CT Pulse Function Pairs (1B)

- Continuous Time Pulse Function Pairs

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7/6/13

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Fourier Transform Types

Continuous Time Fourier **Series**

CTFS

$$C_k = \frac{1}{T_0}\int_{-T_0/2}^{+T_0/2} x_{T_0}(t)\, e^{-jk\omega_0 t}\, dt \quad \Leftrightarrow \quad x_{T_0}(t) = \sum_{k=-\infty}^{\infty} C_k\, e^{+jk\omega_0 t}$$

$$\omega = k\omega_0 \qquad f = \frac{k}{T_0}$$

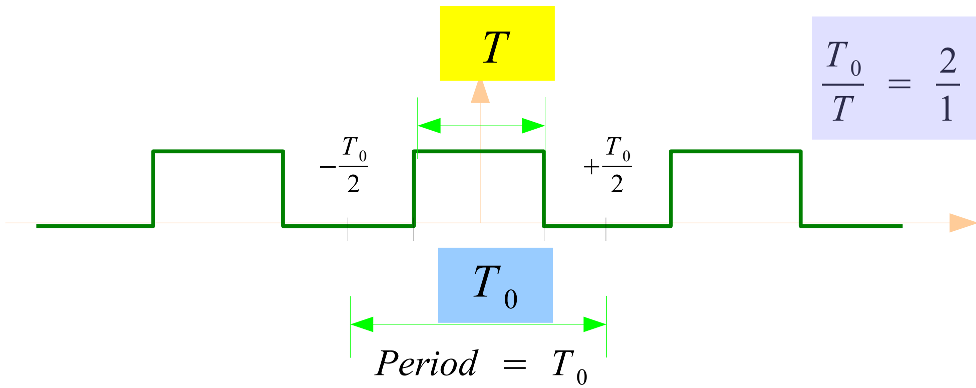
$$T_0 \to \infty, \quad \omega_0 \to d\omega \quad \left(\frac{2\pi}{T_0} \to d\omega\right), \quad k\omega_0 \to \omega \quad \Rightarrow \quad x_{T_0} \to x(t), \quad C_k T_0 \to X(j\omega)$$

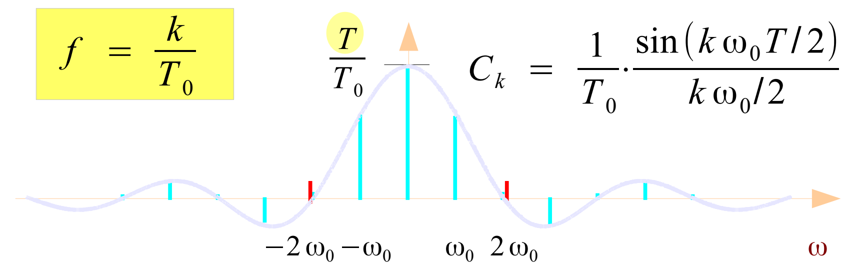
Continuous Time Fourier **Transform**

CTFT

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

CTFS and CTFT

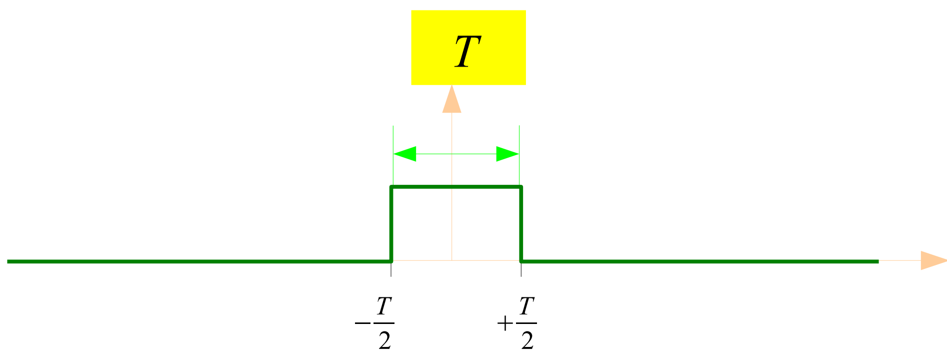


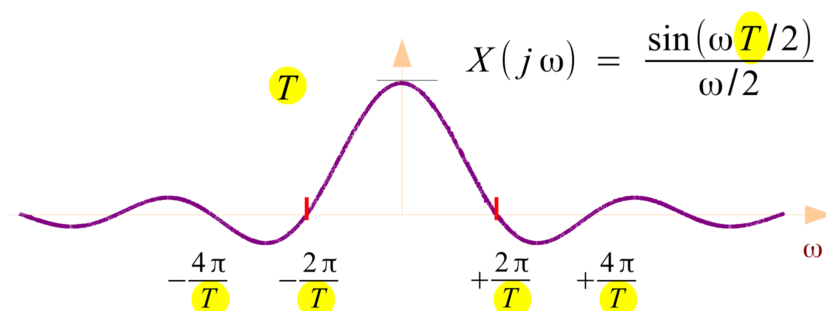


CTFS (Continuous Time Fourier Series)

$$C_k = \frac{1}{T_0} \cdot \frac{\sin(k\omega_0 T/2)}{k\omega_0/2} = \frac{T}{T_0} \cdot \mathbf{sinc}\left(k\frac{T}{T_0}\right)$$

$$C_0 = \frac{T}{T_0}$$





CTFT (Continuous Time Fourier Transform)

$$X(j\omega) = \frac{\sin(\omega T/2)}{\omega/2} = T \cdot \mathbf{sinc}(fT)$$

$$X(j0) = T$$

- Relation between CTFS and CTFT

CTFS and CTFT

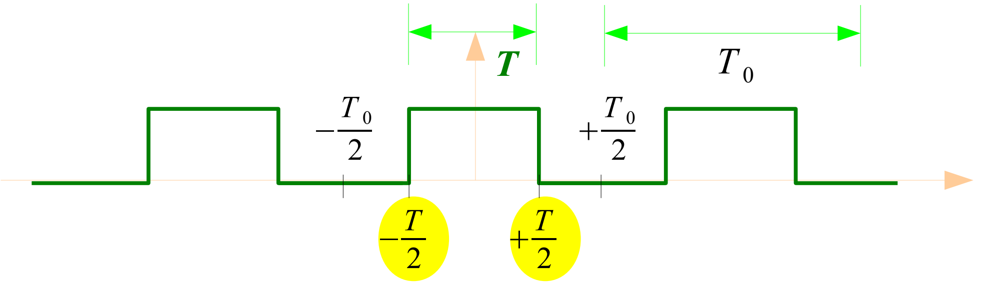
Continuous Time Fourier **Series**

Periodic Continuous Time Signal

$$C_k = \frac{1}{T_0}\int_{-T_0/2}^{+T_0/2} x_{T_0}(t)\, e^{-jk\omega_0 t}\, dt \quad \Longleftrightarrow \quad x_{T_0}(t) = \sum_{k=-\infty}^{\infty} C_k\, e^{+jk\omega_0 t}$$

$$C_k = \frac{1}{T_0}\cdot\frac{\sin(k\omega_0 T/2)}{k\omega_0/2} = \frac{T}{T_0}\cdot \mathit{sinc}\left(k\frac{T}{T_0}\right)$$

