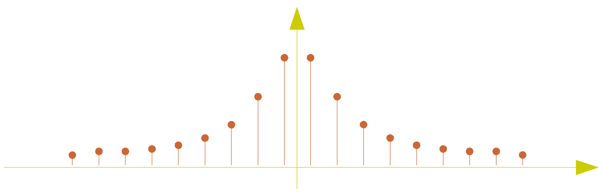
$X(j\hat{\omega}) \approx N \cdot \gamma_k$ **CF** $\left(\frac{1}{2\pi}\right) \cdot N \cdot \left(\frac{2\pi}{N}\right)$

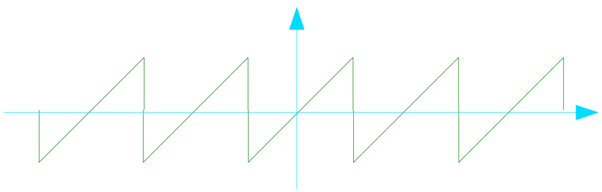
CF $X(j\omega)$ **AF** $\frac{1}{2\pi}\int_{-\infty}^{+\infty} \quad d\omega$

CF $X(j\hat{\omega})$ **PF** $\frac{1}{2\pi}\int_{-\pi}^{+\pi} \quad d\hat{\omega}$

Continuous Time Fourier Series

$$C_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt \quad \Longleftrightarrow \quad x(t) = \sum_{k=-\infty}^{+\infty} C_k\, e^{+jk\omega_0 t}$$





Aperiodic
Discrete Frequency Spectrum

Periodic
Continuous Time Signal

$$\sum_{k=-\infty}^{+\infty}$$

$$C_k$$

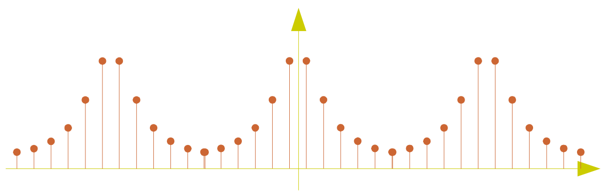
$$\frac{1}{T}\int_0^T \quad dt$$

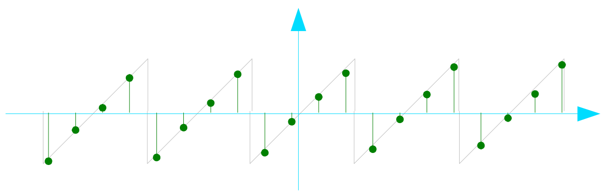
$$x(t)$$

B. DTFS

Discrete Time Fourier Series

$$\gamma[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\hat{\omega}_0 n} \quad \Leftrightarrow \quad x[n] = \sum_{k=0}^{N-1} \gamma[k]\, e^{+jk\hat{\omega}_0 n}$$





Periodic
Discrete Frequency Spectrum

$$\sum_{k=0}^{N-1}$$

$$\gamma[k]$$

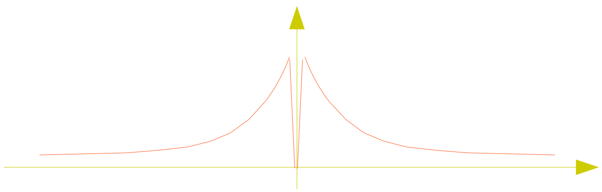
Periodic
Discrete Time Signal

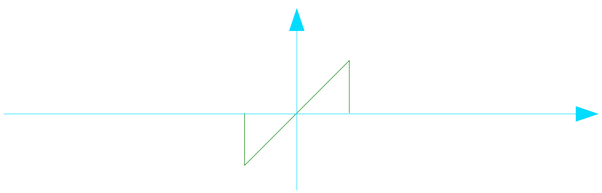
$$\frac{1}{N}\sum_{n=0}^{N-1}$$

$$x[n]$$

Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Longleftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$





Aperiodic
Discrete Frequency Spectrum

Aperiodic
Continuous Time Signal

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} \quad d\omega$$

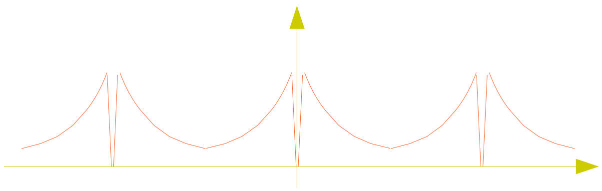
$$X(j\omega)$$

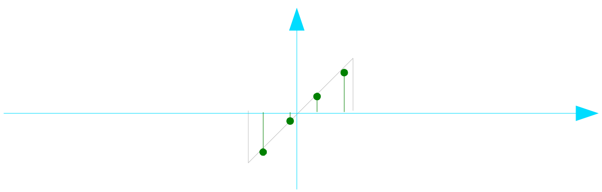
$$\int_{-\infty}^{+\infty} \quad dt$$

$$x(t)$$

Discrete Time Fourier Transform

$$X(j\hat{\omega}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\hat{\omega}n} \quad \Longleftrightarrow \quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(j\hat{\omega})\, e^{+j\hat{\omega}n}\, d\hat{\omega}$$





Periodic
Continuous Frequency Spectrum

$$\frac{1}{2\pi}\int_{-\pi}^{+\pi} \quad d\hat{\omega}$$

$$X(j\hat{\omega})$$

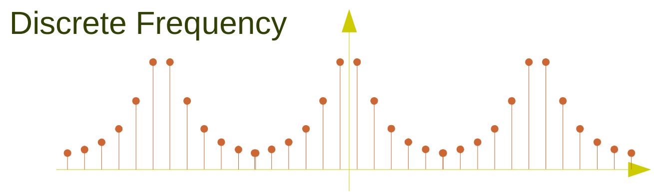
Aperiodic
Discrete Time Signal

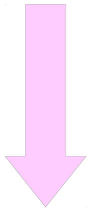
$$\sum_{n=-\infty}^{+\infty}$$

$$x[n]$$

Continuous Time Fourier **Series**

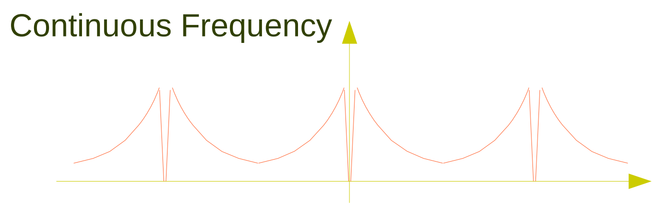
$$C_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt \quad \Longleftrightarrow \quad x(t) = \sum_{k=-\infty}^{+\infty} C_k\, e^{+jk\omega_0 t}$$





Continuous Time Fourier **Transform**

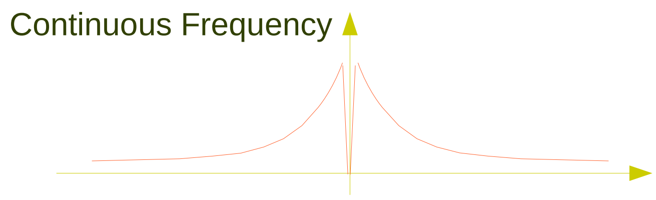
$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Longleftrightarrow \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$



DTFS & DTFT

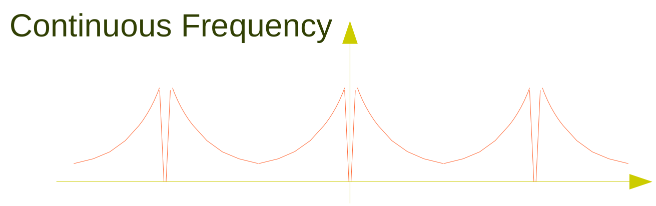
Discrete Time Fourier **Series**

$$\gamma[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\hat{\omega}_0 n} \quad \Longleftrightarrow \quad x[n] = \sum_{k=0}^{N-1} \gamma[k]\, e^{+jk\hat{\omega}_0 n}$$



Discrete Time Fourier **Transform**

$$X(j\hat{\omega}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\hat{\omega} n} \quad \Longleftrightarrow \quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(j\hat{\omega})\, e^{+j\hat{\omega} n}\, d\hat{\omega}$$

