Feedback System (H.1)
Block Diagram

20150613

IVP

$$\begin{cases} \dfrac{d^2}{dt^2} y(t) + 3\dfrac{d}{dt} y(t) + 2y(t) = x(t) \\ y(0) = k_0, \quad y'(0) = k_1 \end{cases}$$

$$\begin{cases} (y''(t)) + 3(y'(t)) + 2y(t) = x(t) \\ y(0) = k_0, \quad y'(0) = k_1 \end{cases}$$

$$(s^2 Y(s) - s y(0) - y'(0)) + 3(sY(s) - y(0)) + 2Y(s) = X(s)$$

$$(s^2 + 3s + 2) Y(s) = s y(0) + y'(0) + 3y(0) + X(s)$$

$$(s^2 + 3s + 2) Y(s) = k_0 s + k_1 + 3k_0 + X(s)$$

$$Y(s) = \underbrace{\frac{\overset{\text{state}}{k_0 s + k_1 + 3k_0}}{(s^2+3s+2)}}_{\text{Zero Input Rsp}} + \underbrace{\frac{\overset{\text{input}}{X(s)}}{(s^2+3s+2)}}_{\text{Zero state Rsp}}$$

Transfer function $\quad H(s) = \dfrac{Y(s)}{X(s)}$ .... "zero state"

Zero.. state

$$= \frac{Y(s)}{X(s)} = \frac{1}{(s^2+3s+2)}$$

$$Y(s) = \cancel{\frac{k_0 s + k_1 + 3k_0}{(s^2+3s+2)}} + \frac{X(s)}{(s^2+3s+2)}$$

Zero state $(y(0) = y'(0) = 0)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s^2+3s+2)}$$

Impulse Response

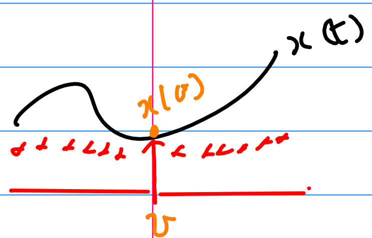
$$\underset{\delta(t)}{x(t)} \longrightarrow \boxed{h(t)} \longrightarrow \underset{h(t)}{y(t)}$$

$$\delta(t-v) \qquad h(t-v)$$

$$a \cdot \delta(t-v) \qquad a \cdot h(t-v)$$

$$x(v)\,\delta(t-v) \qquad x(v)\,h(t-v)$$

$$x(v)\,\delta(t-v) \qquad x(v)\,h(t-v)$$



$$\int_{-\infty}^{+\infty} x(v)\,\delta(t-v)\,dt \qquad \int_{-\infty}^{\infty} x(v)\,h(t-v)\,dv = y(t)$$

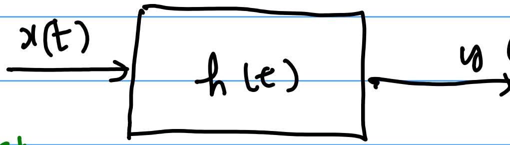
$$x(t) * h(t)$$

$$x(v) = 0 \qquad v < 0$$

$$h(t-v) = 0 \qquad t - v < 0$$

$$t < v$$

$$0 < v < t$$



$$y(t) = \int_0^\infty h(\nu)\, x(t-\nu)\, d\nu \qquad t-\nu \geq 0$$

e^{st}

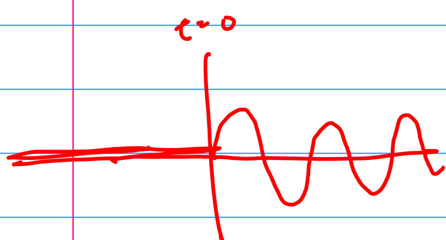
$$y(t) = \int_0^\infty h(\nu)\, e^{s(t-\nu)}\, d\nu$$

