# Feedback System (H.1)
# State Equations

20150613

# Differentiator & Integrator

$$f(t) \Longleftrightarrow F(s)$$

$$f'(t) \Longleftrightarrow s\,F(s)$$

$$\int_{-\infty}^{t} f(t)\,dt \Longleftrightarrow \frac{F(s)}{s}$$

$\boxed{\text{differentiator}}$ 미분기

$$c(t) = \frac{d}{dt} r(t)$$

$$\underset{R(s)}{\overset{r(t)}{\circ}} \xrightarrow{\quad s \quad} \circ$$

$$C(s) = s\,R(s)$$

Transfer function $= \dfrac{Y(s)}{X(s)} = \dfrac{C(s)}{R(s)} = s$

$s = \sigma + j\omega$

$G(s) = s$

Frequency Response $\quad s \leftarrow j\omega$

$$|G(j\omega)| = |j\omega| = \omega$$

$\boxed{\text{integrator}}$ 적분기

$$c(t) = \int_{-\infty}^{t} r(t)\,dt$$

$$\underset{R(s)}{\overset{r(t)}{\circ}} \xrightarrow{\quad \frac{1}{s} \quad} \circ$$

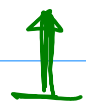
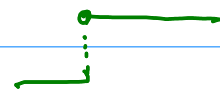
$$C(s) = \frac{R(s)}{s}$$

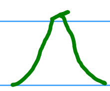
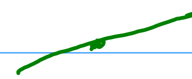
# State

$\boxed{\text{integrator}}$

$$r(t) \xrightarrow{\frac{1}{s}} c(t) = \int_{-\infty}^{t} r(t)\,dt$$

$$R(s) \qquad C(s) = \frac{R(s)}{s}$$

ideal  

pract  $c(t) = \int_{-\infty}^{t} r(t)\,dt$ 

$$\frac{d}{dt} c(t) = r(t)$$

$$r(t) \longrightarrow \boxed{\phantom{\frac{1}{s}}} \longrightarrow c(t) = \int_{-\infty}^{t} r(t)\,dt$$

$$R(s) \longrightarrow \boxed{\frac{1}{s}} \longrightarrow C(s) = R(s) \cdot \frac{1}{s}$$

# State diagram

integrator + 비례기

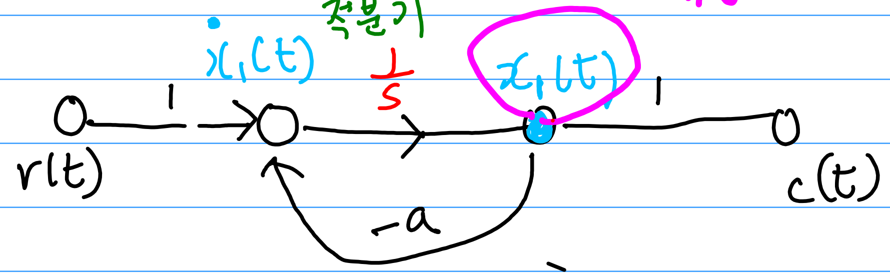
상수

$\frac{1}{s}$

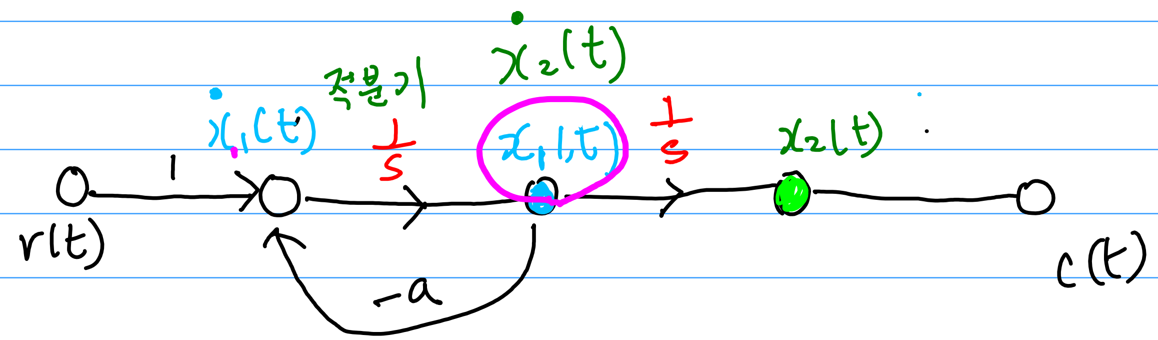
2권 3-17

적분 된 것 → State

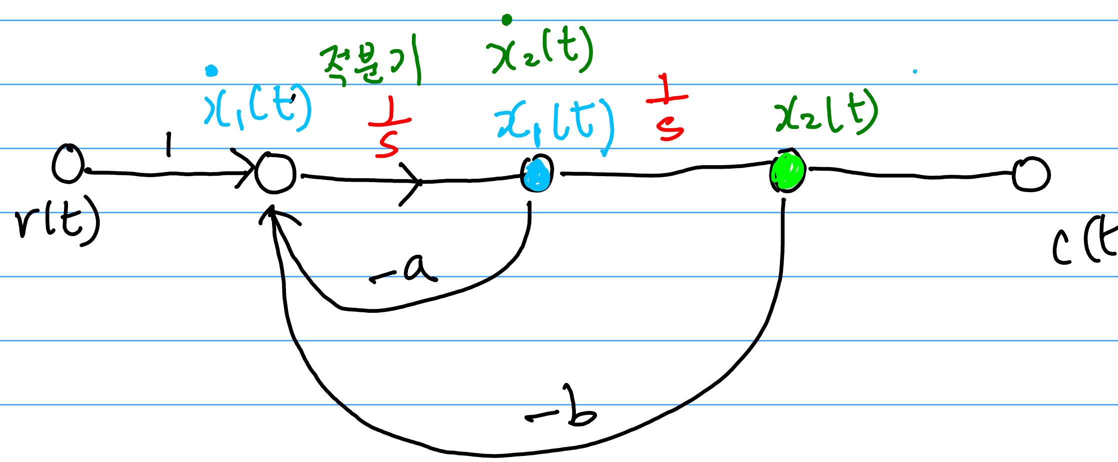
(A)



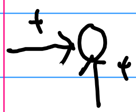

(B)



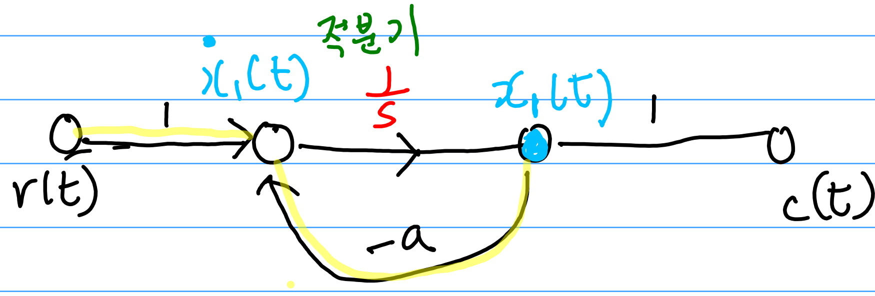

(C)