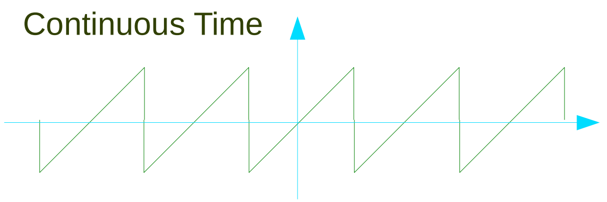


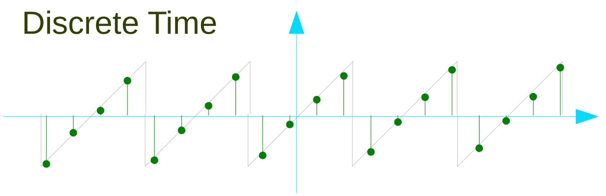
Continuous Time Fourier Series

$$C_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt \quad \Longleftrightarrow \quad x(t) = \sum_{k=-\infty}^{+\infty} C_k\, e^{+jk\omega_0 t}$$

Discrete Time Fourier Series

$$\gamma[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\hat{\omega}_0 n} \quad \Longleftrightarrow \quad x[n] = \sum_{k=0}^{N-1} \gamma[k]\, e^{+jk\hat{\omega}_0 n}$$



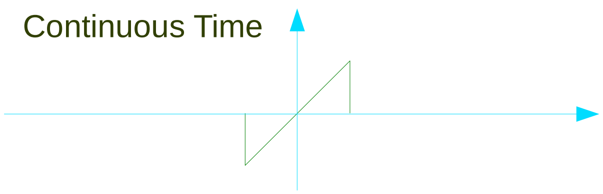


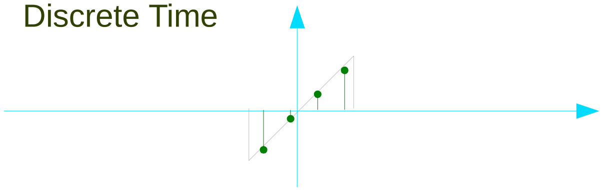
Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

Discrete Time Fourier Transform

$$X(j\hat{\omega}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\hat{\omega} n} \quad \Leftrightarrow \quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(j\hat{\omega})\, e^{+j\hat{\omega} n}\, d\hat{\omega}$$



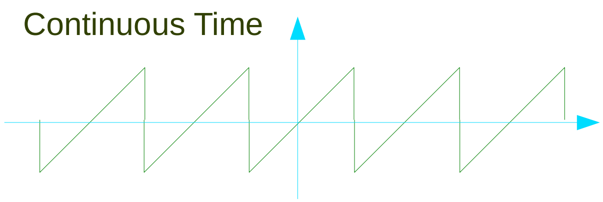


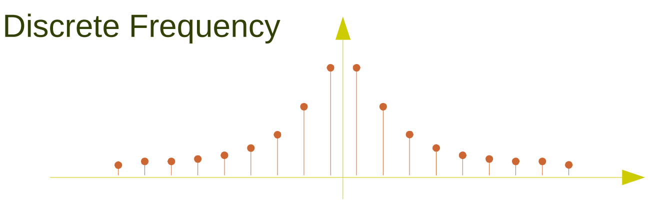
Fourier Analysis Methods

	Discrete Frequency	Continuous Frequency
Continuous Time	**CTFS** C_k $x(t)$ **Periodic** in time $e^{-jk\omega_0 t}$ $k\omega_0$ Aperiodic in freq	**CTFT** $X(j\omega)$ $x(t)$ Aperiodic in time $e^{-j\omega t}$ ω Aperiodic in freq
Discrete Time	**DTFS / DFT** γ_k $x[n]$ **Periodic** in time $e^{-jk\hat{\omega}_0 n}$ $k\hat{\omega}_0$ **Periodic** in freq	**DTFT** $X(j\hat{\omega})$ $x[n]$ Aperiodic in time $e^{-j\hat{\omega} n}$ $\hat{\omega}$ **Periodic** in freq
	Normalized Discrete Frequency	**Normalized Continuous Frequency**