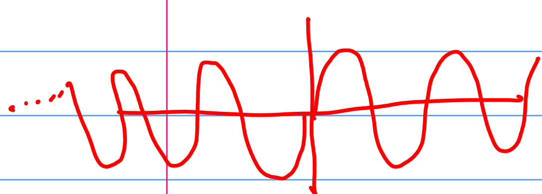
$$= e^{st} \int_0^\infty h(\nu)\, e^{-s\nu}\, d\nu$$

$s = \sigma + j\omega$

e^{st}

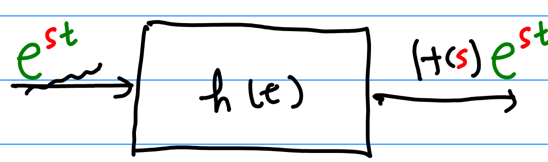
$$e^{st} \cdot \underbrace{\int_0^\infty h(\nu)\, e^{-s\nu}\, d\nu}_{= H(s)}$$





$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \int_0^\infty f(\nu)\, e^{-s\nu}\, d\nu$$



$h(t) \Longleftrightarrow H(s)$

Laplace Transform

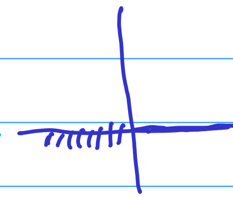
$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$$

$$X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s) = X(s) \cdot H(s)$$

$$y^{(3)}(t) + a\,y^{(2)}(t) + b\,y^{(1)}(t) + c\,y(t) = m\,x^{(1)}(t) + n\,x(t)$$

Zero-state assumed $\quad y(0) = y'(0) = y''(0) = 0$

Transfer Fn 구할 때 $\quad x(0) = 0 \quad x(t)$



$$s^3 Y(s) + a s^2 Y(s) + b s Y(s) + c Y(s) = m s X(s) + n X(s)$$

$$(s^3 + a s^2 + b s + c)\, Y(s) = (m s + n)\, X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{m s + n}{s^3 + a s^2 + b s + c}$$

$$y'' + 3y' + 2y = x$$

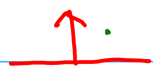
$\boxed{\text{Zero state}}$

$$s^2 Y(s) + 3sY(s) + 2Y(s) = X(s)$$

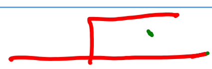
$$(s^2 + 3s + 2)\,Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)(s+2)}$$

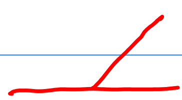
$$x(t) \longrightarrow \boxed{G(s)} \longrightarrow y(t)$$

impulse 함수  impulse 응답 $g(t)$

$\Downarrow$ 적분 $\qquad\qquad$ $\Downarrow$ 적분

step 함수  step 응답 $\int g(t)\,dt$

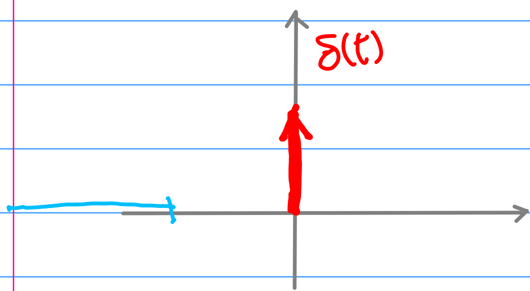
$\Downarrow$ $\qquad\qquad$ $\Downarrow$ 적분

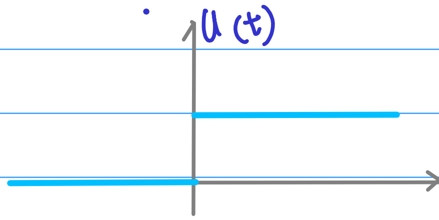
ramp 함수  ramp 응답 $\int_0^t \int_0^t g(\tau)\,d\tau\,dt$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