SFG



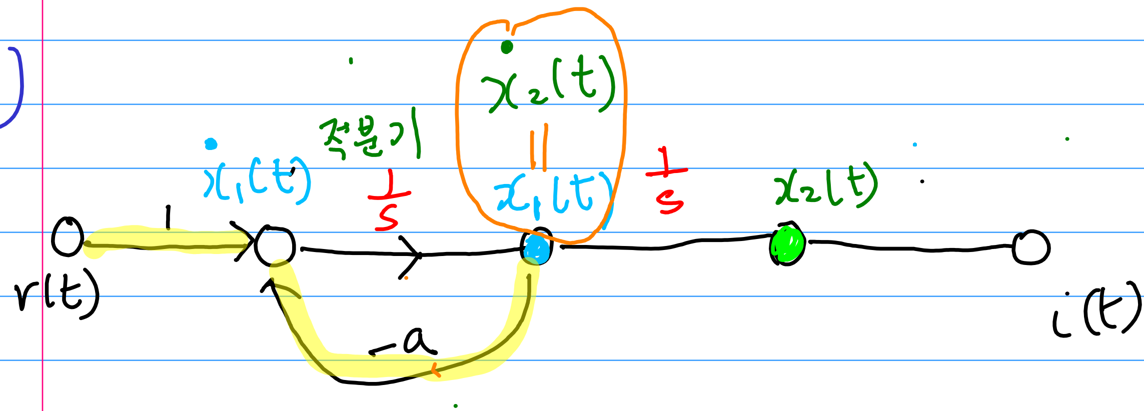
(A)




$$\underbrace{\dot{x}_1(t)}_{\text{state의 미분한것}} = -a \underbrace{x_1(t)}_{\text{state}} + r(t) \qquad \boxed{\text{state eq}}$$

$$c(t) = x_1(t) \qquad \boxed{\text{output eq}}$$

(13)




$$\dot{x}_1(t) = -a\,x_1(t) + r(t)$$
$$\dot{x}_2(t) = x_1(t) \quad ★$$

state eq

• 있는 상태변수

• 이 없는 상태변수

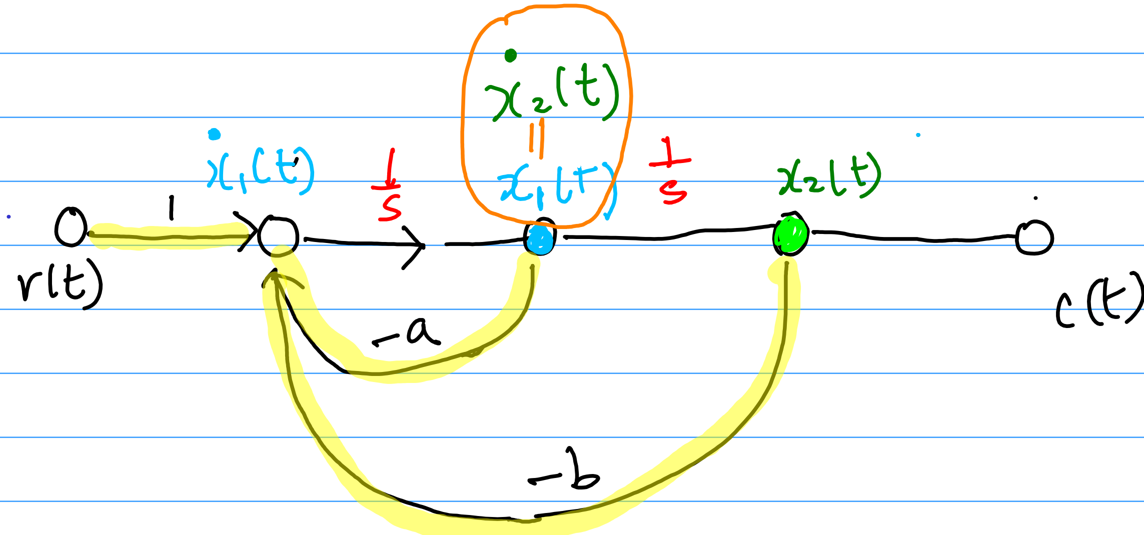
① $\dot{x}_2(t) = x_1(t)$ (OK)

② $\dot{x}_2(t) = \frac{1}{s} \cdot x_1(t)$ ~~(OK)~~

$$c(t) = x_2(t)$$

output equation

(c)




$$\begin{cases} \dot{x}_1(t) = -a x_1(t) - b x_2(t) + r(t) \\ \dot{x}_2(t) = x_1(t) \end{cases}$$ $\boxed{\text{state eq}}$

$$c(t) = x_2(t)$$ $\boxed{\text{output eq}}$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -a & -b \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(t)$$

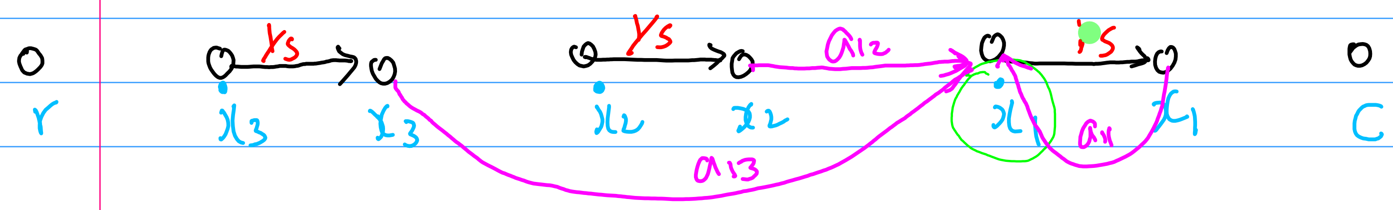
P54)

상수행렬

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix} + \begin{bmatrix}b_1\\ b_2\\ b_3\end{bmatrix} r(t)$$

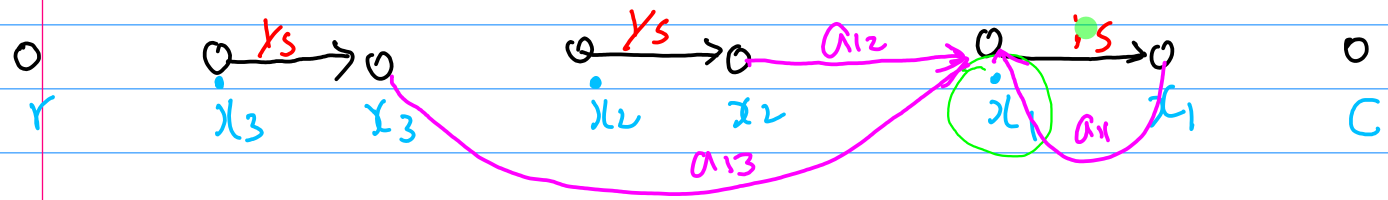
$\cdots$ $\boxed{\text{state eq}}$

$$c(t) = \underline{\begin{bmatrix} c_1 & c_2 & c_3\end{bmatrix}}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix}$$




$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} r(t)$$

$$\dot{x}_1(t) = a_{11}\, x_1(t) + a_{12}\, x_2(t) + a_{13}\, x_3(t) + b_1\, r(t)$$




$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} r(t)$$

$$\dot{x}_1(t) = a_{11}\, x_1(t) + a_{12}\, x_2(t) + a_{13}\, x_3(t) + b_1\, r(t$$

# Transfer function $\longrightarrow$ State Space

$$\text{Transfer function} = \frac{Y(s)}{X(s)} = \frac{C(s)}{R(s)}$$

$$\boxed{G(s) = \frac{b_0}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)}} = \frac{C(s)}{R(s)}$$

$$(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)\,C(s) = b_0\,R(s)$$

$$(s^n C(s) + a_{n-1}s^{n-1}C(s) + \cdots + a_1 s C(s) + a_0 C(s)) = b_0\,R(s)$$

$$\frac{d^n c}{dt^n} + a_{n-1}\underbrace{\frac{d^{n-1}c}{dt^{n-1}}}_{=x_n} + \cdots + a_1\underbrace{\frac{dc}{dt}}_{=x_2} + a_0\underbrace{c(t)}_{=x_1} = b_0\,r(t)$$