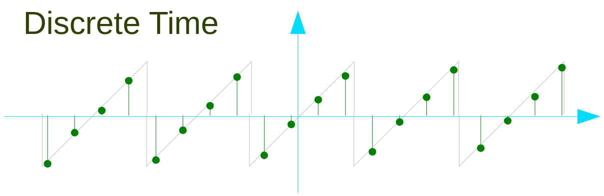
Types of Fourier Transforms

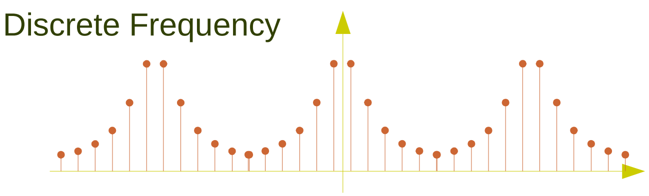
Continuous Time
Fourier Series
CTFS



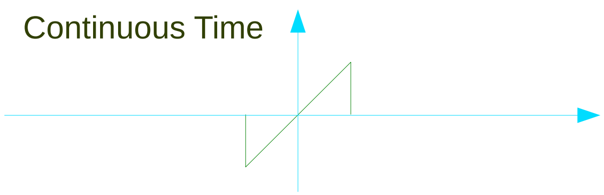


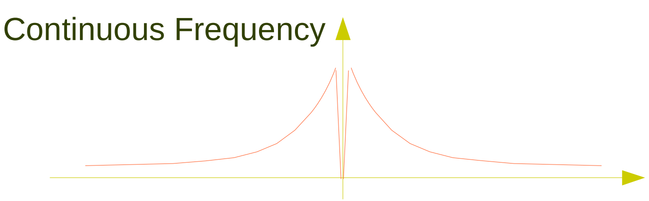
Discrete Time
Fourier Series
DTFS / DFT



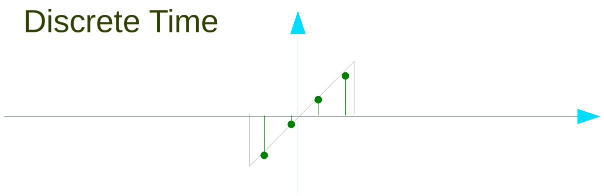


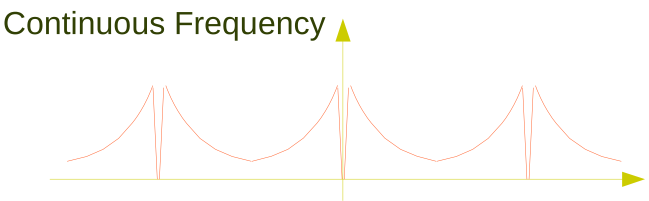
Continuous Time
Fourier Transform
CTFT





Discrete Time
Fourier Transform
DTFT



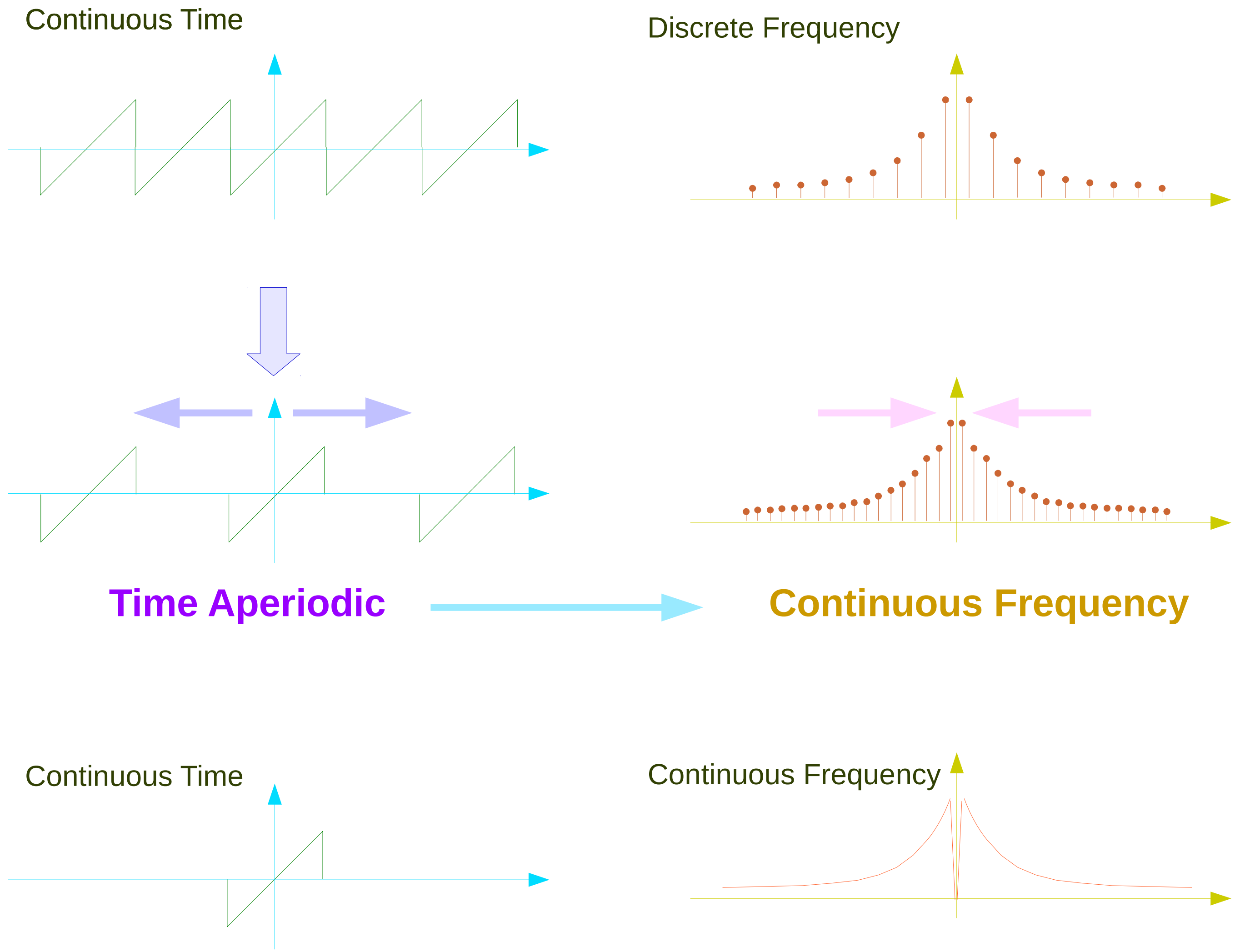


1. CTFS → CTFT

Continuous Time
Fourier Series
CTFS

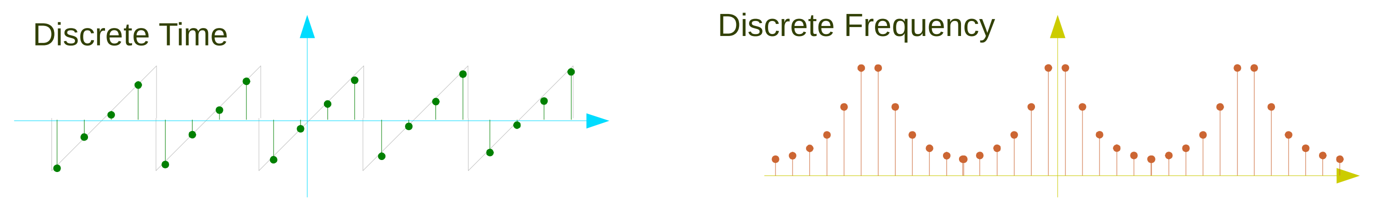
CTFS CTFT
DTFS DTFT

Continuous Time
Fourier Transform
CTFT

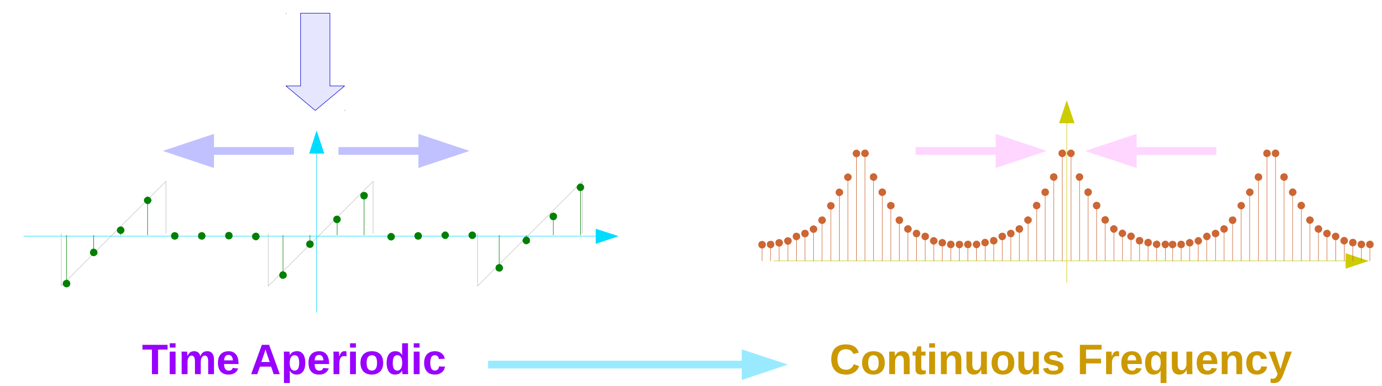