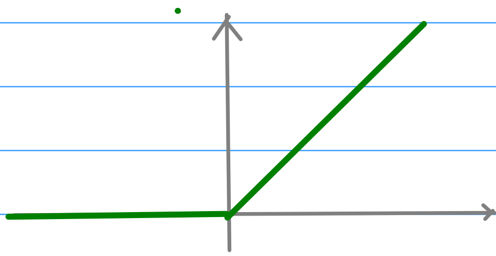
$$x'(t) \longrightarrow \boxed{h(t)} \longrightarrow y'(t)$$

$$x''(t) \longrightarrow \boxed{h(t)} \longrightarrow y''(t)$$

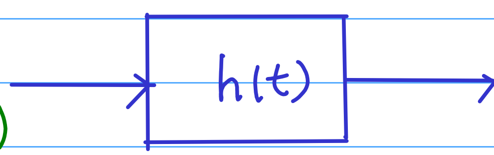




$$u(t) = \int_{-\infty}^{t} \delta(t)\, dt$$



$$r(t) = \int_{-\infty}^{t} u(t)\, dt$$



$\delta(t)$ → $h(t)$

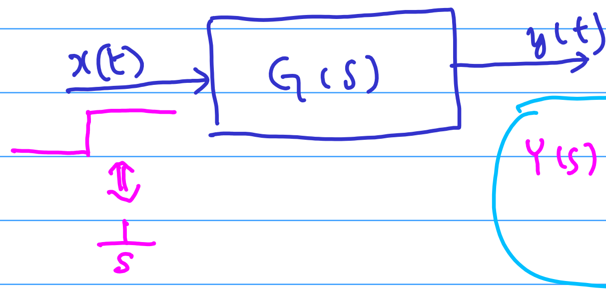
$u(t)$ → $\int_{-\infty}^{t} h(t)\, dt$

$r(t)$ → $\int_{-\infty}^{t}\int_{-\infty}^{t} h(t)\, dt\, dt$

$$s^2 Y(s) + 3s Y(s) + 2Y(s) = X(s)$$

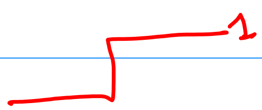
$$(s^2 + 3s + 2)\, Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)(s+2)}$$

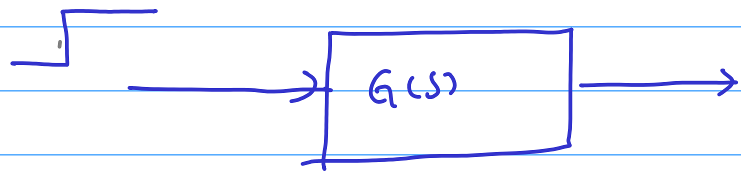


$$Y(s) = G(s)\cdot X(s) = \frac{1}{s\,(s+1)(s+2)}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+2)}\right)$$



$$= \underbrace{A\cdot 1}_{y_p} + \underbrace{B e^{-1\cdot t} + C e^{-2\cdot t}}_{y_h}$$



$$\frac{k_0}{s} + \frac{k_1}{s+\sigma_1} + \frac{k_2}{s+\sigma_2}$$

$$\Updownarrow$$

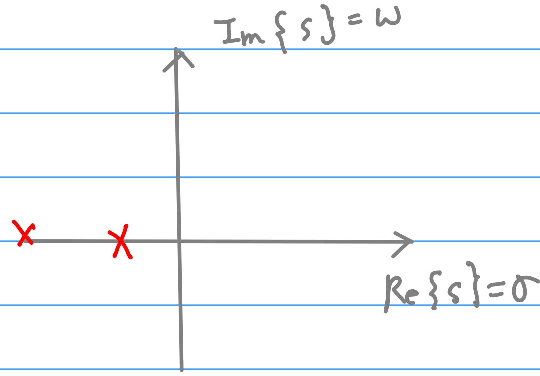
$$k_0 + k_1 e^{-\sigma_1 t} + k_2 e^{-\sigma_2 t}$$