CTFS



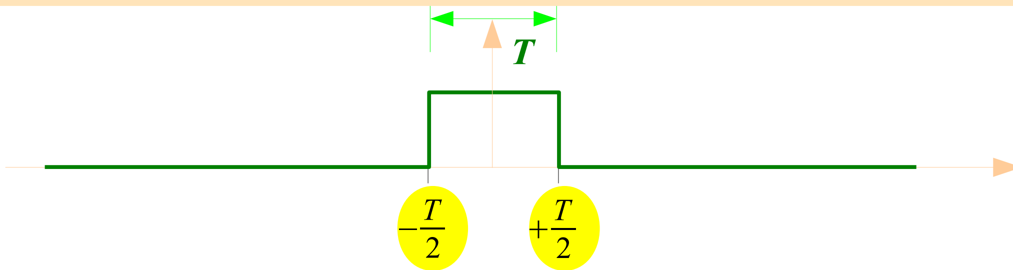
Continuous Time Fourier **Transform**

Aperiodic Continuous Time Signal

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Longleftrightarrow \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

$$X(j\omega) = \frac{\sin(\omega T/2)}{\omega/2} = T\cdot \mathit{sinc}(fT)$$

CTFT

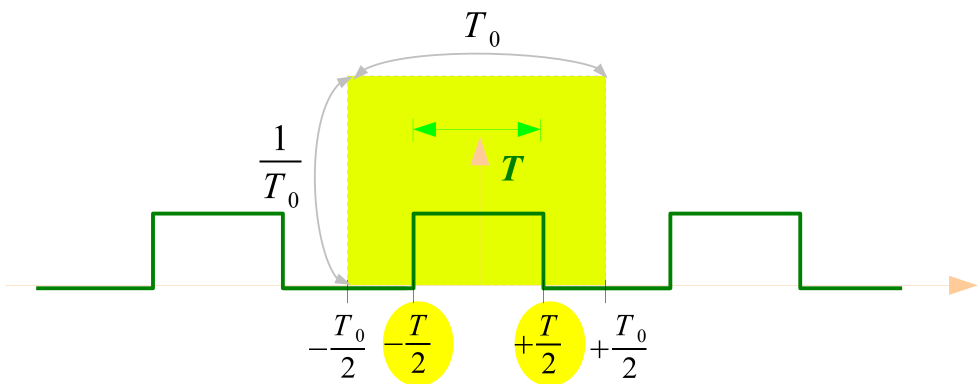


CTFS and CTFT – in the time domain

CTFS (Continuous Time Fourier Series)

$$x_{T_0}(t) = \sum_{k=-\infty}^{\infty} C_k \, e^{+jk\omega_0 t}$$

$$x_{T_0}(t) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} C_k \, T_0 \, e^{+jk\omega_0 t} \left(\frac{2\pi}{T_0}\right)$$



$T_0 \to \infty$

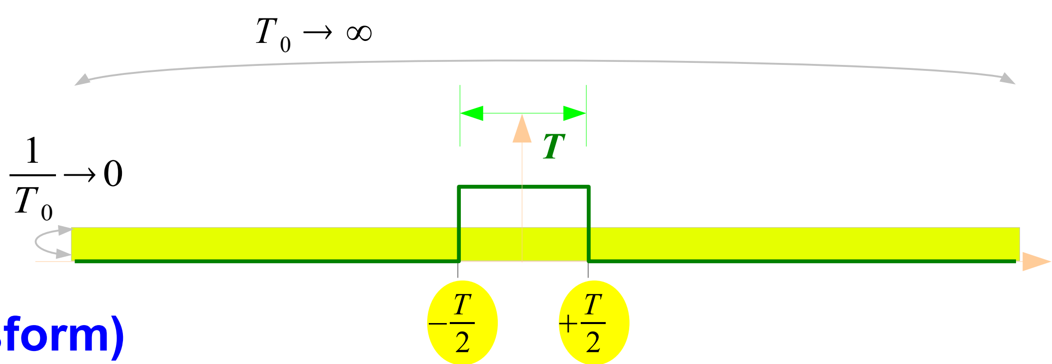
$\omega_0 \to 0$

$k\omega_0 \to \omega$

$\frac{2\pi}{T_0} \to d\omega$

$x_{T_0}(t) \to x(t)$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) \, e^{+j\omega t} \, d\omega$$



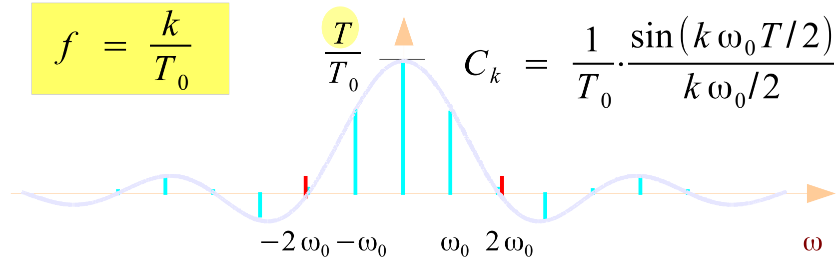
CTFT (Continuous Time Fourier Transform)

CTFS and CTFT – in the frequency domain

CTFS (Continuous Time Fourier Series)

$$C_k = \frac{1}{T_0}\int_{-T_0/2}^{+T_0/2} x_{T_0}(t)\, e^{-jk\omega_0 t}\, dt$$