ODE

$$\boxed{\frac{d^n c}{dt^n} + a_{n-1}\frac{d^{n-1} c}{dt^{n-1}} + \cdots + a_1 \frac{dc}{dt} + a_0\, c(t) = b_0\, r(t)}$$

$$\frac{d^{n-1} c}{dt^{n-1}} = x_n, \quad \frac{dc}{dt} = x_2, \quad c(t) = x_1$$

state

$$\boxed{\begin{aligned} x_1(t) &= c(t) \\ x_2(t) &= \frac{d}{dt} c(t) \\ &\vdots \\ x_n(t) &= \frac{d^{n-1}}{dt^{n-1}} c(t) \end{aligned}}$$

$$\dot{x}_1(t) = \frac{d}{dt} c(t) = x_2(t)$$

$$\dot{x}_2(t) = \frac{d^2}{dt^2} c(t) = x_3(t)$$

$$\vdots$$

$$\dot{x}_n(t) = \frac{d^n}{dt^n} c(t) = ?$$

$$-a_{n-1} x_n - \cdots - a_1 x_2 - a_0 x_1 + b_0 r(t)$$

$$= -a_0 x_1 - a_1 x_2 - \cdots - a_{n-1} x_n + b_0 r(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_{n-1}(t) \\ \dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & \cdots & \cdots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_0 \end{bmatrix} r(t)$$

$$c(t) = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix}$$

Transfer function $\longrightarrow$ State Space

$$\text{Transfer function} = \frac{Y(s)}{X(s)} = \frac{C(s)}{R(s)}$$

$$\boxed{G(s) = \frac{b_0}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)}} = \frac{C(s)}{R(s)}$$

$$\boxed{G(s) = \frac{(b_{n-1}s^{n-1} + \cdots + b_1 s + b_0)}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)}} = \frac{C(s)}{R(s)}$$

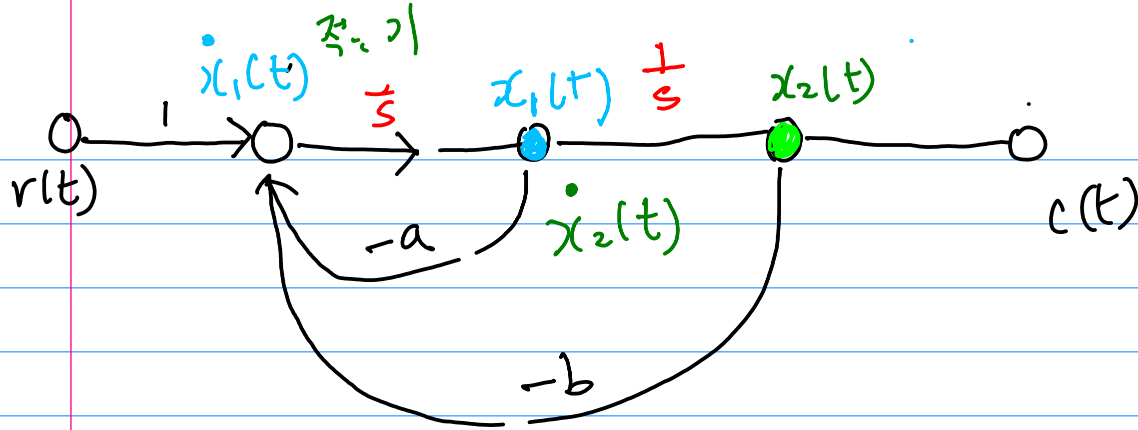
$$\rightarrow \boxed{\frac{1}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)}} \rightarrow \boxed{(b_{n-1}s^{n-1} + \cdots + b_1 s + b_0)} \rightarrow$$

Transfer fn $\longrightarrow$ State Space

State Space $\longrightarrow$ Transfer fn.

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$

$$y(t) = C\,x(t) + D\,u(t)$$




$$\begin{cases} \dot{x}_1(t) = -a\,x_1(t) - b\,x_2(t) + r(t) \\ \dot{x}_2(t) = x_1(t) \end{cases}$$ $\boxed{\text{state eq}}$

$$c(t) = x_2(t)$$ $\boxed{\text{output eq}}$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -a & -b \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(t)$$

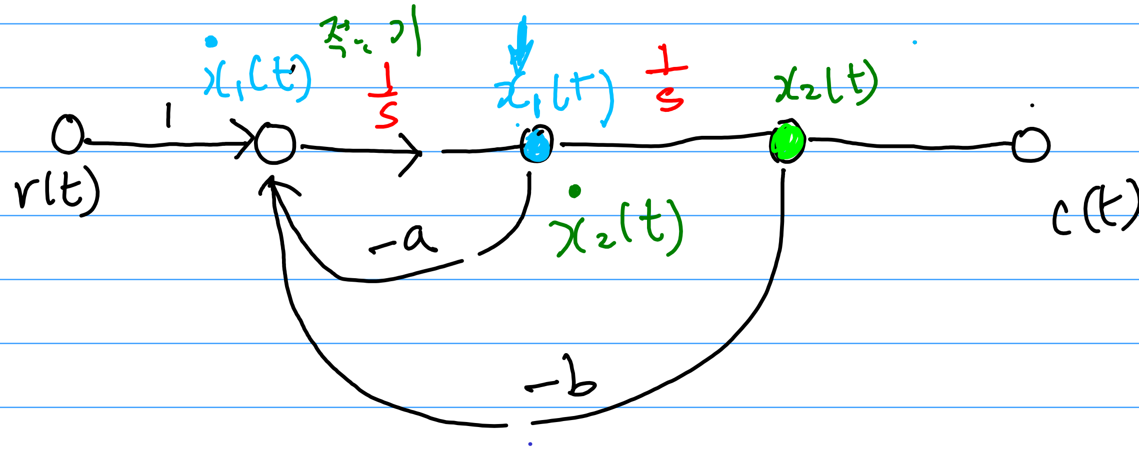
$$c(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + 0 \cdot r(t)$$

$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{r}(t)$$

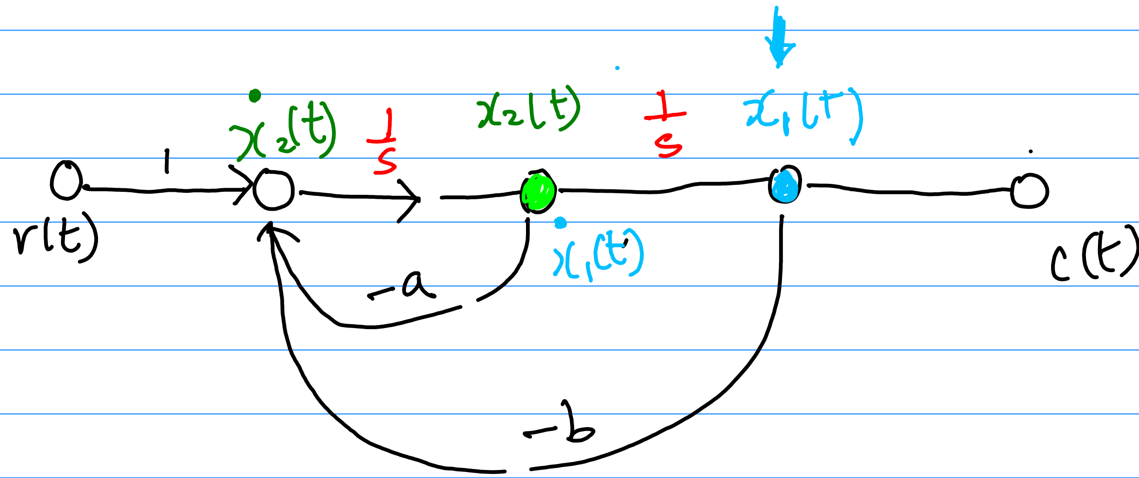
$$\mathbf{c}(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,r(t)$$

(c)