$$G(s) = \frac{1}{s^2+3s+2}$$

$\sigma_1, \sigma_2 \Leftarrow$ 분모 $= 0$ $\quad (s^2+3s+2)=0$

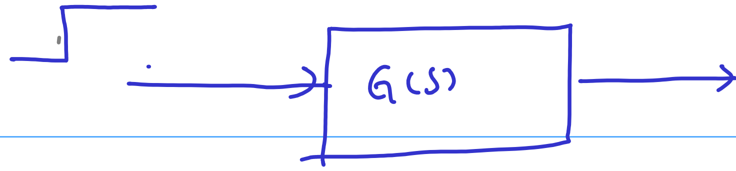
분모가 0이 되는 s들은 pole

$$s = \sigma + j\omega$$



σ_1, σ_2 는 서로 다른 두 실근

$\sigma_1 = \sigma_2$ 중근

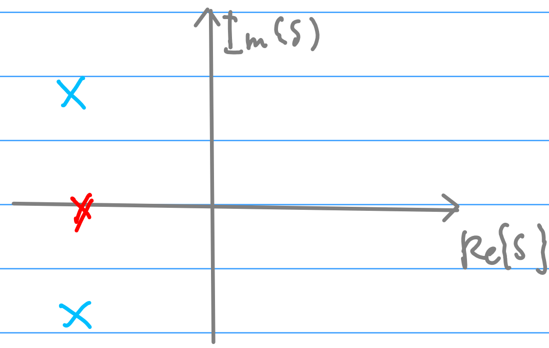


$$\frac{1}{s(s-\sigma_1)^2} = \frac{k_0}{s} + \frac{k_1}{(s-\sigma_1)} + \frac{k_2}{(s-\sigma_1)^2}$$