2. DTFS → DTFT

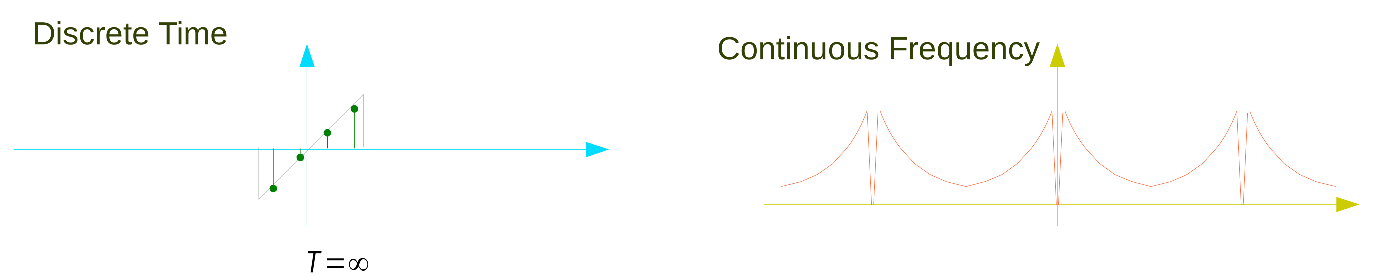
Discrete Time
Fourier Series
DTFS / DFT



CTFS CTFT
DTFS DTFT



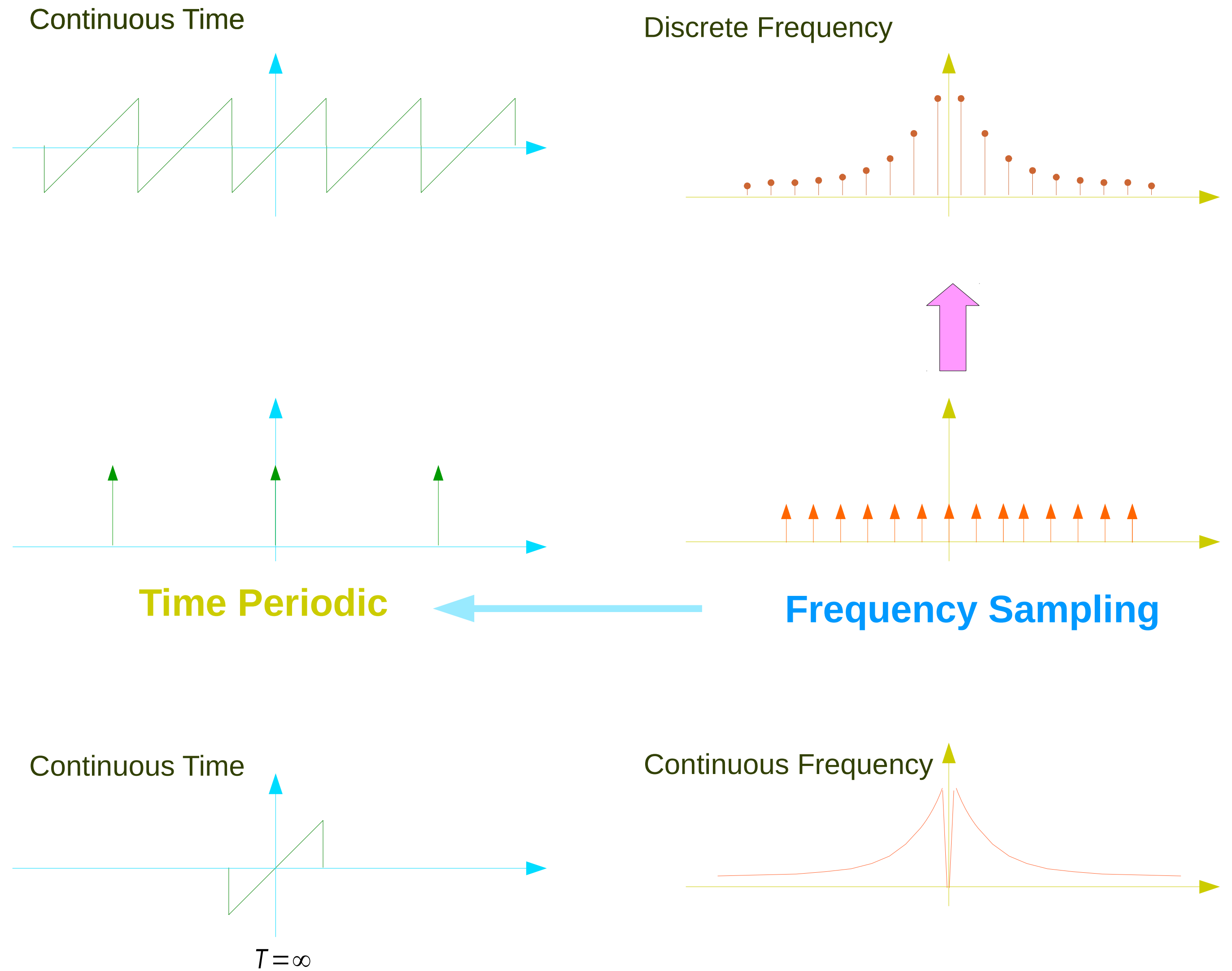
Discrete Time
Fourier Transform
DTFT



3. CTFS ← CTFT

Continuous Time
Fourier Series
CTFS

CTFS	**CTFT**
DTFS	**DTFT**



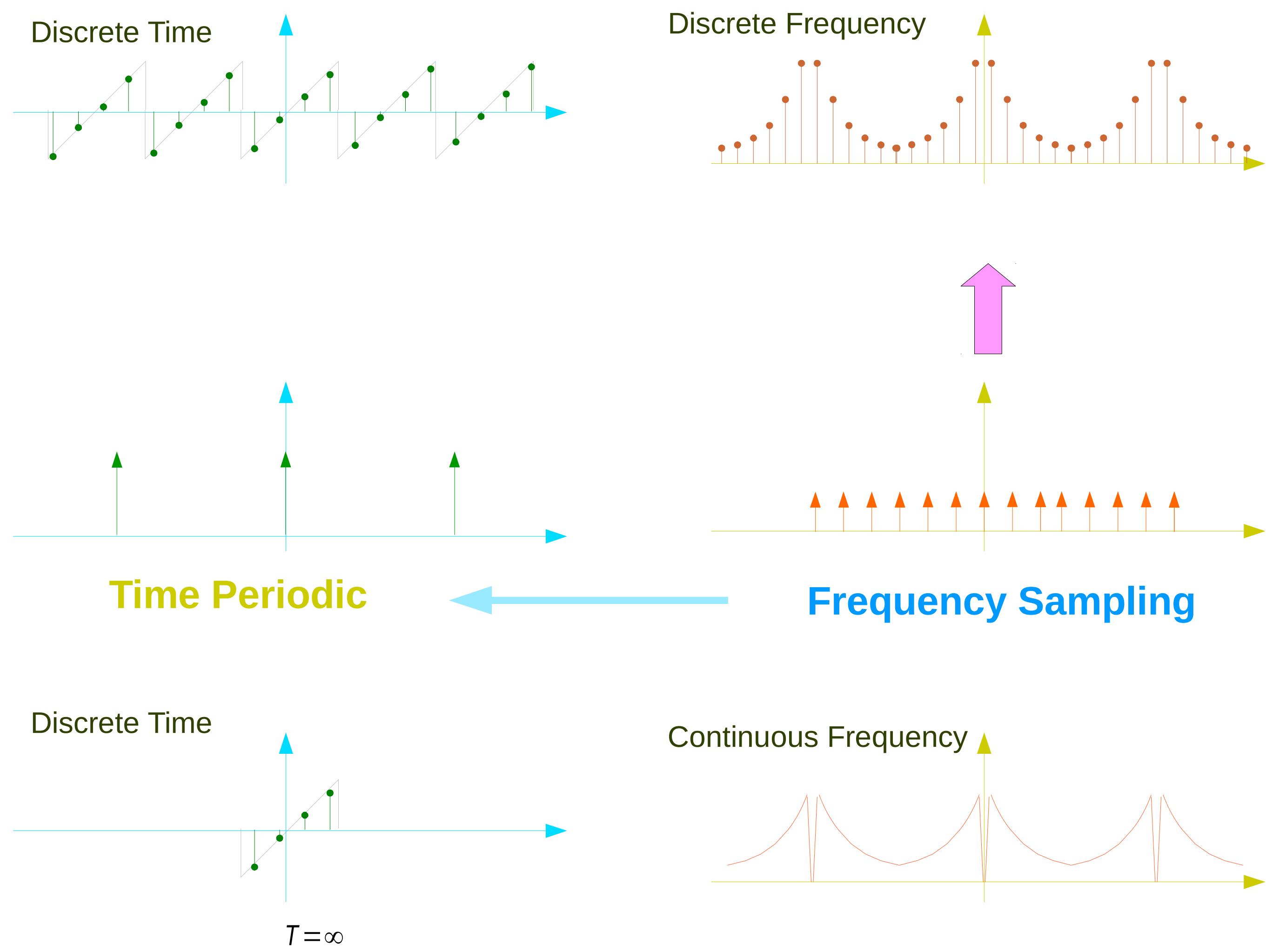
Continuous Time
Fourier Transform
CTFT

4. DTFS ← DTFT

Discrete Time Fourier Series
DTFS / DFT

CTFS CTFT
DTFS DTFT

Discrete Time Fourier Transform
DTFT

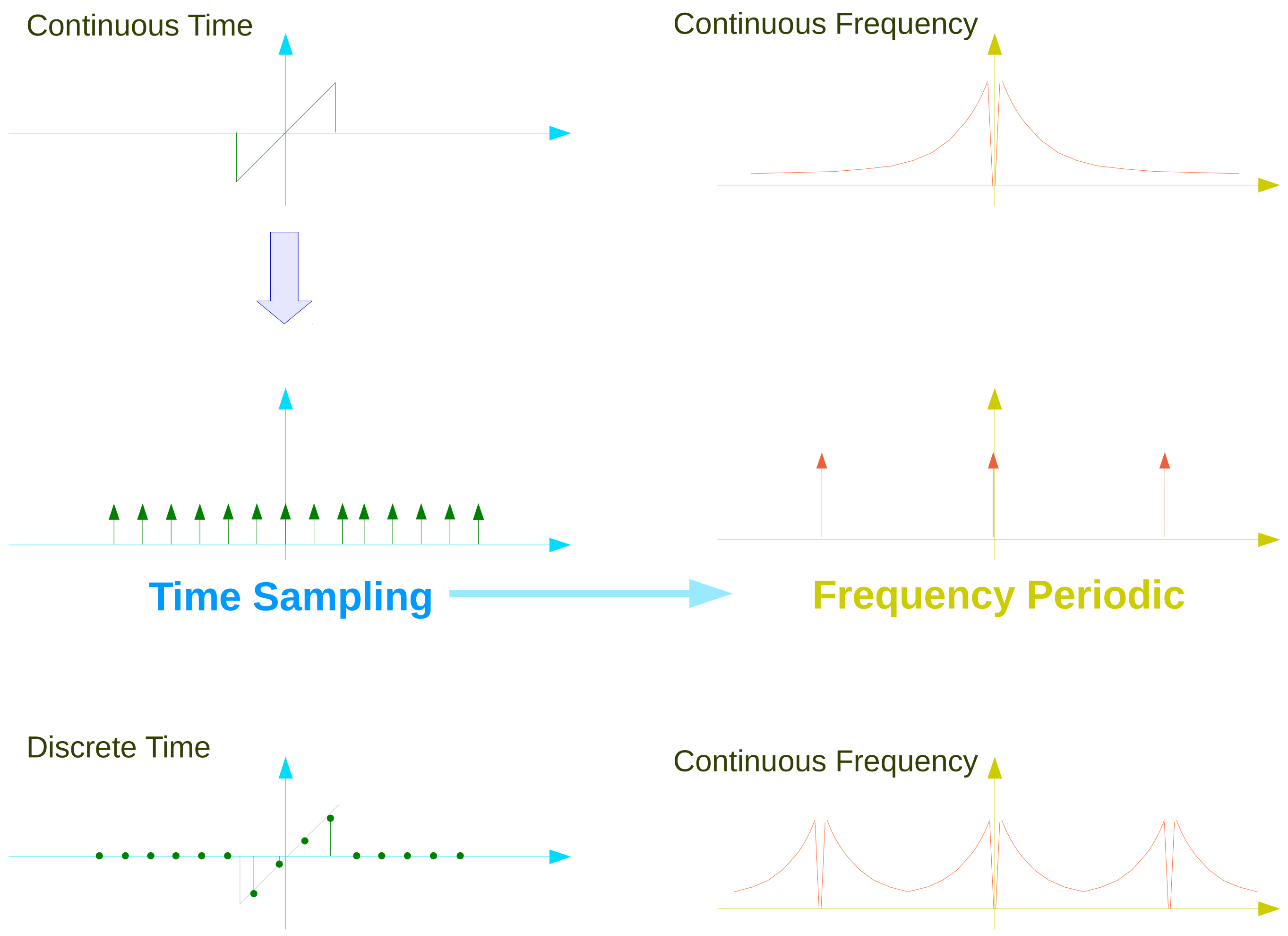


5. CTFT → DTFT

Continuous Time
Fourier Transform
CTFT

CTFS CTFT
DTFS DTFT

Discrete Time
Fourier Transform
DTFT

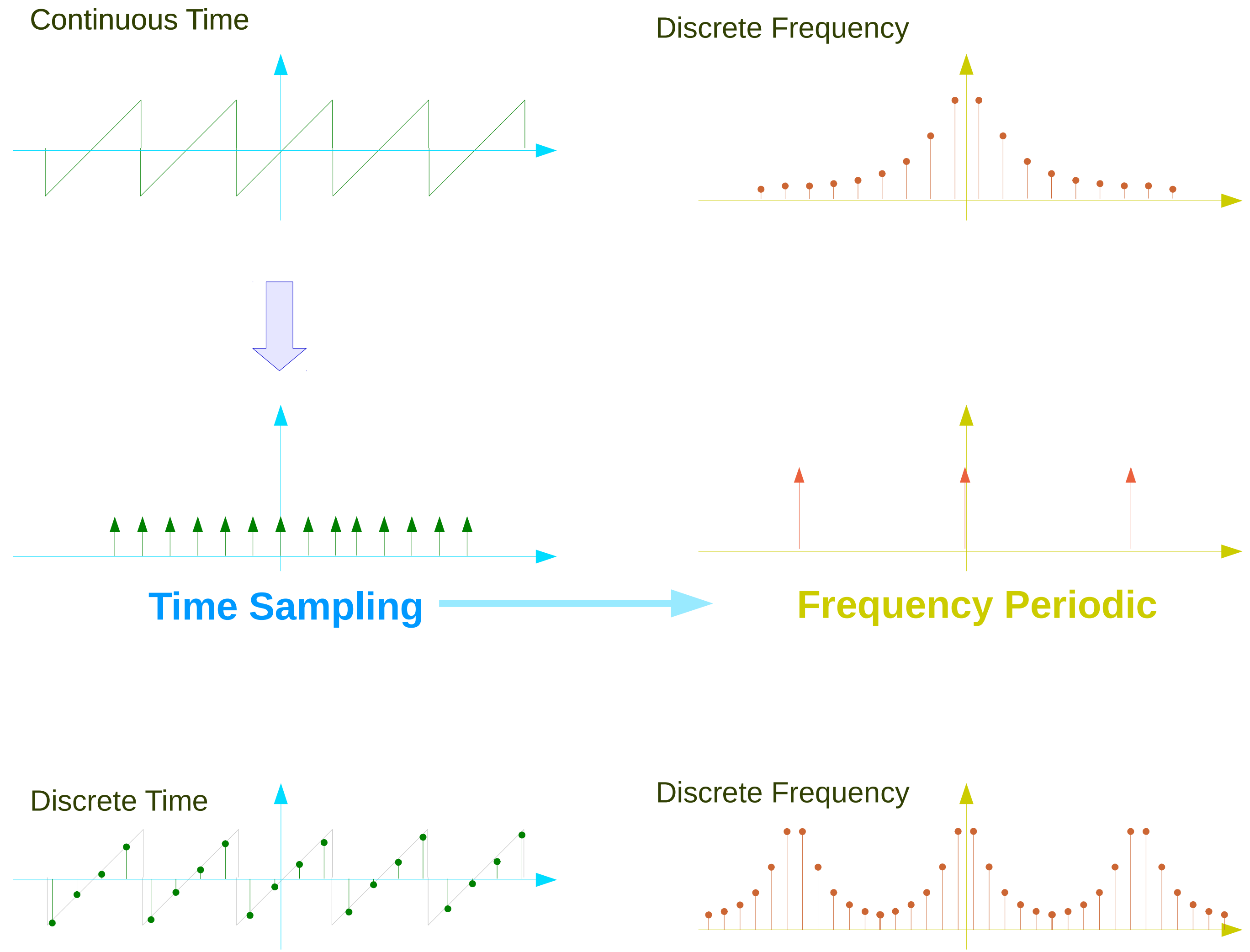