$$f = \frac{k}{T_0}$$

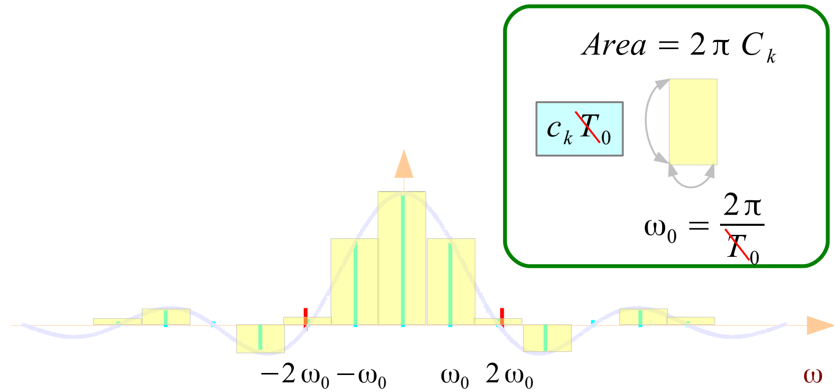


$$C_k T_0 = \int_{-T_0/2}^{+T_0/2} x_{T_0}(t)\, e^{-jk\omega_0 t}\, dt$$

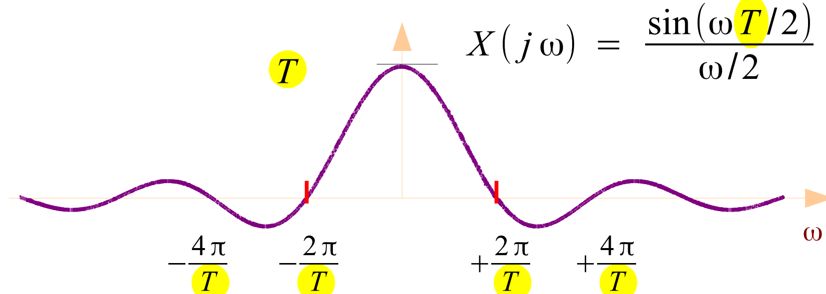
$T_0 \to \infty$

$k\omega_0 \to \omega$

$x_{T_0}(t) \to x(t)$

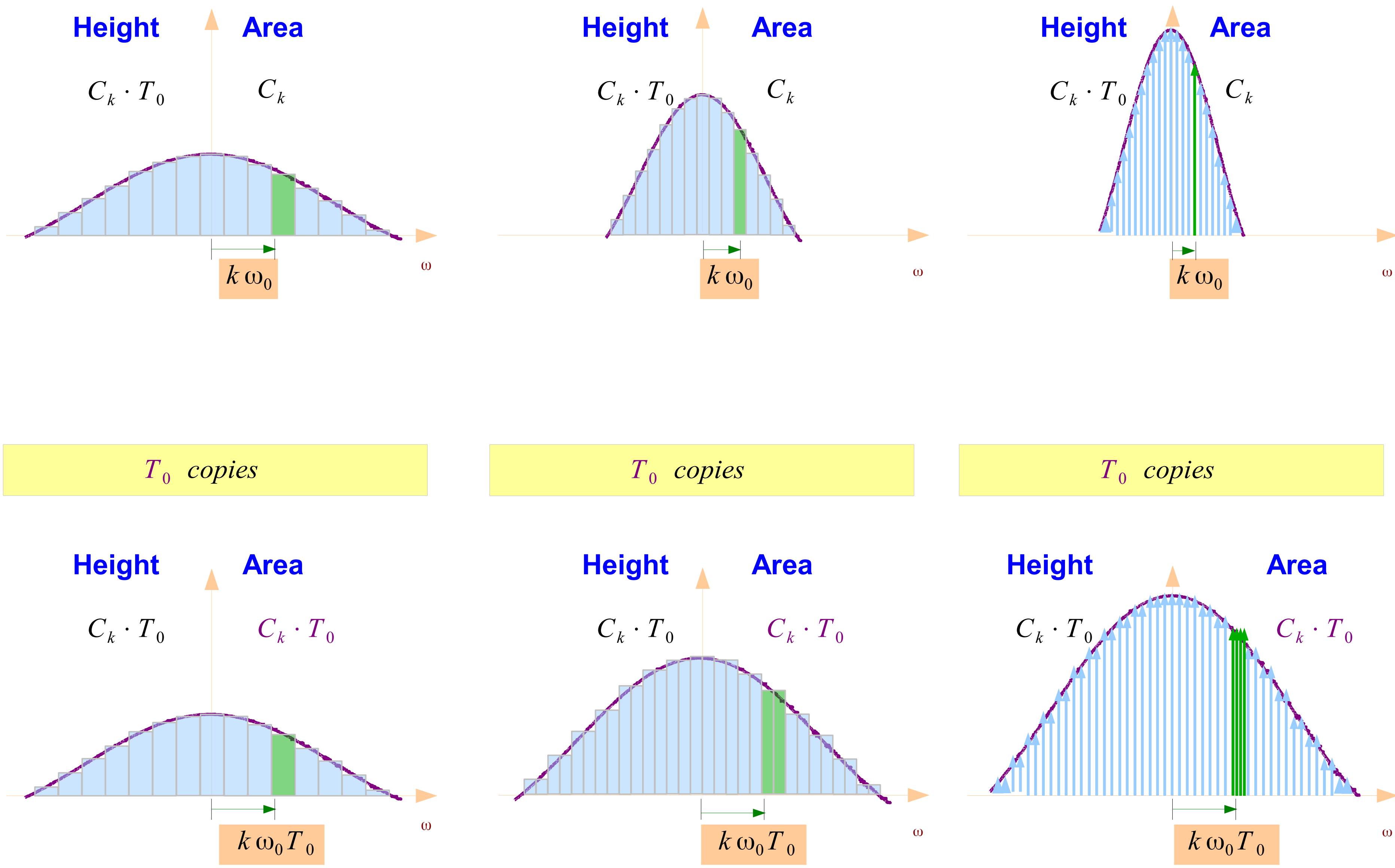


$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

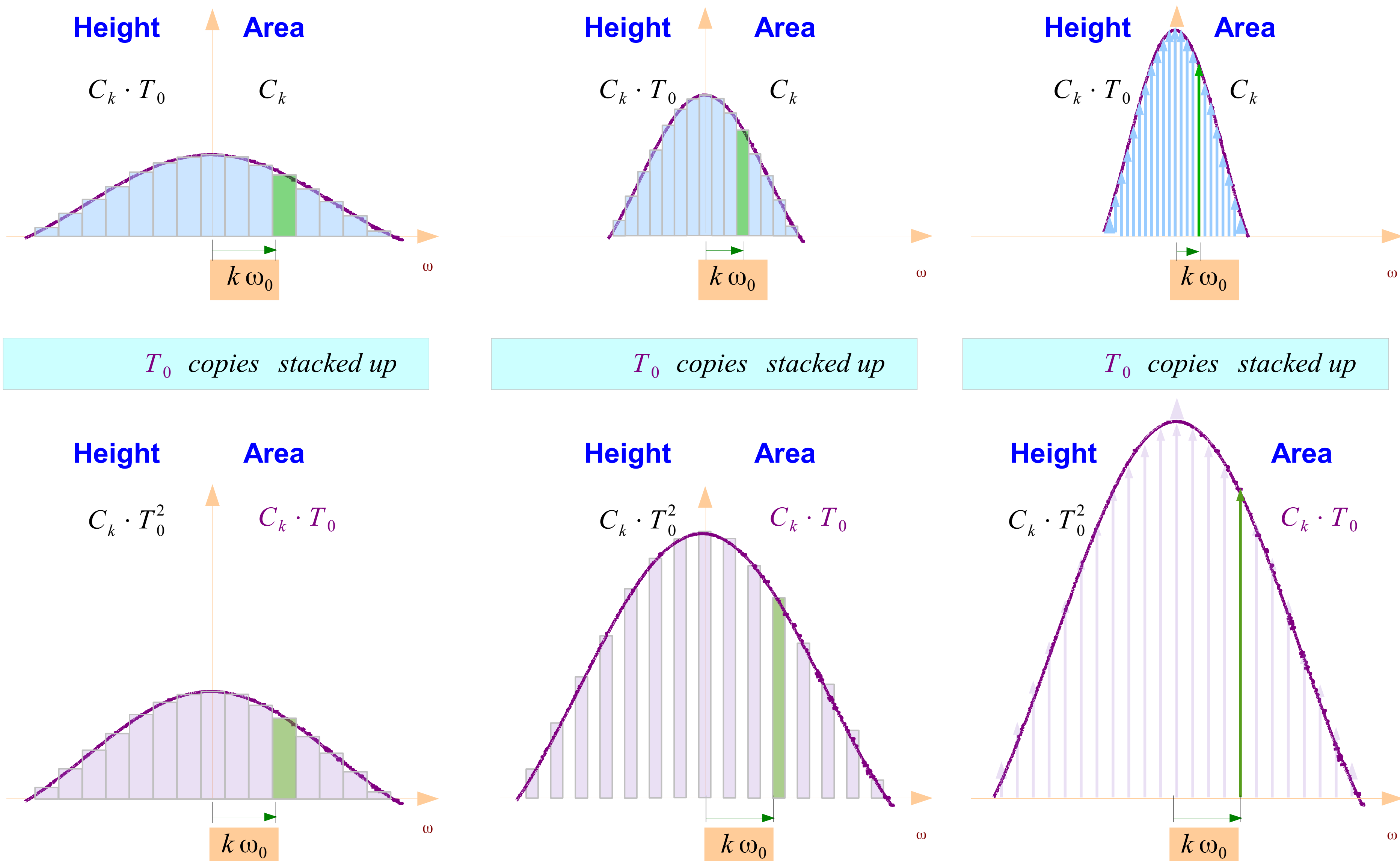


CTFT (Continuous Time Fourier Transform)