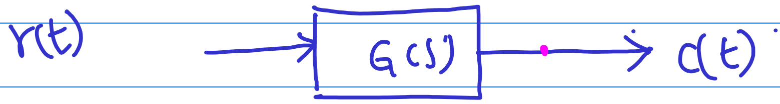
$$\Updownarrow$$

$$k_0 + k_1 e^{-\sigma_1 t} + k_2 t e^{-\sigma_1 t}$$

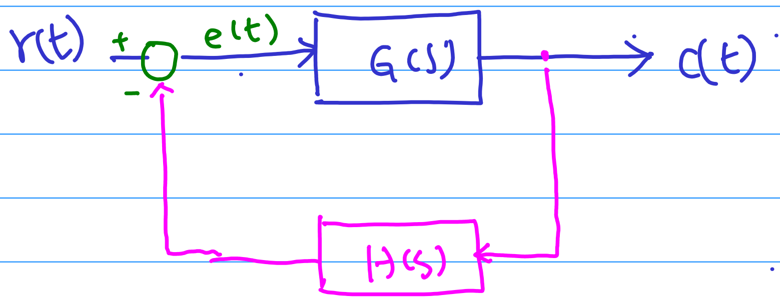
$$G(s) = \frac{1}{s^2+3s+2}$$



Feedback

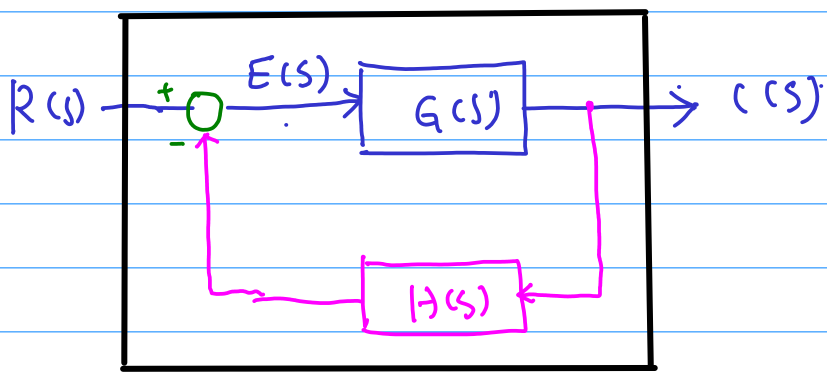


$$C(s) = G(s) \cdot R(s)$$



$$C(s) = G(s) \cdot E(s)$$

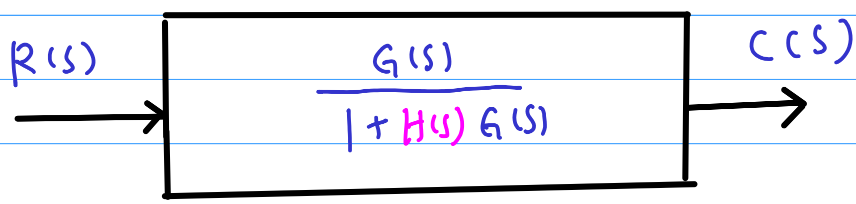
$$E(s) = R(s) - H(s)\,C(s)$$

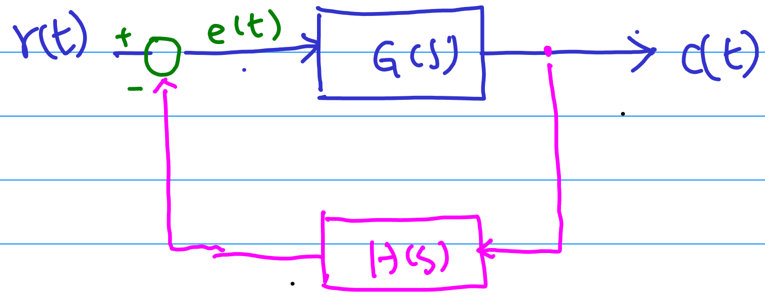


$$\frac{C(s)}{R(s)} = \frac{G(s) \cdot E(s)}{E(s) + H(s)\,C(s)}$$

$$= \frac{G(s) \cdot E(s)}{E(s) + H(s)\,G(s) \cdot E(s)}$$

$$= \frac{G(s)}{1 + H(s)\,G(s)}$$





$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - H(s)C(s)$$

$$E(s) + H(s)C(s) = R(s)$$

$$\frac{E(s)}{R(s)} = \frac{E(s)}{E(s) + H(s)C(s)}$$

$$= \frac{E(s)}{E(s) + H(s)G(s)E(s)}$$

$$= \frac{1}{1 + H(s)G(s)}$$

$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - H(s)C(s)$$

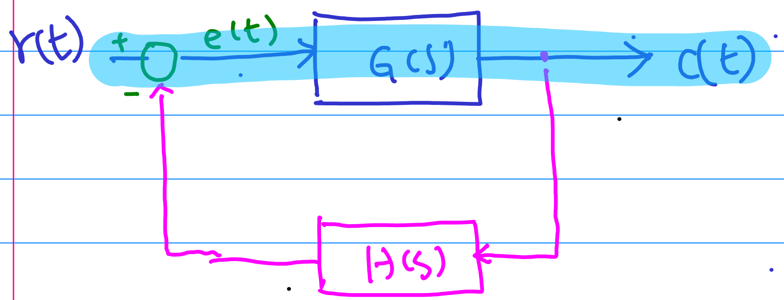
$$\frac{E(s)}{R(s)} = \frac{R(s) - H(s)C(s)}{R(s)}$$