①




②



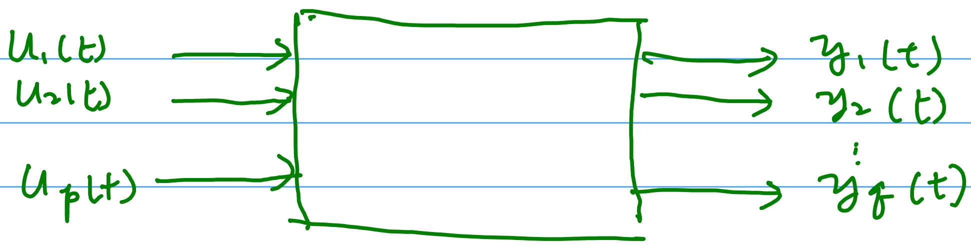

$$\dot{x}_2(t) = \underline{-b\,x_1(t) - a\,x_2(t)} + r(t)$$

$$\dot{x}_1(t) = x_2(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

$$\frac{d^2 c(t)}{dt^2} + a\frac{dc(t)}{dt} + b\,c(t) = r(t)$$

# MIMO




$$\mathbf{u}(t) = \begin{array}{cc} & 1 \\ p & \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_p(t) \end{bmatrix} \end{array} \qquad \mathbf{y}(t) = \begin{array}{cc} & 1 \\ q & \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_q(t) \end{bmatrix} \end{array}$$

$$\mathbf{U}(s) = \begin{array}{cc} & 1 \\ p & \begin{bmatrix} U_1(s) \\ U_2(s) \\ \vdots \\ U_p(s) \end{bmatrix} \end{array} \qquad \mathbf{Y}(s) = \begin{array}{cc} & 1 \\ q & \begin{bmatrix} Y_1(s) \\ Y_2(s) \\ \vdots \\ Y_q(s) \end{bmatrix} \end{array}$$

$p = 1 \dashrightarrow u(t) \qquad q = 1 \cdots y(t)$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -a & -b \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + 0 \cdot u(t)$$

$$\begin{bmatrix} sX_1(s) \\ sX_2(s) \end{bmatrix} = \underbrace{\begin{bmatrix} -a & -b \\ 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} U(s)$$

$$Y(s) = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + \underbrace{0}_{D}$$

$$s\,\mathbf{X}(s) = \mathbf{A}\,\mathbf{X}(s) + \mathbf{B}\,U(s)$$

$$Y(s) = \mathbf{C}\,\mathbf{X}(s) + \mathbf{D}\,U(s)$$

$$sX(s) = AX(s) + Bu(s)$$

$$Y(s) = CX(s) + Du(s)$$

$$sX(s) - AX(s) = Bu(s)$$

$$sIX(s) - AX(s) = Bu(s)$$

$$(sI - A)X(s) = Bu(s)$$

$$X(s) = (sI - A)^{-1} Bu(s)$$

$$sX(s) = AX(s) + Bu(s)$$
$$Y(s) = CX(s) + Du(s)$$

$$X(s) = (sI - A)^{-1} Bu(s)$$
$$Y(s) = CX(s) + Du(s)$$
$$= C(sI - A)^{-1} Bu(s) + Du(s)$$
$$Y(s) = \{C(sI - A)^{-1}B + D\}u(s)$$

$$\frac{Y(s)}{u(s)} = \{\underbrace{\underbrace{C}_{1\times 2}\underbrace{(sI - A)^{-1}}_{2\times 2}\underbrace{B}_{2\times 1}}_{1\times 1} + D\}$$

$$\underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}}_{n\times 1} = \underbrace{\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}}_{n\times n} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}}_{n\times 1} + \underbrace{\begin{bmatrix} & & \\ & B & \\ & & \end{bmatrix}}_{n\times p} \times \underbrace{\begin{bmatrix} r_1(t) \\ \vdots \\ r_p(t) \end{bmatrix}}_{p\times 1}$$

$$\begin{bmatrix} c_1(t) \\ \vdots \\ c_q(t) \end{bmatrix} = \begin{bmatrix} & & \\ & C & \\ & & \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} & & \\ & D & \\ & & \end{bmatrix} \begin{bmatrix} r_1(t) \\ \vdots \\ r_p(t) \end{bmatrix}$$

q개 출력 p개 입력

$q\times p$

$$\frac{Y(s)}{U(s)} = \underbrace{\left\{ \overset{q\times n}{C}\,(\overset{n\times n}{sI - A})^{-1}\,\overset{n\times p}{B} + \overset{q\times p}{D} \right\}}_{q\times p}$$

q개의 출력 $\begin{cases} c_1(t) \\ c_2(t) \\ \vdots \\ c_q(t) \end{cases}$ p개 입력 $\begin{cases} r_1(t) \\ r_2(t) \\ \vdots \\ r_p(t) \end{cases}$

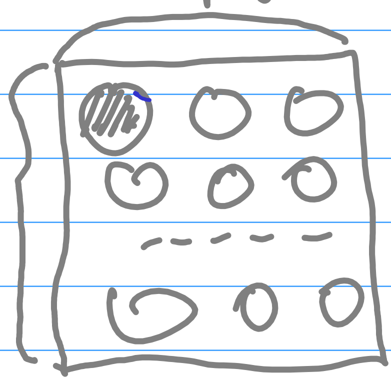
$$\underbrace{\frac{Y(s)}{U(s)}}_{q \times p} = \Big\{ \overset{\substack{q\text{개 출력} \\ \uparrow}}{\underset{}{C}}{}^{q \times n} (sI - A)^{-1}_{n \times n} \cdot \overset{p\text{개 입력}}{B}{}^{n \times p} + D^{q \times p} \Big\}$$

$$\underbrace{C(sI-A)^{-1}B + D}_{q \times p}$$

q 개의 출력 $\begin{cases} y_1(t) \\ y_2(t) \\ \vdots \\ y_q(t) \end{cases}$ p개 입력 $\begin{cases} u_1(t) \\ u_2(t) \\ \vdots \\ u_p(t) \end{cases}$

p 입력

q 출력



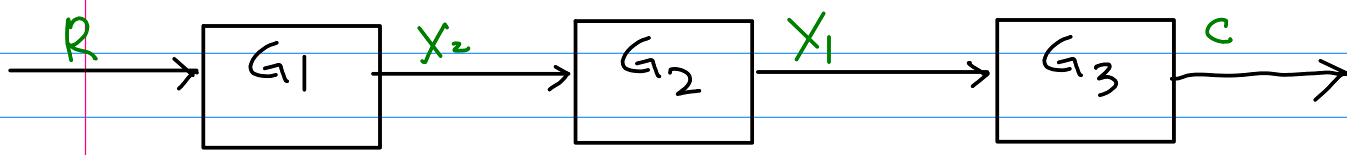
$\frac{Y_1(s)}{U_1(s)}$ $\frac{Y_1(s)}{U_2(s)}$ ...

$\frac{Y_2(s)}{U_1(s)}$

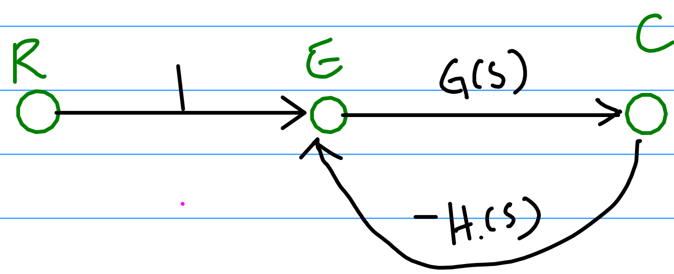
$$p\text{개 입력} \left\{ \begin{array}{l} u_1(t) \\ u_2(t) \\ \vdots \\ u_p(t) \end{array} \right. \qquad \left. \begin{array}{l} y_1(t) \\ y_2(t) \\ \vdots \\ y_q(t) \end{array} \right\} q\text{개의 출력}$$