Impulse Train Weights (1)

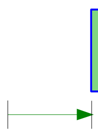


Impulse Train Weights (2)



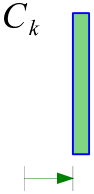
Impulse Train Weights (3)

Height $C_k \cdot T_0$ **Area** C_k



$k\omega_0$ $\frac{1}{T_0} = \frac{1}{a}$

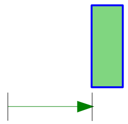
Height $C_k \cdot T_0$ **Area** C_k



$k\omega_0$ $\frac{1}{T_0} = \frac{1}{2a}$

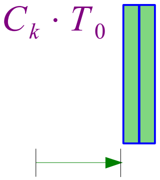
Weight C_k

Height $C_k \cdot T_0$ **Area x no** $C_k \cdot T_0$



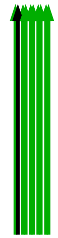
$k\omega_0 T_0$ $\frac{1}{T_0} = \frac{1}{a}$

Height $C_k \cdot T_0$ **Area x no** $C_k \cdot T_0$



$k\omega_0 T_0$ $\frac{1}{T_0} = \frac{1}{2a}$

Weight C_k



Height $C_k \cdot T_0^2$ **Area x 1** $C_k \cdot T_0$

$k\omega_0$ $\frac{1}{T_0} = \frac{1}{a}$

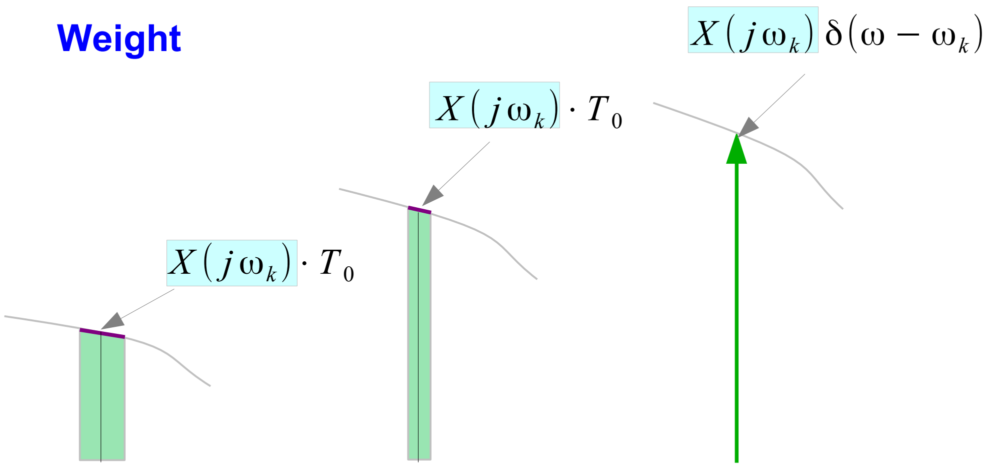
Height $C_k \cdot T_0^2$ **Area x 1** $C_k \cdot T_0$

$k\omega_0$ $\frac{1}{T_0} = \frac{1}{2a}$

Weight $C_k \cdot T_0$

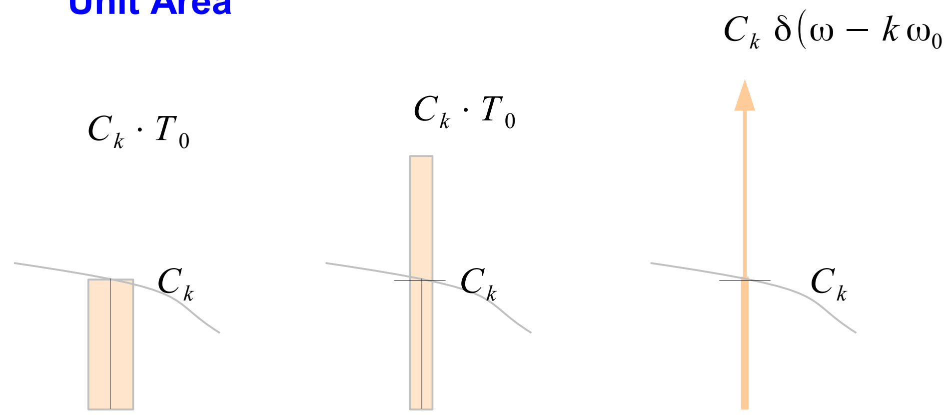
Impulse Train's Sampling Property

Weight



$\frac{1}{T_0} = \frac{1}{a}$ $\quad$ $\frac{1}{T_0} = \frac{1}{2a}$ $\quad$ $\frac{1}{T_0} \to 0$

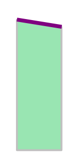
Unit Area



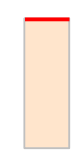
$\frac{1}{T_0} = \frac{1}{a}$ $\quad$ $\frac{1}{T_0} = \frac{1}{2a}$ $\quad$ $\frac{1}{T_0} \to 0$

Sampling Property

$$X(j\omega_k) = \int_{-\infty}^{+\infty} X(j\omega)\,\delta(\omega-\omega_k)\,d\omega$$



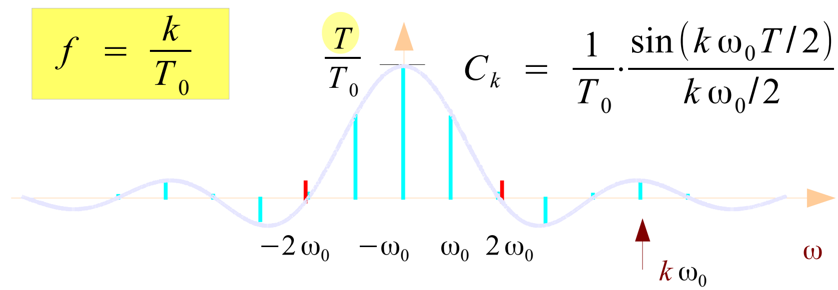
$$X(j\omega') \neq C_k\cdot T_0\cdot\frac{1}{T_0} = C_k$$