6. CTFS → DTFS

Continuous Time
Fourier Series
CTFS

CTFS CTFT
DTFS DTFT

Discrete Time
Fourier Series
DTFS / DFT

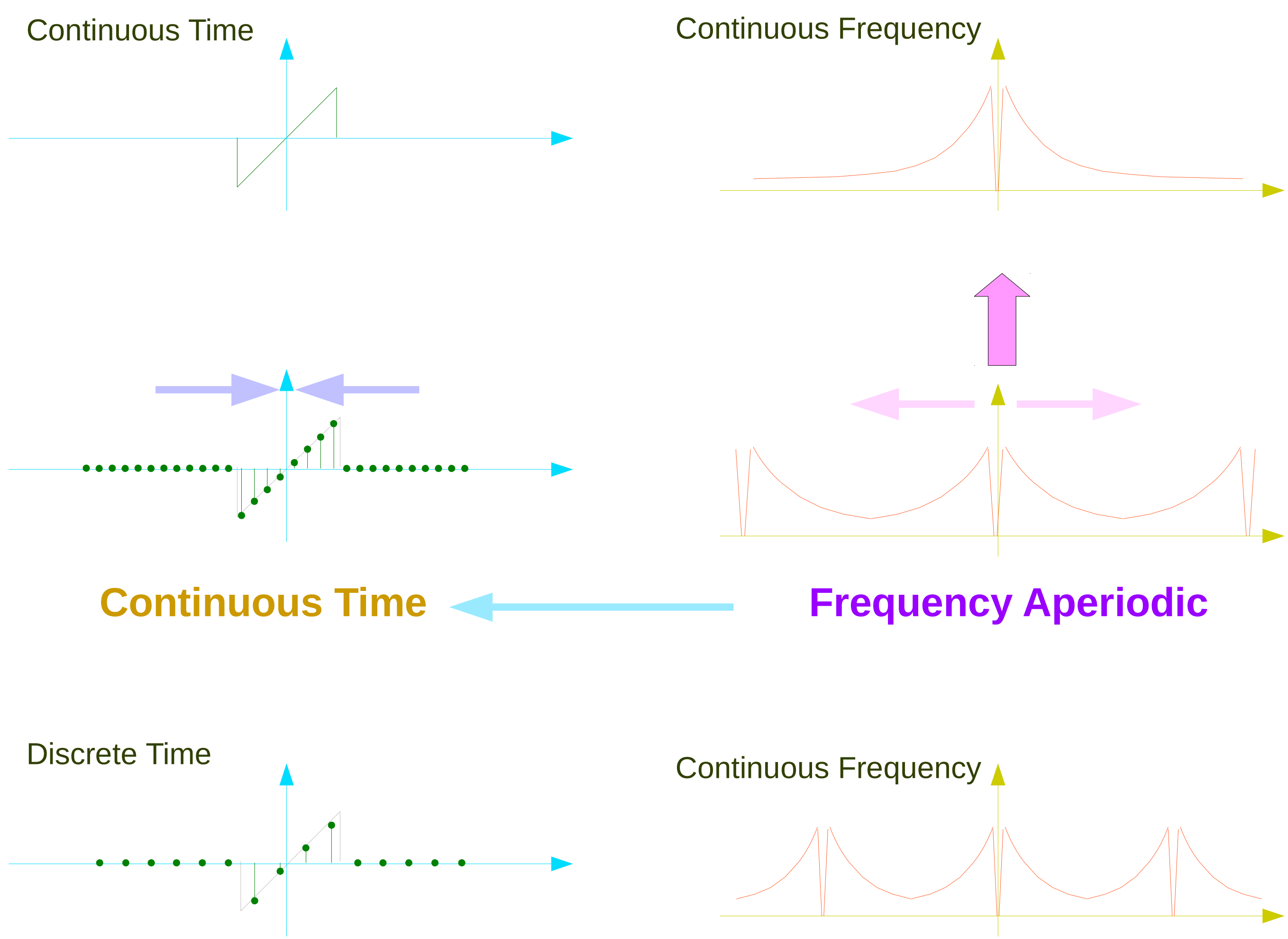


7. CTFT ← DTFT

Continuous Time
Fourier Transform
CTFT

CTFS CTFT
DTFS DTFT

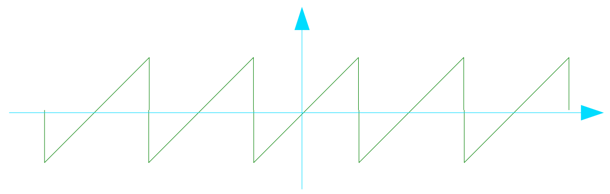
Discrete Time
Fourier Transform
DTFT



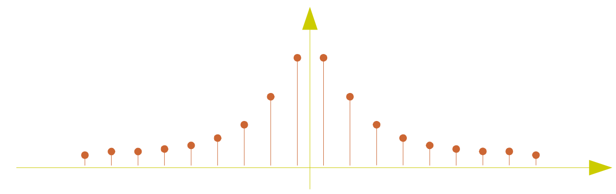
8. CTFS ← DTFS

Continuous Time
Fourier Series
CTFS

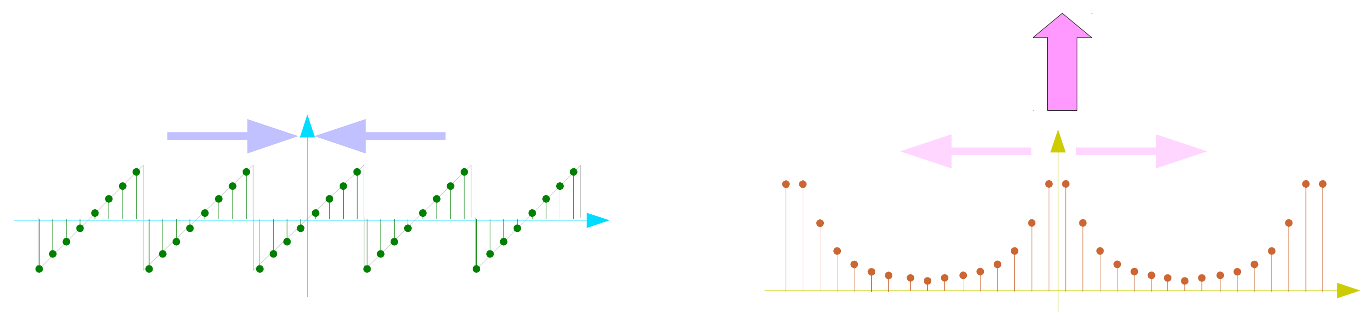
Continuous Time



Discrete Frequency



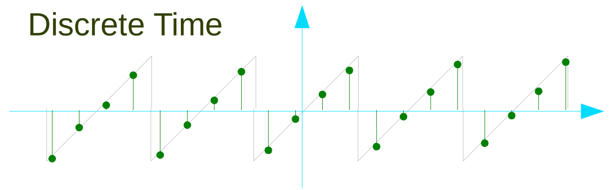
CTFS CTFT
DTFS DTFT



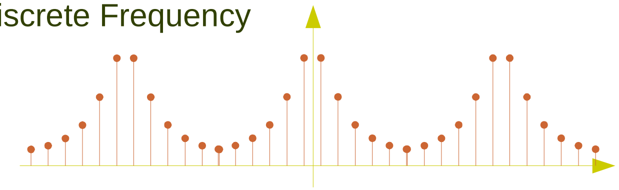
Continuous Time ← Frequency Aperiodic

Discrete Time
Fourier Series
DTFS / DFT

Discrete Time