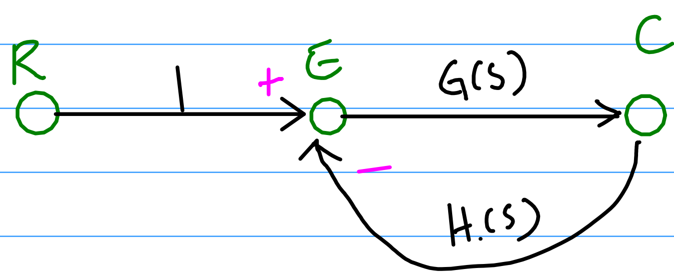
# Block Diagram & Signal Flow Grap

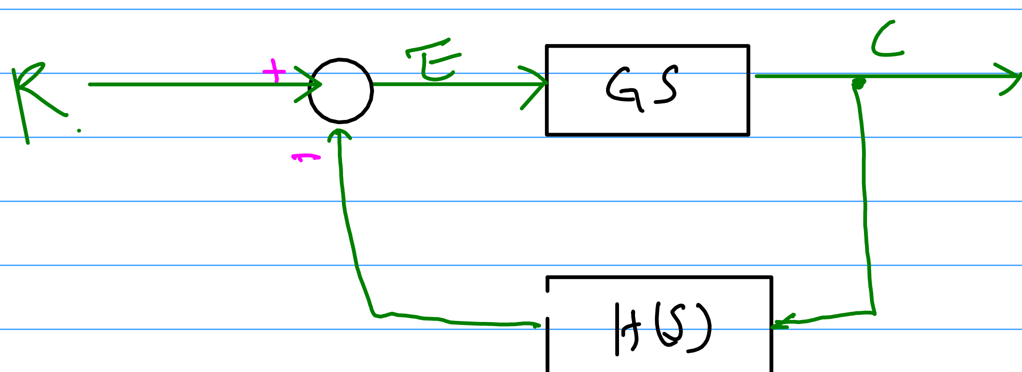
## Block Diagram



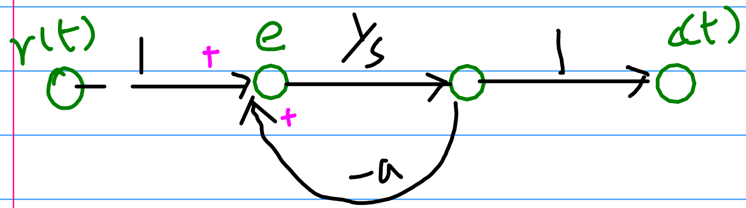

## Signal Flow Graph.

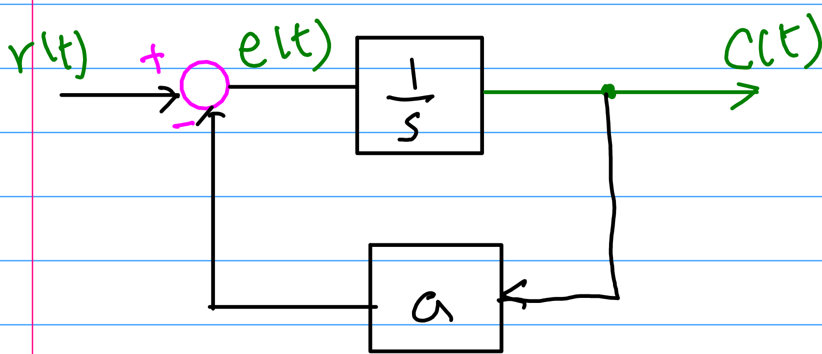



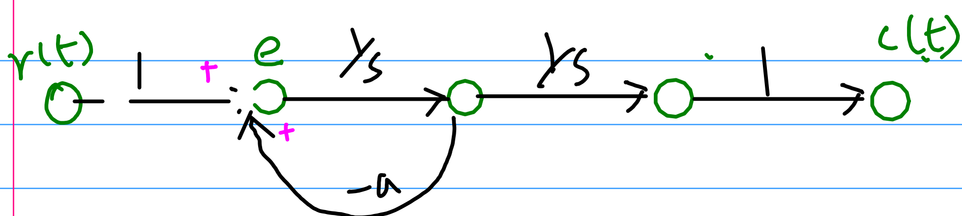
P48 . 2(2) 3.16



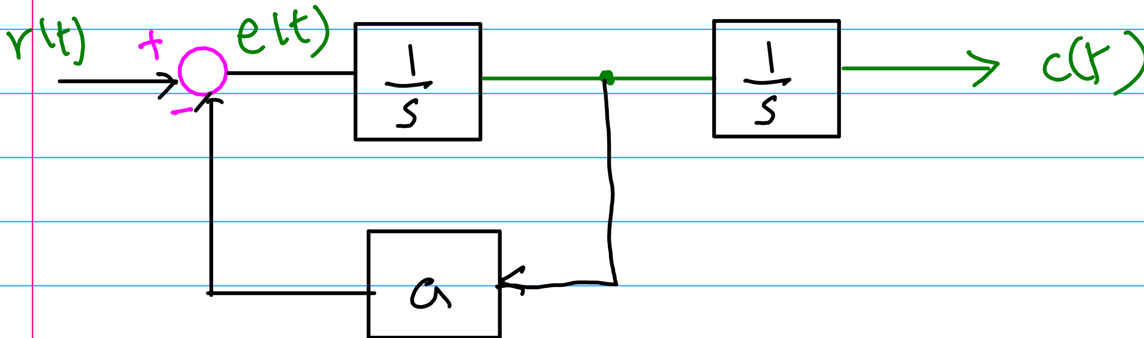

$$\frac{\frac{1}{s}}{1+\frac{a}{s}} = \frac{1}{s+a}$$

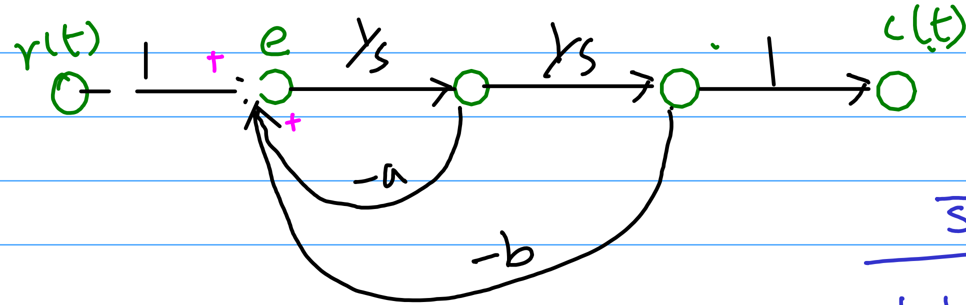



$$\frac{1}{s+a} \times \frac{1}{s}$$

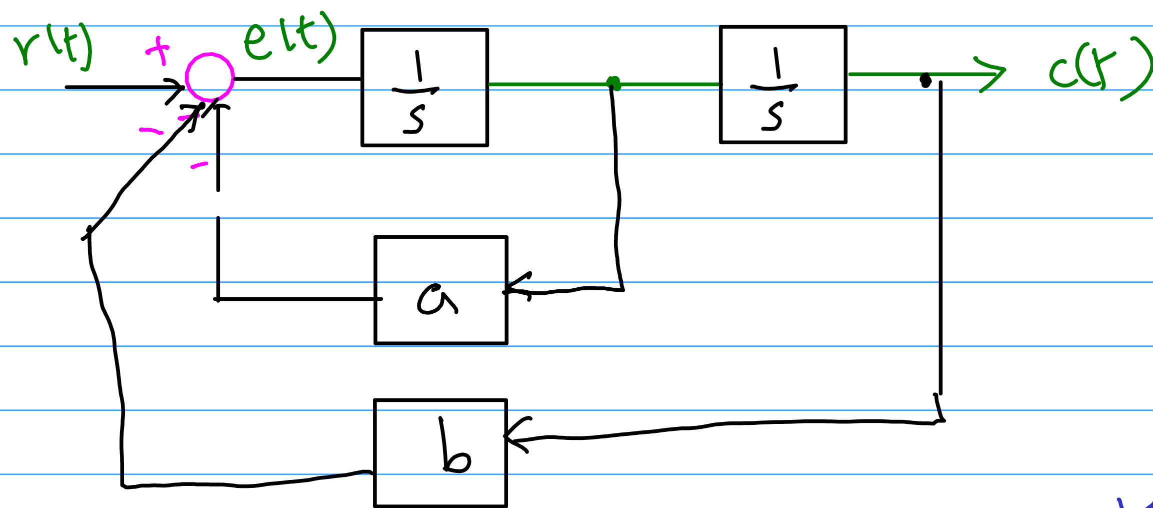



$$= \frac{1}{s(s+a)}$$



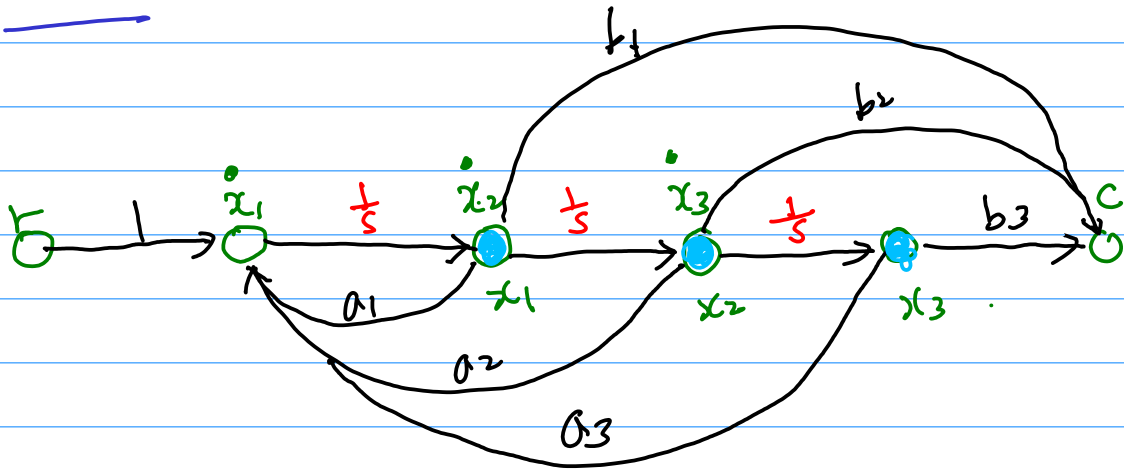



$$\frac{\frac{1}{s(s+a)}}{1+\frac{b}{s(s+a)}}$$




$$\frac{1}{s^2+as+b}$$

# P51




$$\dot{x}_1 = r + a_1 x_1 + a_2 x_2 + a_3 x_3$$

$$\dot{x}_2 = x_1$$

$$\dot{x}_3 = x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} r$$

$$c = b_1 x_1 + b_2 x_2 + b_3 x_3$$