$$= 1 - \frac{H(s)G(s)E(s)}{R(s)}$$

$$\frac{E(s)}{R(s)} + H(s)G(s)\frac{E(s)}{R(s)} = 1$$

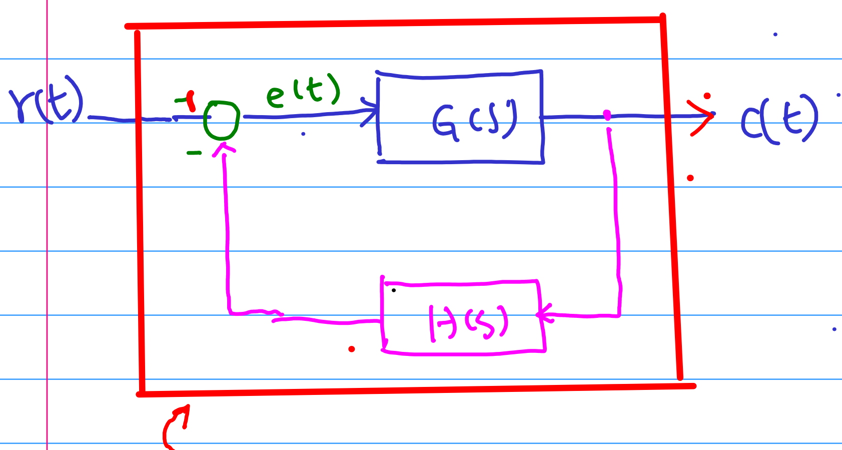
$$(1 + H(s)G(s))\frac{E(s)}{R(s)} = 1$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + H(s)G(s)}$$

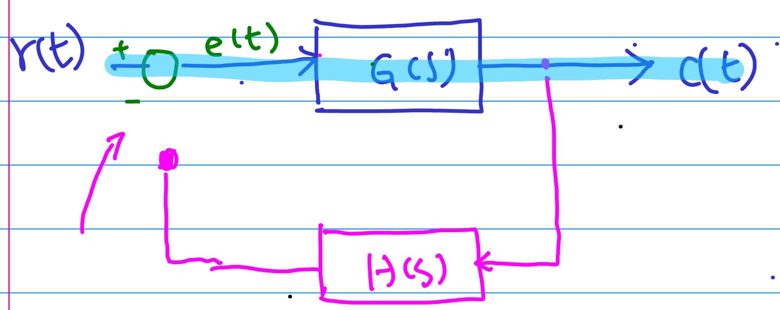


forward transfer fn : G(s)

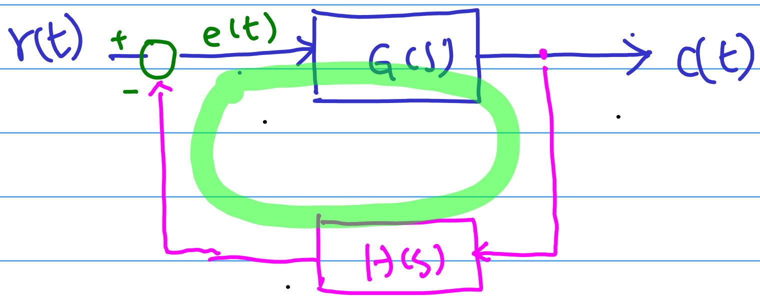
feedback transfer fn : H(s)



closed loop transfer fn : $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$

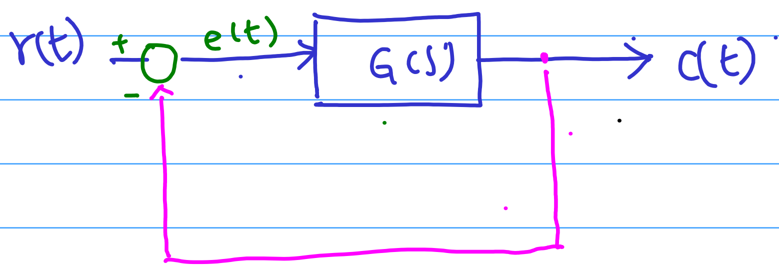


Open loop transfer fn : G(s)