Discrete Frequency



****

CTFS CTFT
DTFS DTFT

Time Aperiodic → Continuous Frequency

CTFS CTFT
DTFS DTFT

Time Periodic ← Frequency Sampling

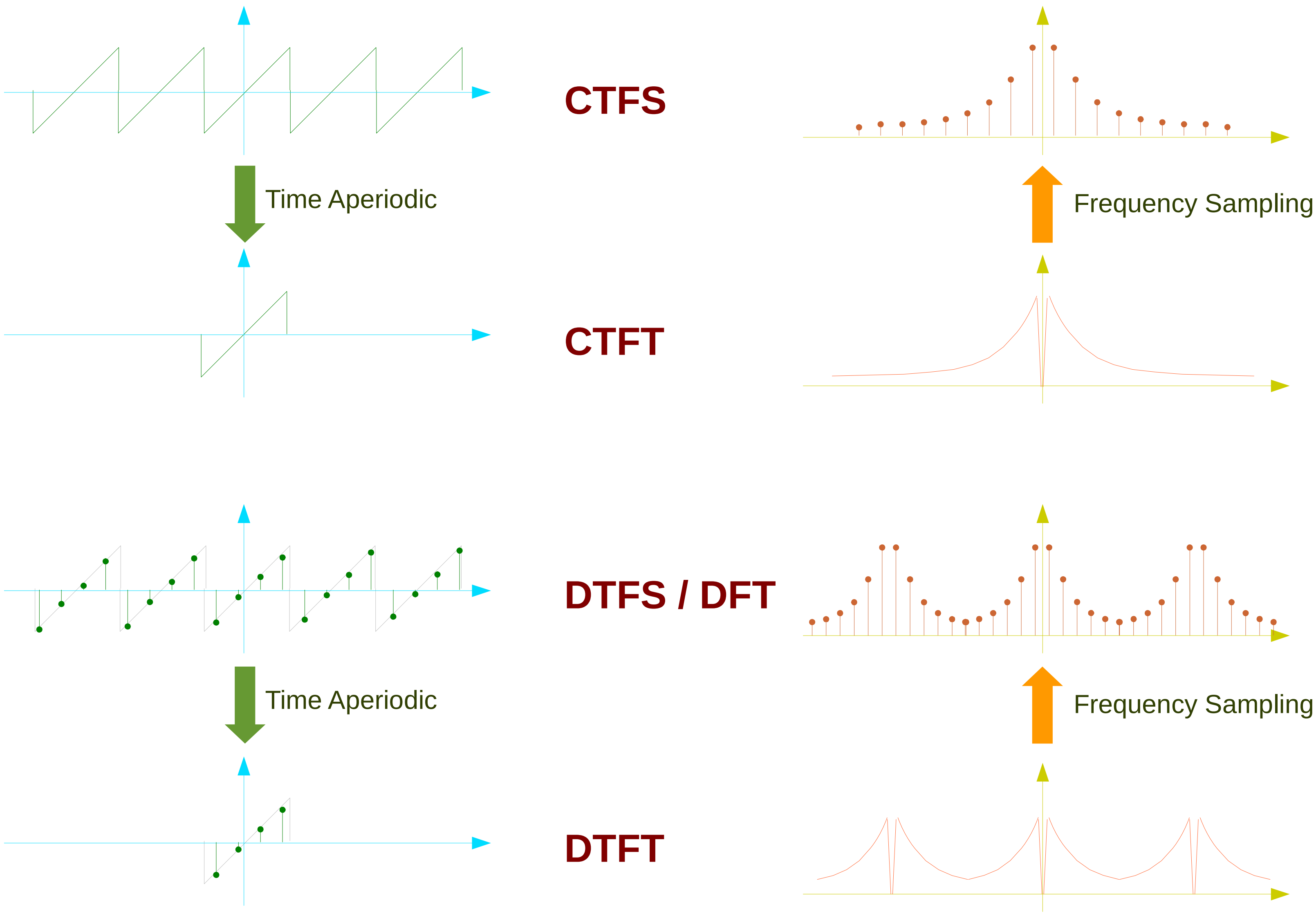
CTFS CTFT
DTFS DTFT

Time Sampling → Frequency Periodic

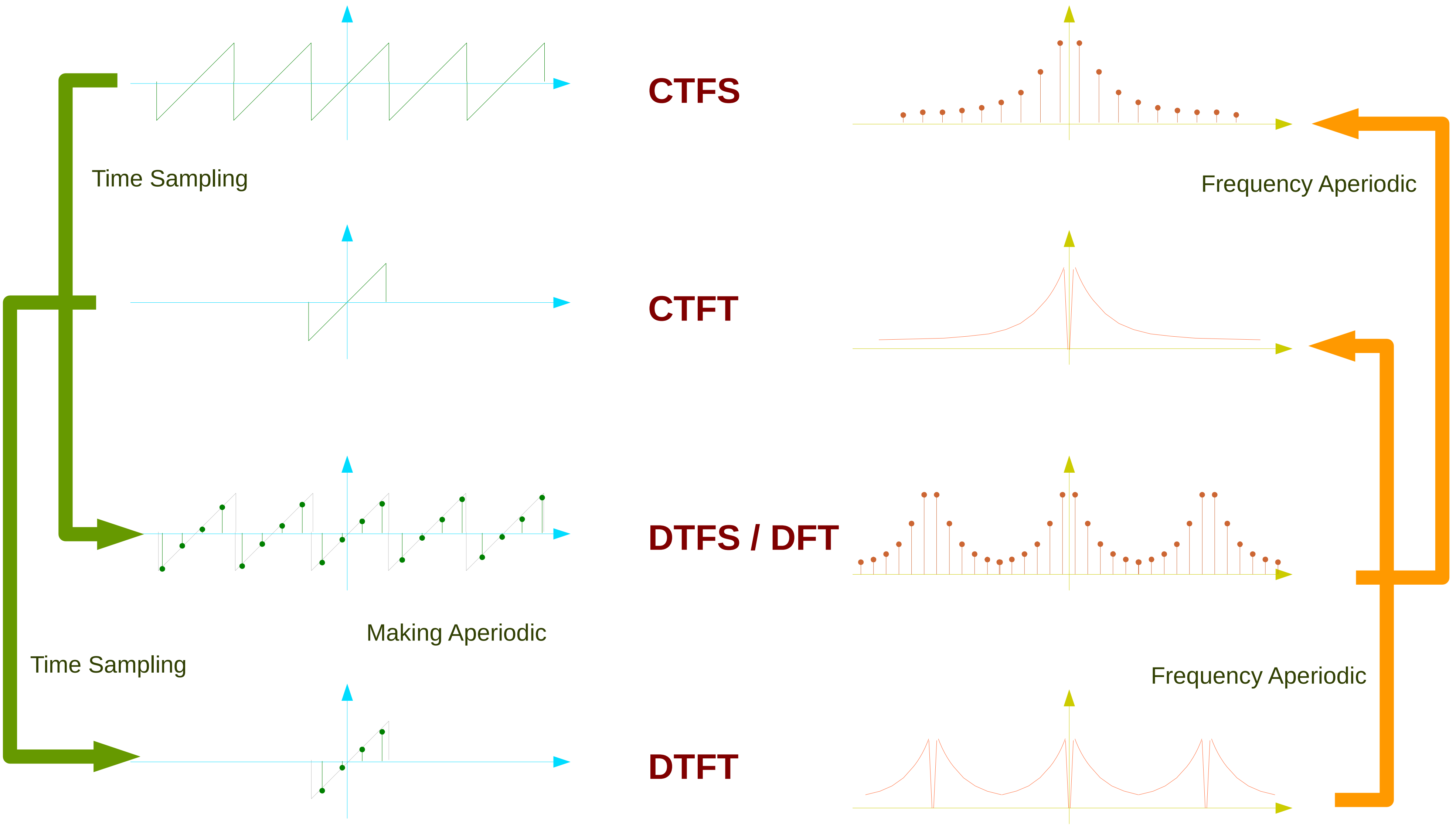
CTFS CTFT
DTFS DTFT

Continuous Time ← Frequency Aperiodic

*****



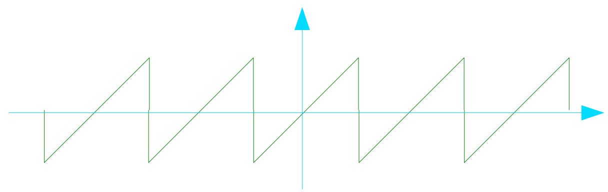
CTFS
Time Aperiodic
Frequency Sampling
CTFT
DTFS / DFT
Time Aperiodic
Frequency Sampling
DTFT


CTFS
Time Sampling
Frequency Aperiodic
CTFT
DTFS / DFT
Making Aperiodic
Time Sampling
Frequency Aperiodic
DTFT

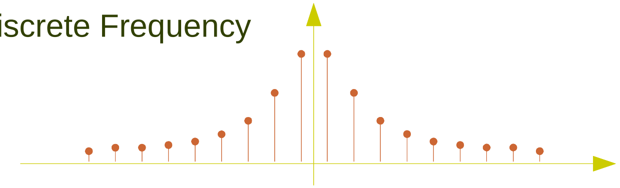
9 CTFS → DTFT

Continuous Time
Fourier Series