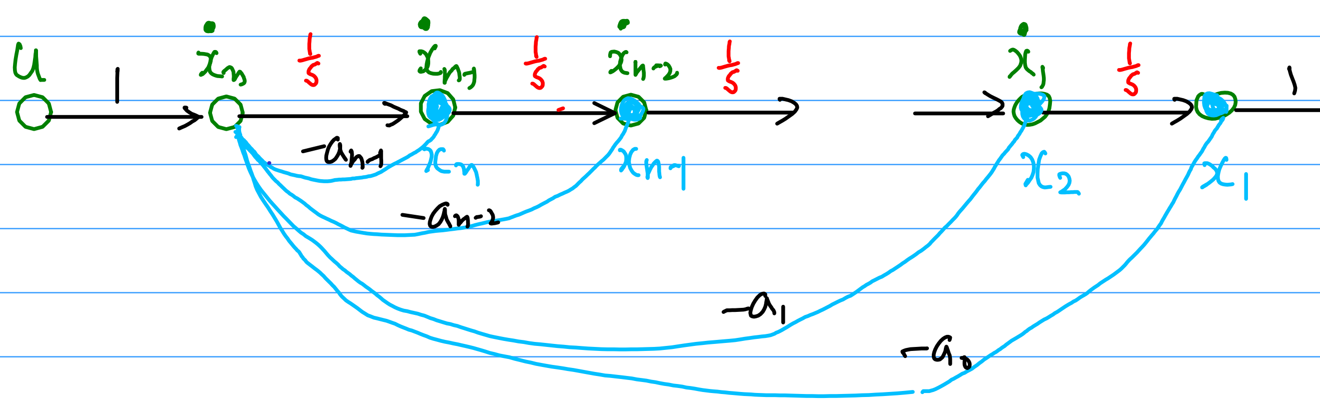
$$c = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

P 183

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \\ \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} & 1 & & & 0 \\ & & 1 & & \\ & & & \ddots & \\ & 0 & & & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\vdots$$
$$\dot{x}_n = -(a_0 x_1 + a_1 x_2 + \cdots + a_{n-1} x^n) + u$$

$$y^{(n)} + a_{n-1} \overset{x_n}{y^{(n-1)}} + \cdots + a_1 \overset{x_2}{y^{(1)}} + a_0 \overset{x_1}{y} = u$$



$$y^{(n)} + a_{n-1} \overset{x_n}{y^{(n-1)}} + \cdots + a_1 \overset{x_2}{y^{(1)}} + a_0 \overset{x_1}{y} = u$$

$$s^n Y(s) + a_{n-1} s^{n-1} Y(s) \qquad + a_1 s Y(s) + a_0 Y(s) = U(s)$$

$$\left(s^n + a_{n-1} s^{n-1} \qquad + a_1 s + a_0\right) Y(s) = U(s)$$

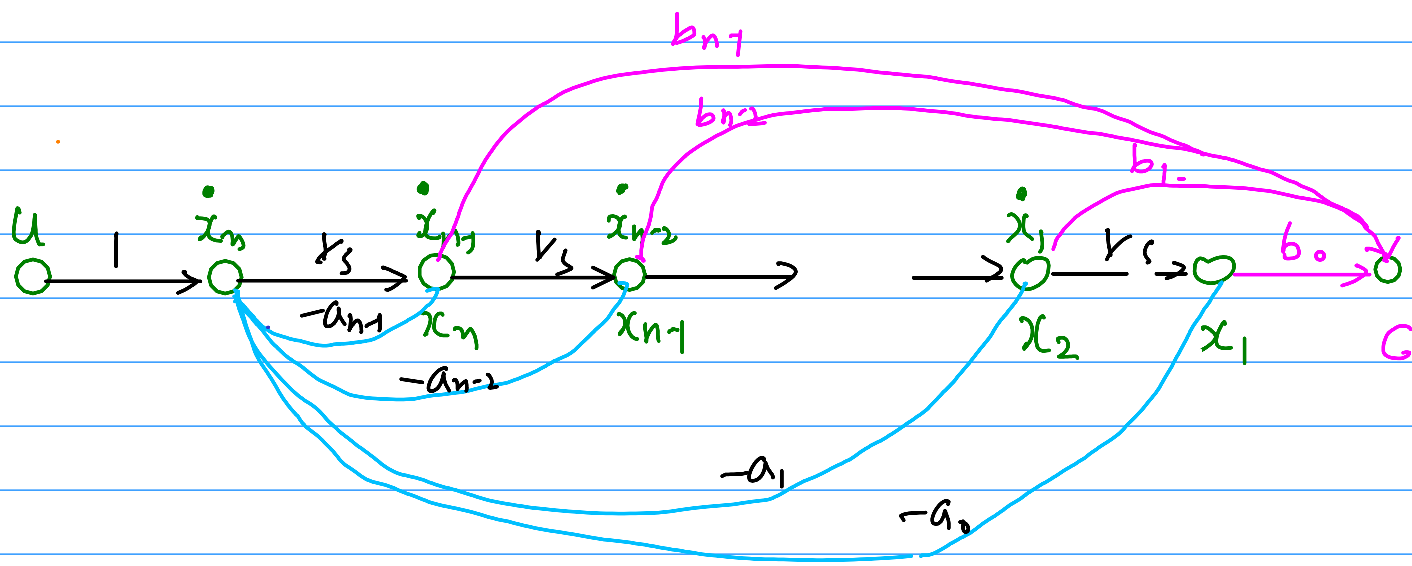
$$\frac{Y(s)}{U(s)} = \frac{1}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