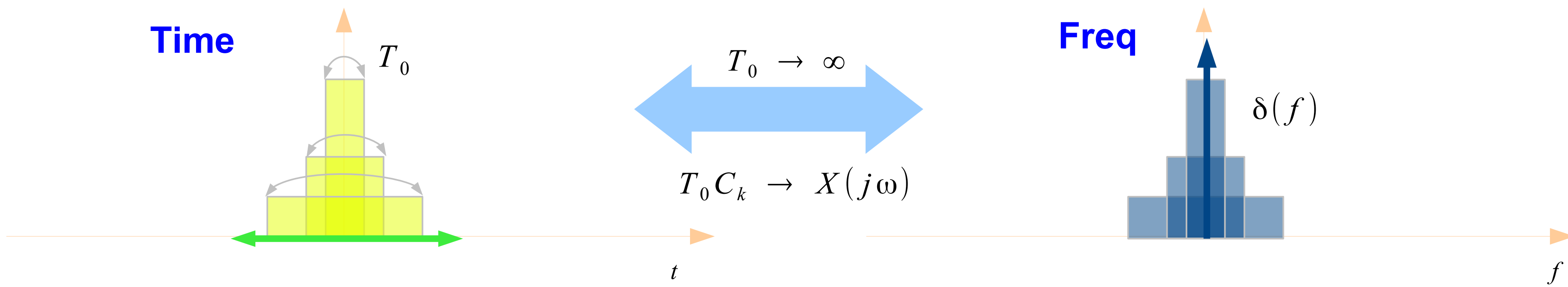


CTFS and CTFT Frequency Components

$$C_k = \frac{1}{T_0} \cdot \frac{\sin(k\omega_0 T/2)}{k\omega_0/2} = \frac{T}{T_0} \cdot \textit{\textbf{sinc}}\left(k\frac{T}{T_0}\right)$$

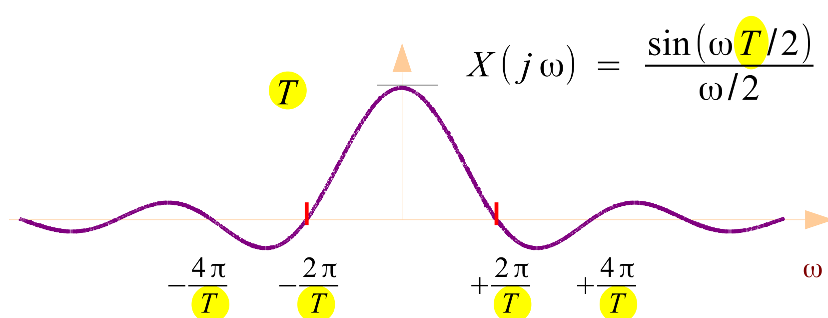
$$C_0 = \frac{T}{T_0}$$





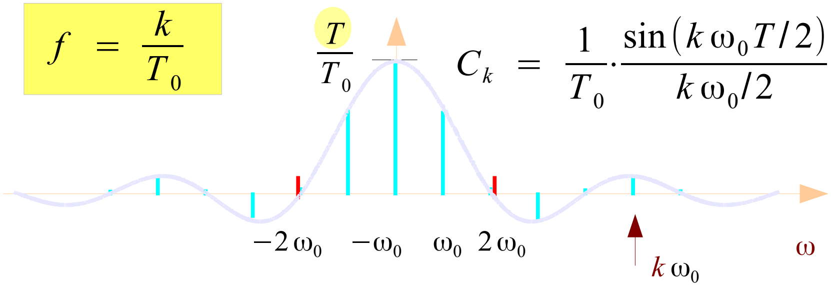
$$X(j\omega) = \frac{\sin(\omega T/2)}{\omega/2} = T \cdot \textit{\textbf{sinc}}(fT)$$

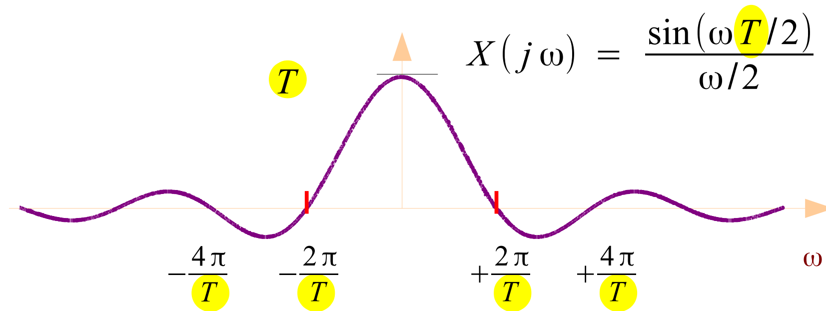
$$X(j0) = T$$



CTFS and CTFT Frequency Components

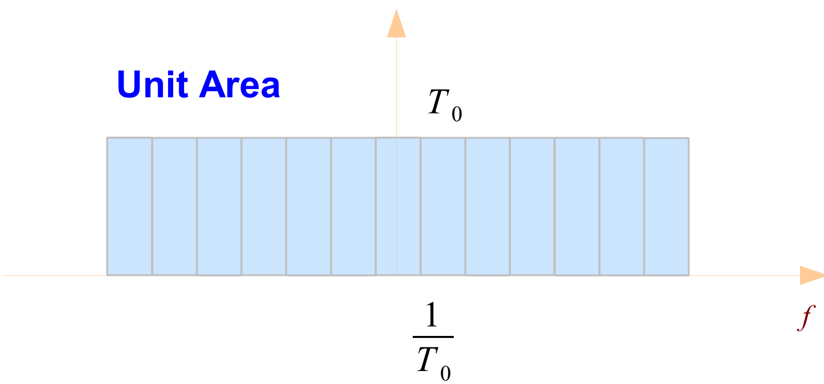
$$f = \frac{k}{T_0}$$

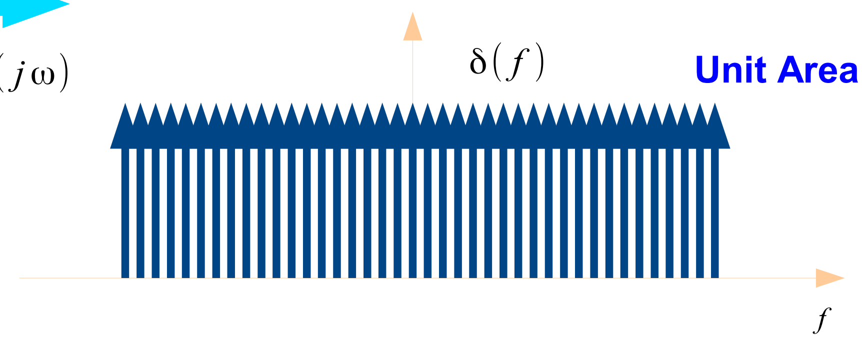




$$T_0 \rightarrow \infty$$

$$T_0 C_k \rightarrow X(j\omega)$$





$$C_k = \frac{1}{T_0}\cdot\frac{\sin(k\omega_0 T/2)}{k\omega_0/2} = \boxed{\frac{T}{T_0}\cdot \mathbf{sinc}\left(k\frac{T}{T_0}\right)}$$