CTFS

Continuous Time



Discrete Frequency



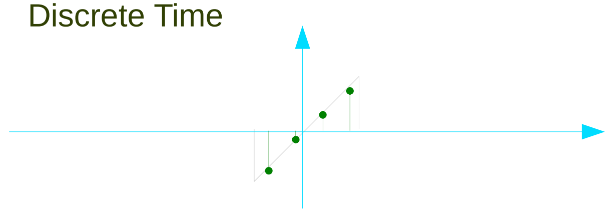
CTFS	**CTFT**
DTFS	**DTFT**

Time Aperiodic → **Continuous Frequency**

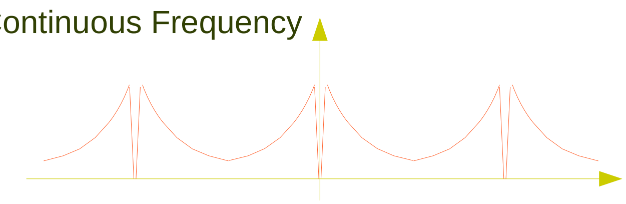
Time Sampling → **Frequency Periodic**

Discrete Time
Fourier Transform
DTFT

Discrete Time



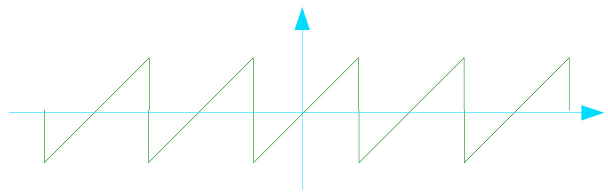
Continuous Frequency



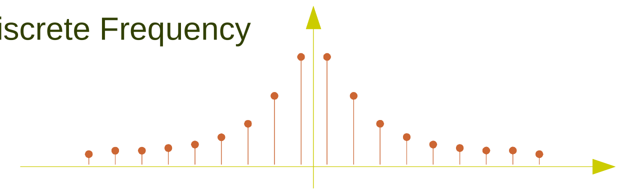
10 CTFS ← DTFT

Continuous Time
Fourier Series
CTFS

Continuous Time



Discrete Frequency



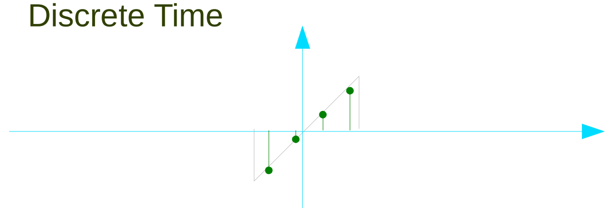
CTFS	**CTFT**
DTFS	**DTFT**

Time Periodic ← **Frequency Sampling**

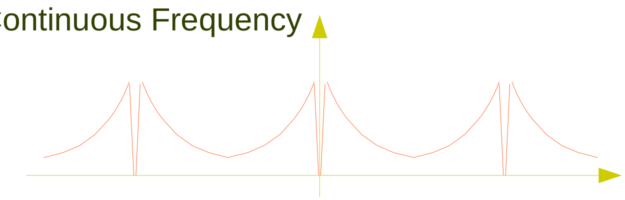
Continuous Time ← **Frequency Aperiodic**