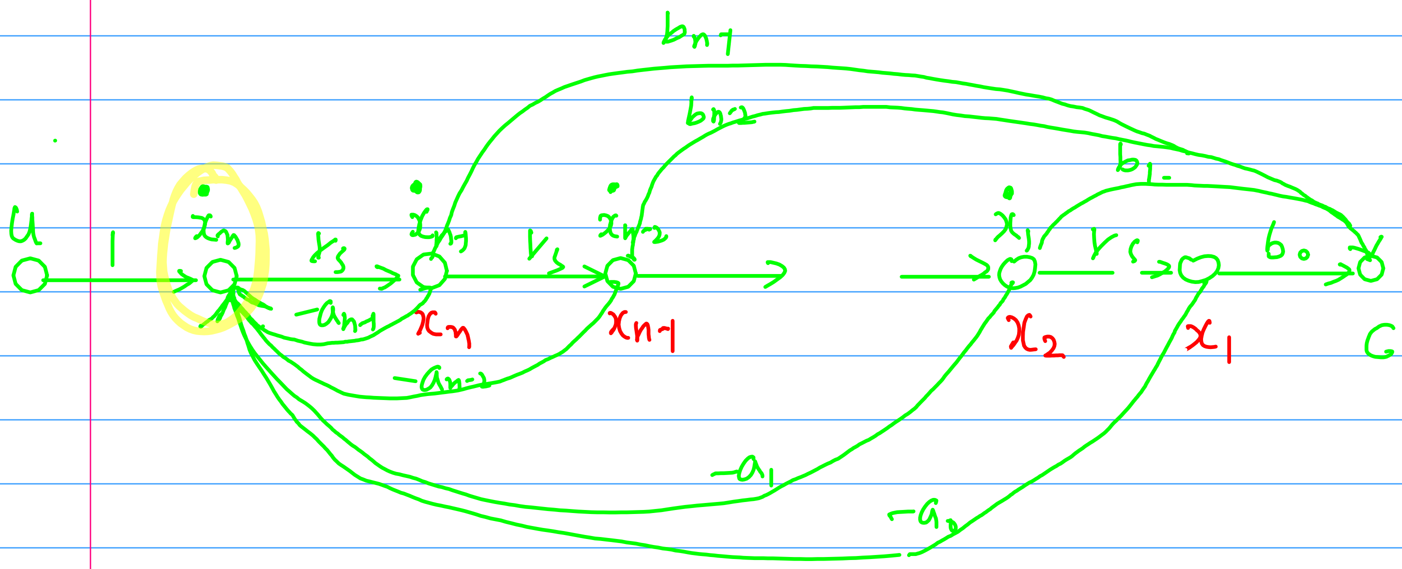
$$\frac{C(s)}{U(s)} = \frac{b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \quad b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$$= \frac{Y(s)}{U(s)} \left(b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \quad b_1 s + b_0\right)$$

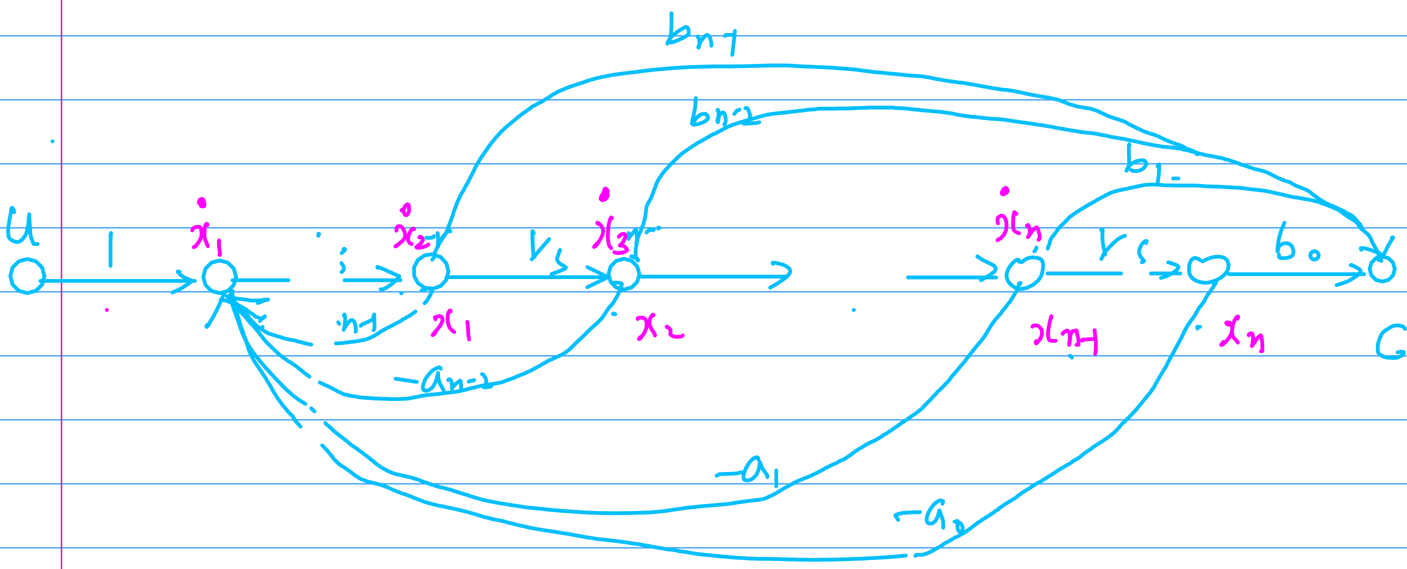
$$c(t) = [\,b_0 \quad b_1 \qquad\qquad b_{n-1}\,] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= b_0 x_1 + b_1 x_2 + \dots \quad b_{n-1} x_n$$






$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & & 0 \\ & & \ddots & \\ 0 & & & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u.$$




$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \vdots\\ \dot x_n\end{bmatrix} = \begin{bmatrix} -a_{n-1} & -a_{n-2} & \cdots & -a_1 & -a_0 \\ 1 & & & 0 & \\ & 1 & & & \\ & & \ddots & & \\ 0 & & & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix} + \begin{bmatrix}1\\ 0\\ \vdots\\ 0\end{bmatrix}u.$$

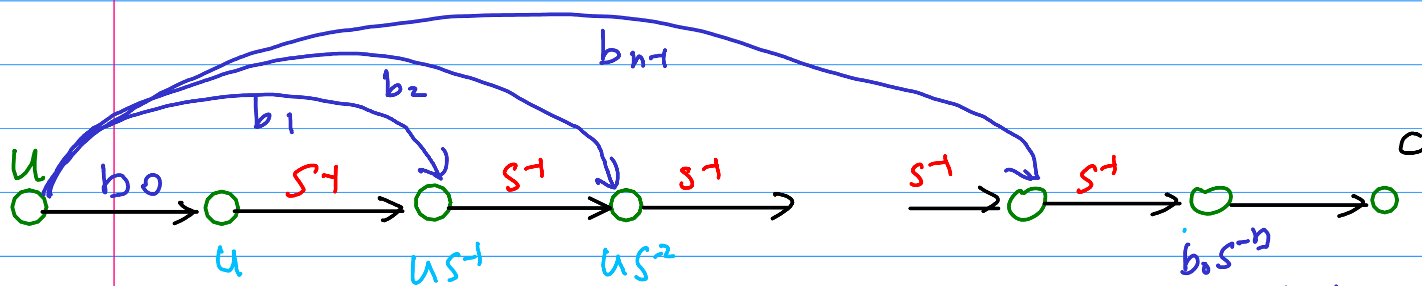
$$\frac{C(s)}{U(s)} = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \quad b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0}$$

$$\frac{C(s)}{U(s)} = \frac{b_{n-1}s^{-1} + b_{n-2}s^{-2} + \quad b_1 s^{-n+1} + b_0 s^{-n}}{1 + a_{n-1}s^{-1} + \dots + a_1 s^{-n+1} + a_0 s^{-n}}$$

$$\left(1 + a_{n-1}s^{-1} + \dots + a_1 s^{-n+1} + a_0 s^{-n}\right) C(s) = \left(b_{n-1}s^{-1} + b_{n-2}s^{-2} + \quad b_1 s^{-n+1} + b_0 s^{-n}\right) U(s)$$

$$C(s) = -\left(a_{n-1}s^{-1} + a_{n-2}s^{-2} + \dots + a_1 s^{-n+1} + a_0 s^{-n}\right) C(s) + \left(b_{n-1}s^{-1} + b_{n-2}s^{-2} + \dots + b_1 s^{-n+1} + b_0 s^{-n}\right) U(s)$$