$$C_0 = \frac{T}{T_0}$$

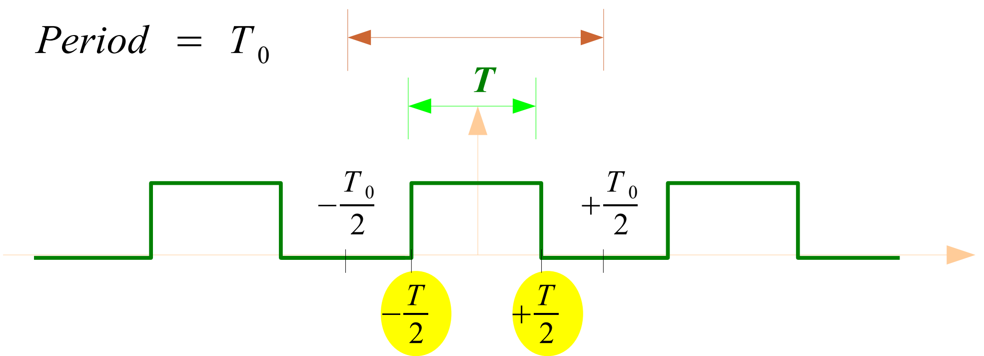
$$X(j\omega) = \frac{\sin(\omega T/2)}{\omega/2} = \boxed{T\cdot \mathbf{sinc}(f\,T)}$$

$$X(j\,0) = T$$

CTFS → CTFT

CTFS

$$C_k = \frac{1}{T_0} \cdot \frac{\sin(k\omega_0 T/2)}{k\omega_0/2} = \frac{T}{T_0} \cdot \mathbf{sinc}\left(k\frac{T}{T_0}\right)$$