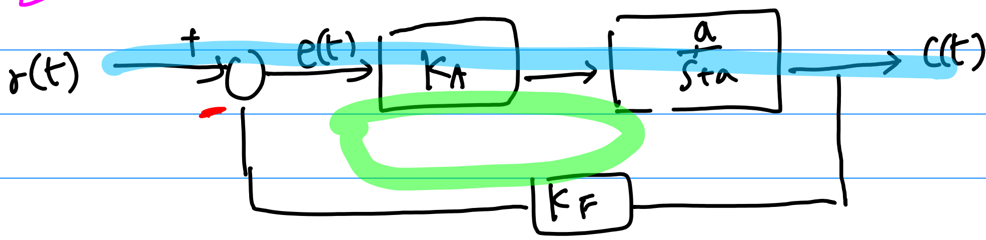


loop transfer fn : G(s)H(s)

Unit feedback H(s) = 1



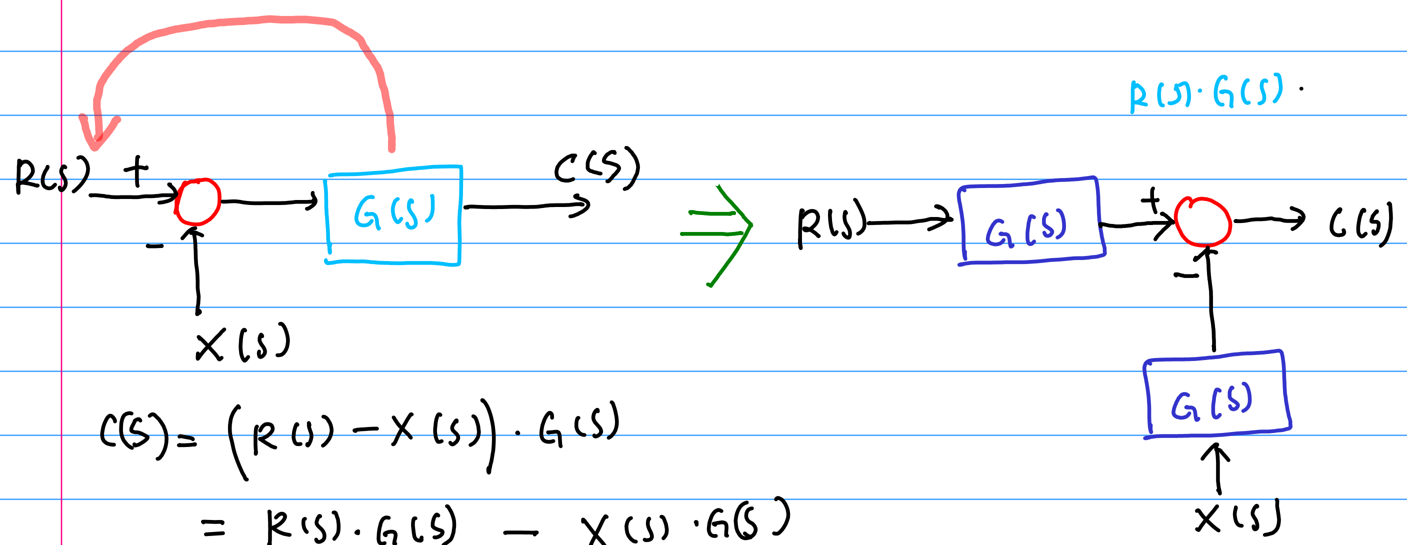
2장 2-10



$$\frac{K_A a}{(s+a)} \qquad\qquad -\frac{K_A K_F a}{(s+a)}$$

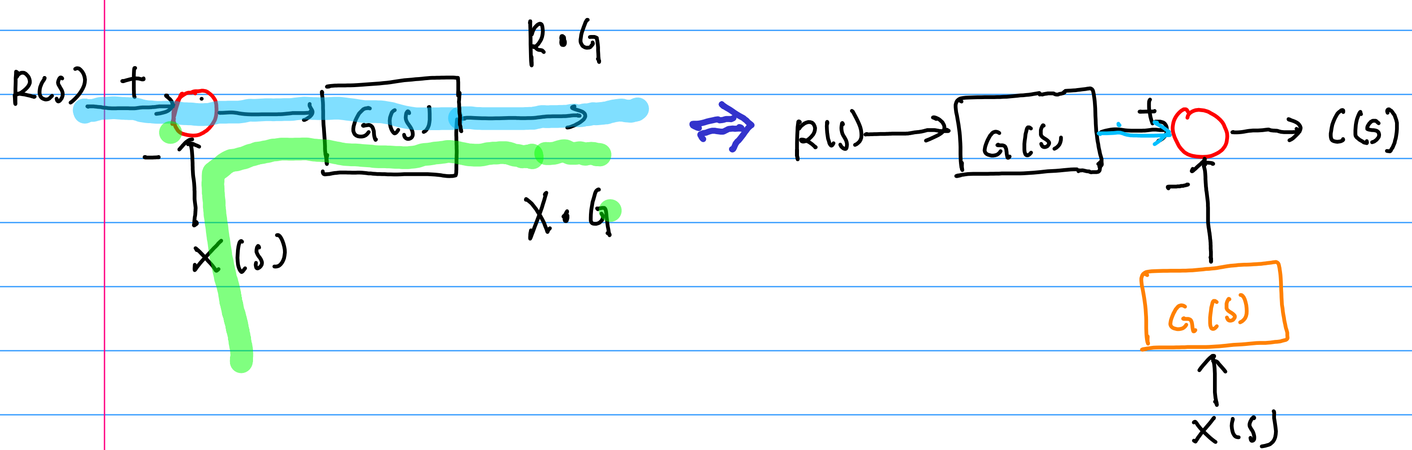