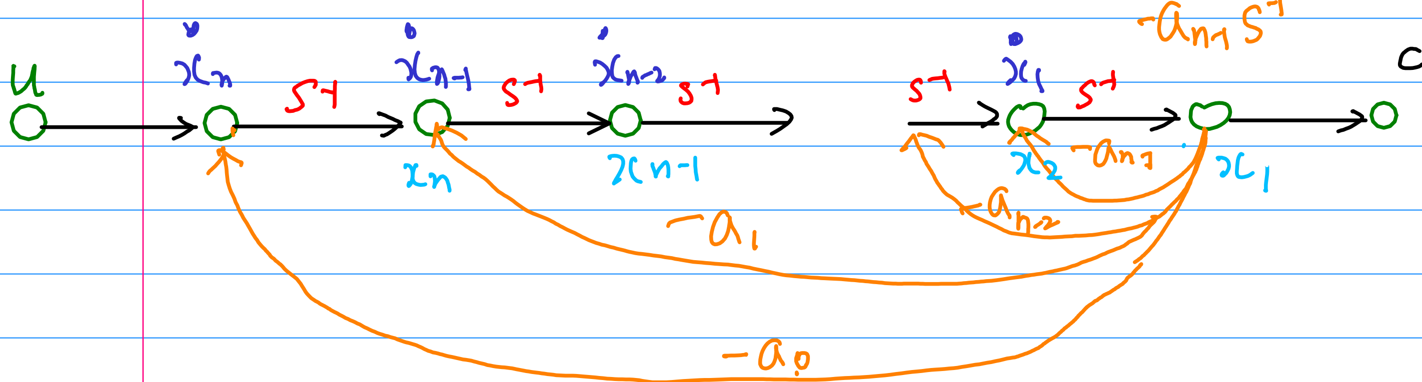
$$C(s) = -\left(a_{n-1}s^{-1} + a_{n-2}s^{-2} + \cdots + a_1 s^{-n+1} + a_0 s^{-n}\right)C(s)$$
$$+\left(b_{n-1}s^{-1} + b_{n-2}s^{-2} + \cdots + b_1 s^{-n+1} + b_0 s^{-n}\right)U(s)$$

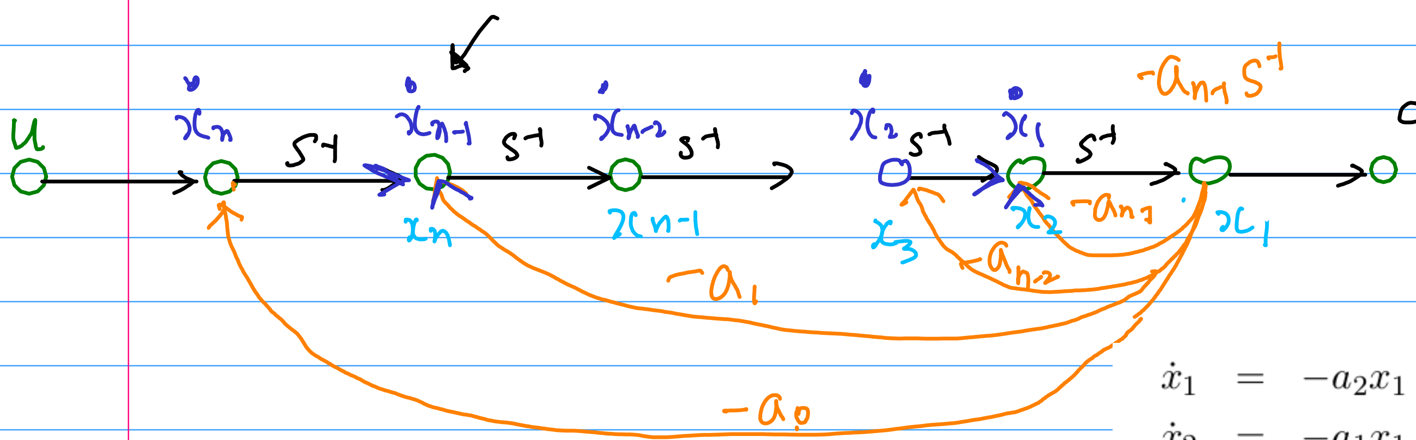



$b_0 s^{-n}$
$+ b_1 s^{-n+1}$
$+ b_2 s^{-n+2}$
$\vdots$
$+ b_{n-1} \cdot s^{-1}$

$-a_0 s^{-n}$
$-a_1 s^{-n+1}$

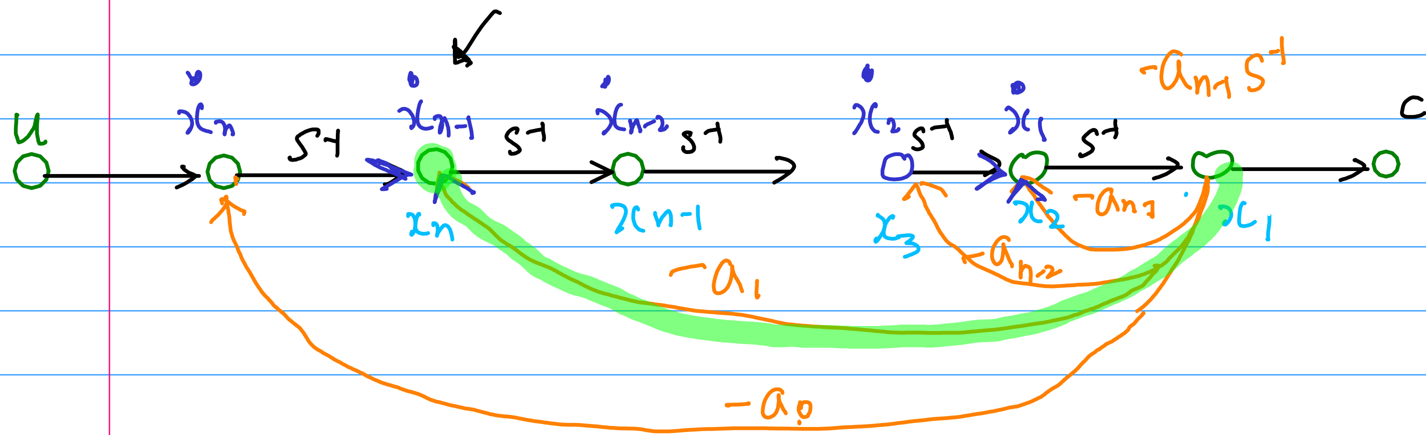
$-a_{n-2} s^{-2}$
$-a_{n-1} s^{-1}$






$$
\begin{aligned}
\dot{x}_1 &= -a_2 x_1 + x_2 + b_2 \\
\dot{x}_2 &= -a_1 x_1 + x_3 + b_1 \\
\dot{x}_3 &= -a_0 x_1 + b_0 u \\
y &= x_1
\end{aligned}
$$

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -a_{n-1} & 1 & & & \\ -a_{n-2} & 0 & 1 & & \\ \vdots & & & \ddots & \\ -a_1 & & & & 1 \\ -a_0 & 0 & 0 & \cdots 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix} u.
$$




$$\dot{x}_n = -a_0 x_1$$

$$\dot{x}_{n-1} = -a_1 x_1 + s^{-1}(\dot{x}_n)$$

$$-a_1 x_1 + x_n$$

$$\dot{x}_2 = -a_{n-1} \cdot x_1 + s^{-1}(\dot{x}_3)$$

$$= -a_{n-1} \cdot x_1 + x_3$$