$$x_{T_0}(t) = \sum_{n=-\infty}^{+\infty} x(t - nT_0)$$

$\omega = k\omega_0$

$f = \frac{k}{T_0}$

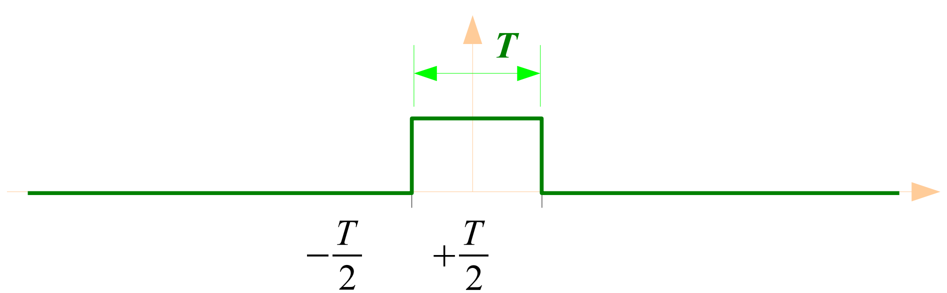
$T_0 \to \infty$
$\omega_0 \to 0$

$$\omega_0 = \frac{2\pi}{T_0}$$

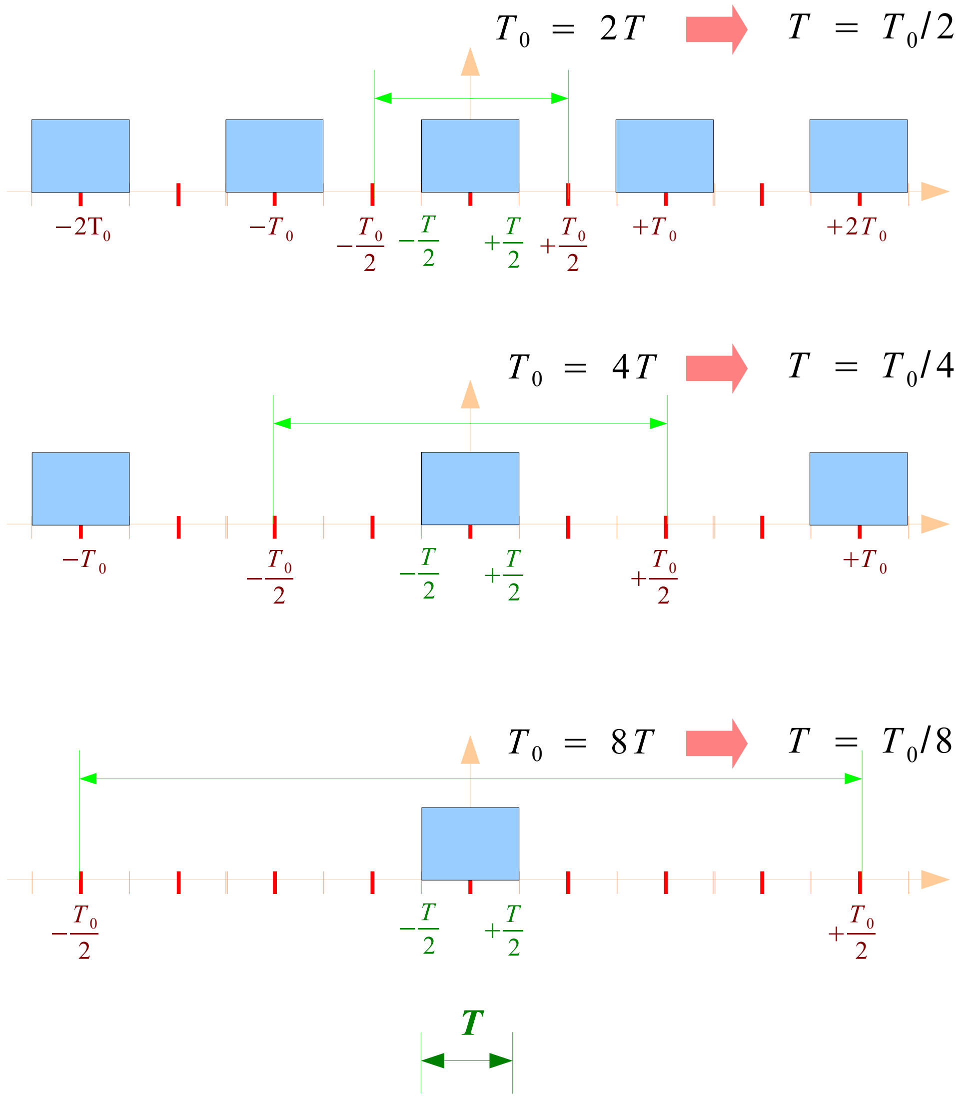
$$x(t)$$

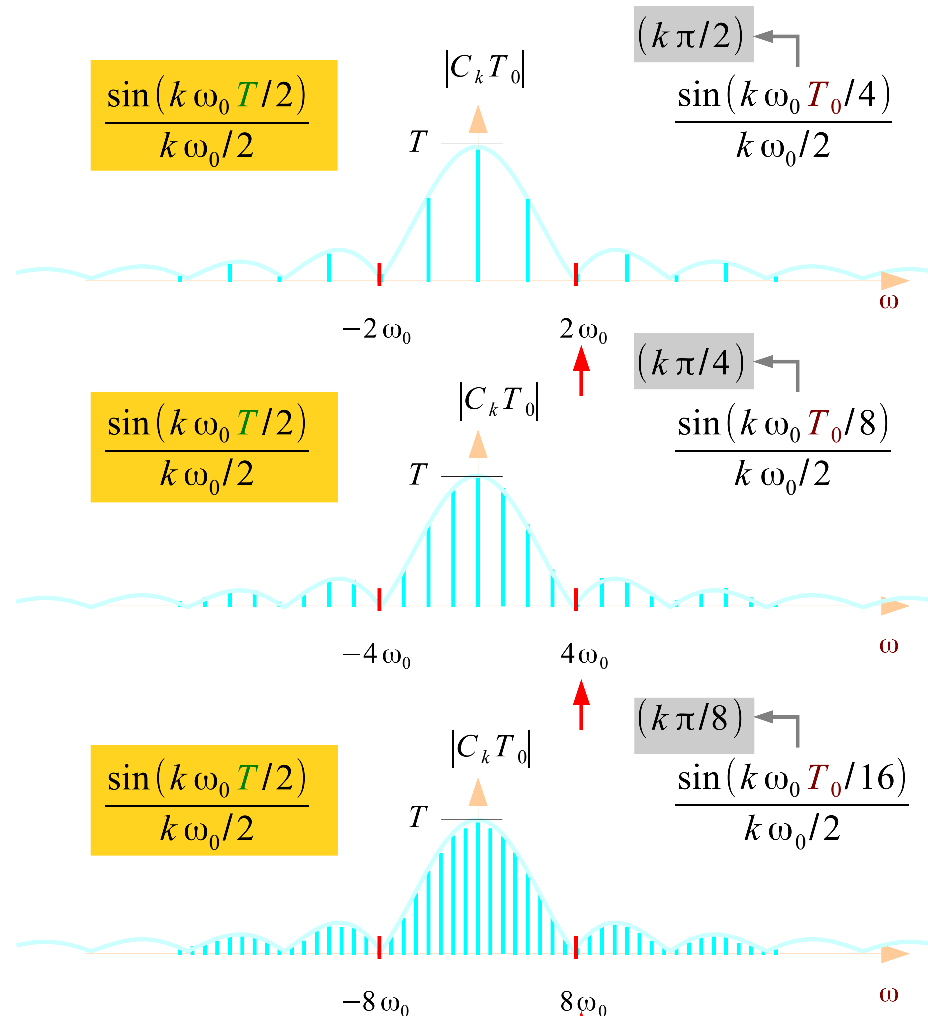
CTFT

$$X(j\omega) = \frac{\sin(\omega T/2)}{\omega/2} = T \cdot \mathbf{sinc}(f T)$$



CTFT and CTFS as $T_0 \to \infty$ (1)



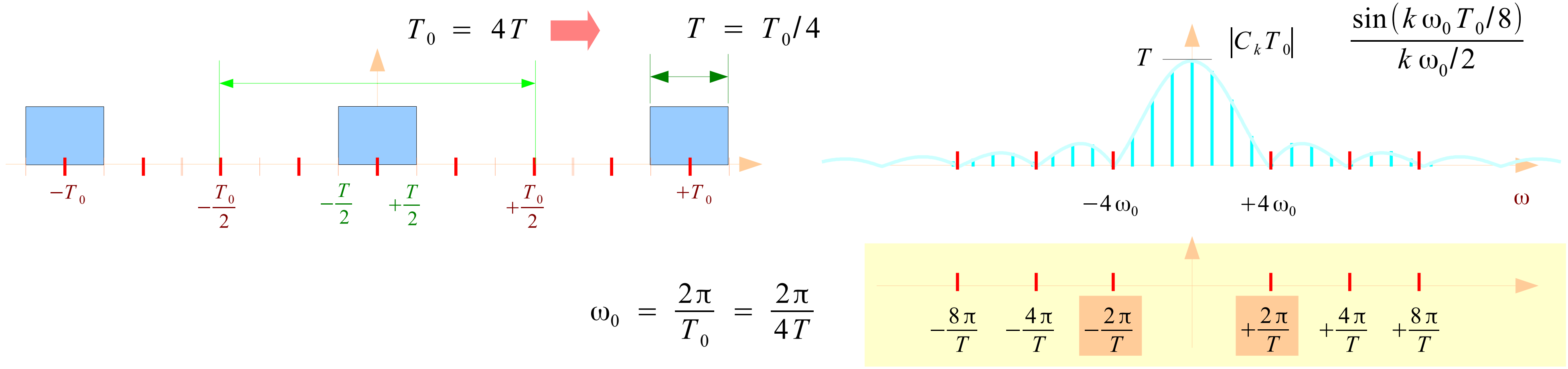


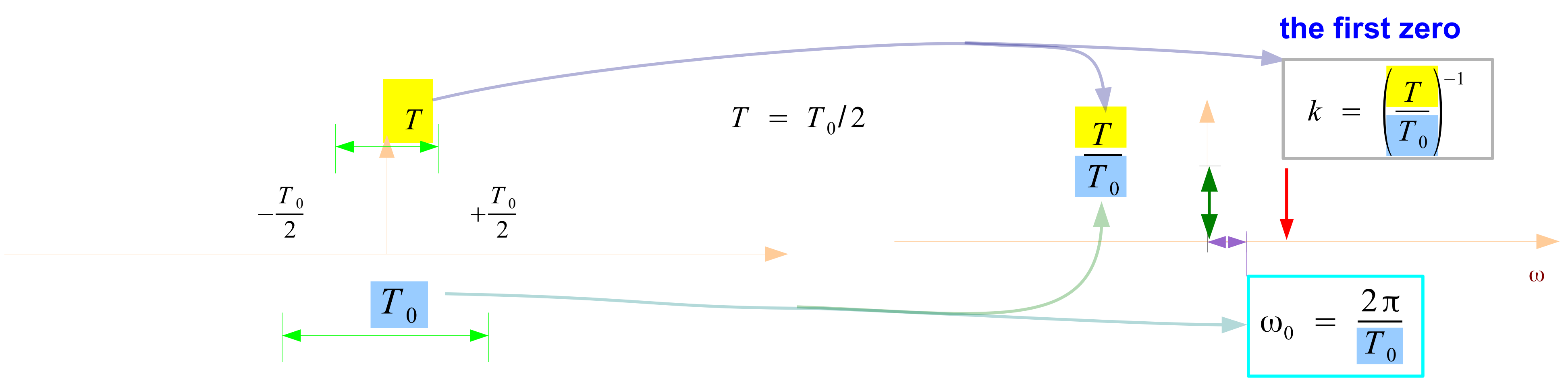
the first zero

$$\omega_0 = \frac{2\pi}{T_0} \Rightarrow \frac{k\omega_0 T}{2} = k\pi\left(\frac{T}{T_0}\right)$$

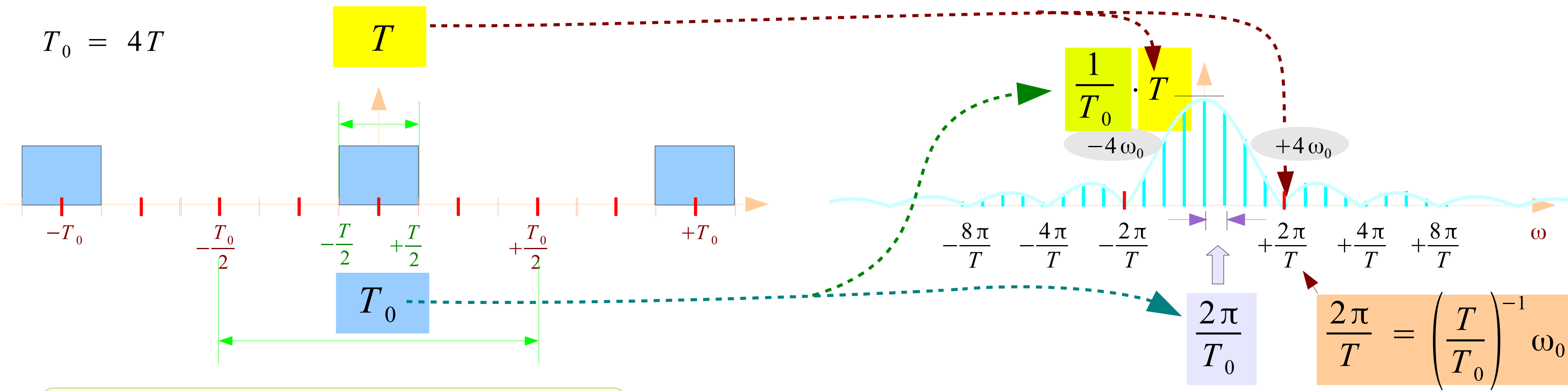
$$k = \left(\frac{T}{T_0}\right)^{-1}$$

CTFT and CTFS as $T_0 \to \infty$ (2)