Discrete Time
Fourier Transform
DTFT

Discrete Time

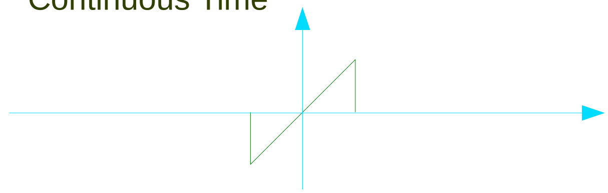


Continuous Frequency



Continuous Time
Fourier Transform
CTFT

Continuous Time



Continuous Frequency

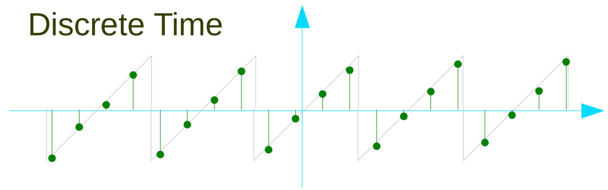
CTFS	**CTFT**
DTFS	**DTFT**

Time Periodic ← **Frequency Sampling**

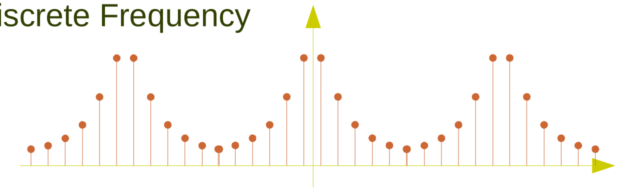
Time Sampling → **Frequency Periodic**

Discrete Time
Fourier Series
DTFS / DFT

Discrete Time

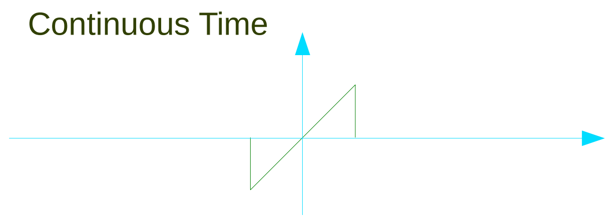


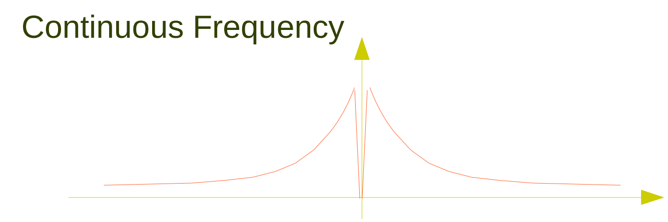
Discrete Frequency



12. CTFT ← DTFS

Continuous Time
Fourier Transform
CTFT



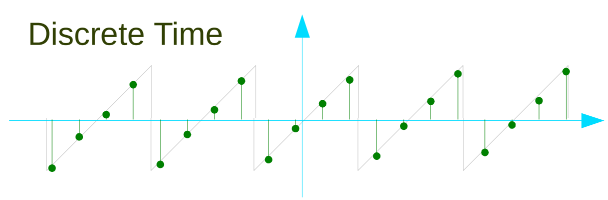


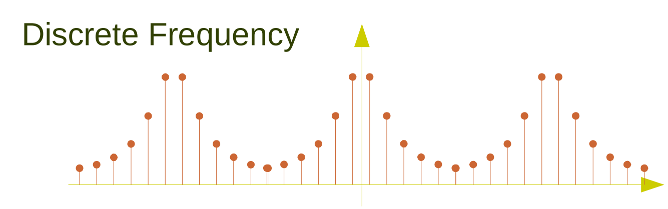
CTFS	**CTFT**
DTFS	**DTFT**

Time Aperiodic → **Continuous Frequency**

Continuous Time ← **Frequency Aperiodic**

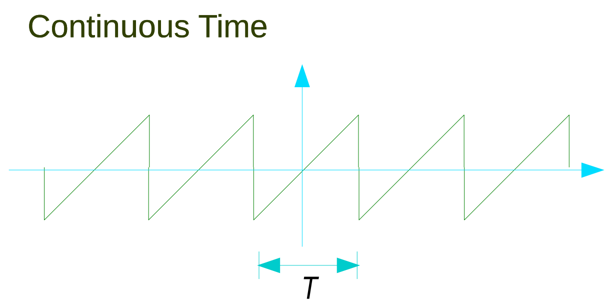
Discrete Time
Fourier Series
DTFS / DFT

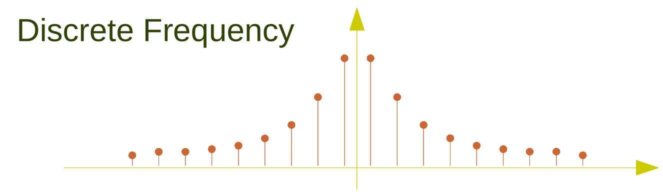




CTFS & DTFT

Continuous Time
Fourier Series
CTFS



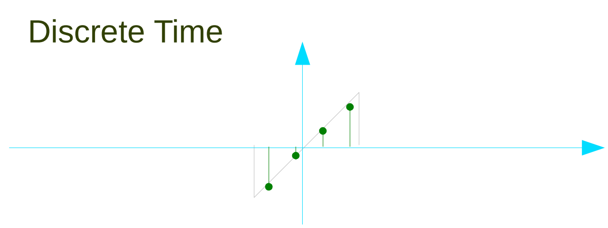


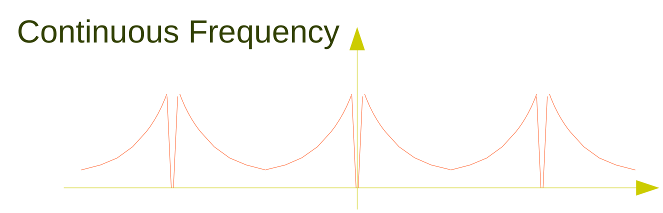
Periodic in time ← **Sampled in frequency**

CTFS	**CTFT**
DTFS	**DTFT**

Sampled in time → **Periodic in frequency**

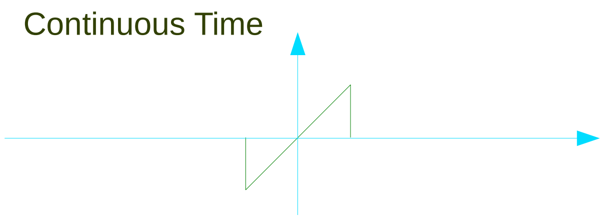
Discrete Time
Fourier Transform
DTFT

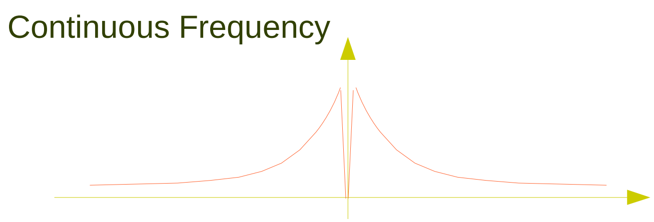




CTFT & DTFS

Continuous Time
Fourier Transform
CTFT



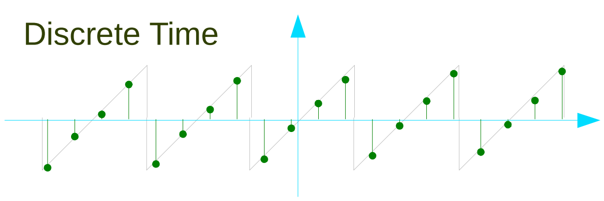


Sampling in time → **Periodic in frequency**

CTFS **CTFT**
DTFS **DTFT**

Periodic in time ← **Sampled in frequency**

Discrete Time
Fourier Series
DTFS / DFT



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