CTFT of a Rect(t/T) function (3)



$$C_k T_0 = \frac{\sin(k\omega_0 T/2)}{k\omega_0/2}$$

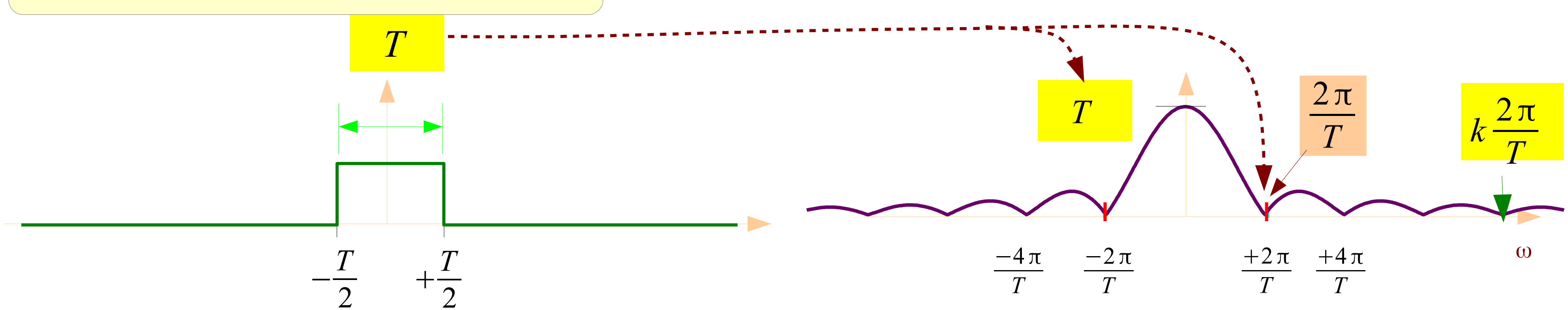
$T_0 \to \infty$

$k\omega_0 \to \omega$

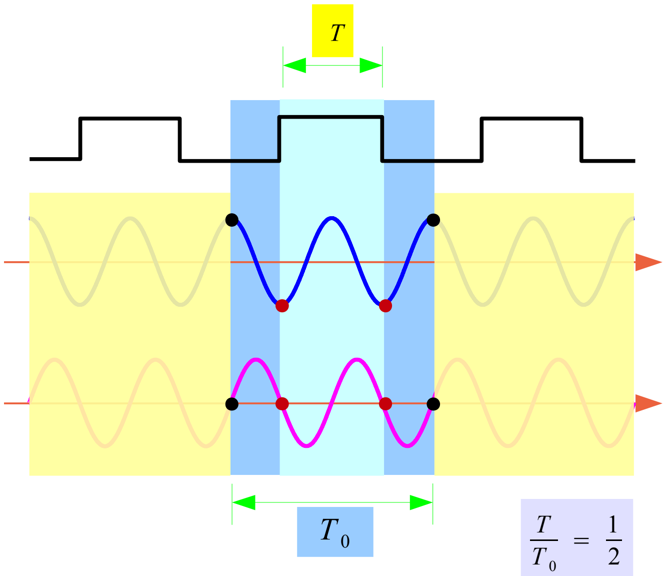
$$X(j\omega) = \frac{\sin(\omega T/2)}{\omega/2}$$

$$C_k = \frac{1}{T_0} \frac{\sin(k\omega_0 T/2)}{k\omega_0/2} = \frac{T}{T_0} \textbf{sinc}\left(k \frac{T}{T_0}\right)$$

$$X(j\omega) = \frac{\sin(\omega T/2)}{\omega/2} = T\,\textbf{sinc}(f\,T)$$



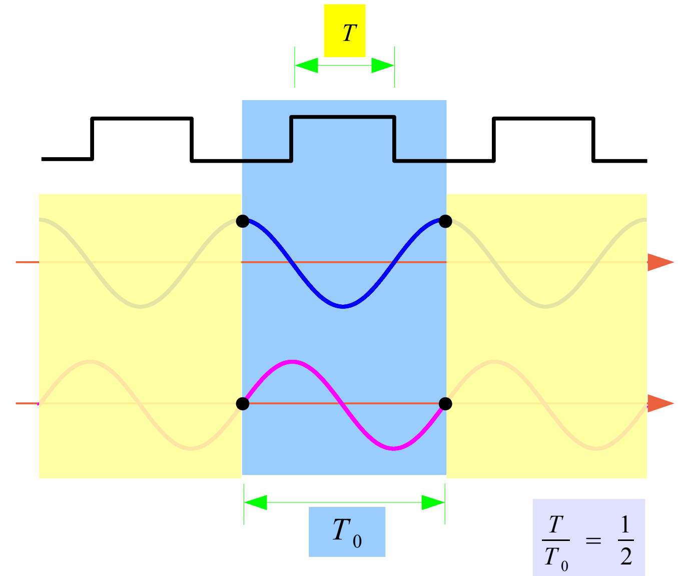
CTFT of a Rect(t/T) function (4)



$$\omega = \frac{2\pi}{T}$$

$\cos \omega t$

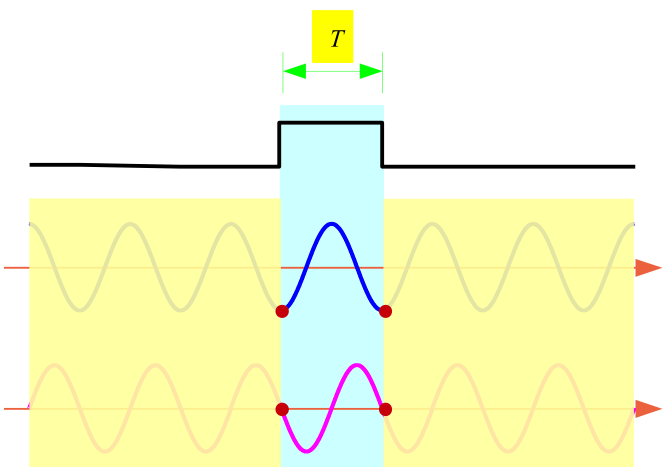
$\sin \omega t$



$$\omega_0 = \frac{2\pi}{T_0}$$

$\cos \omega_0 t$

$\sin \omega_0 t$



$$\omega = \frac{2\pi}{T}$$

$\cos \omega t$

$\sin \omega t$

$$\omega = k\,\omega_0 = \left(\frac{T}{T_0}\right)^{-1} \omega_0 = 2\,\omega_0$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] M. J. Roberts, Fundamentals of Signals and Systems
[4] S. Haykin, An Introduction to Analog & Digital Communications