$$G_{cl}(s) = \frac{}{1 - } = \frac{\frac{K_A a}{(s+a)}}{1 - \left(-\frac{K_A K_F a}{(s+a)}\right)} = \frac{K_A a}{s+a+K_A K_F a}$$

① Moving $\boxed{G(s)}$ backward

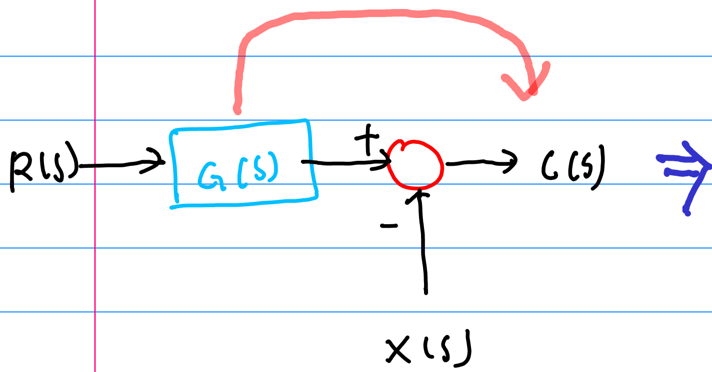


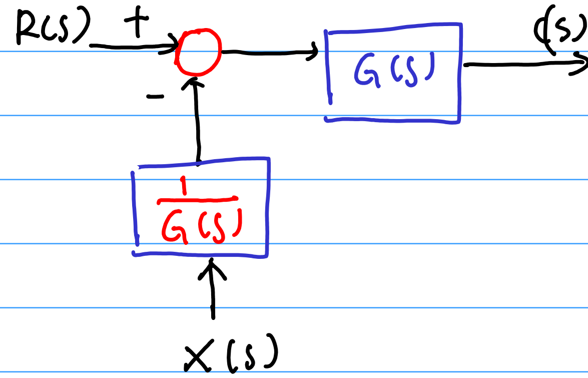
$$C(s) = \left(R(s) - X(s)\right)\cdot G(s)$$

$$= R(s)\cdot G(s) - X(s)\cdot G(s)$$



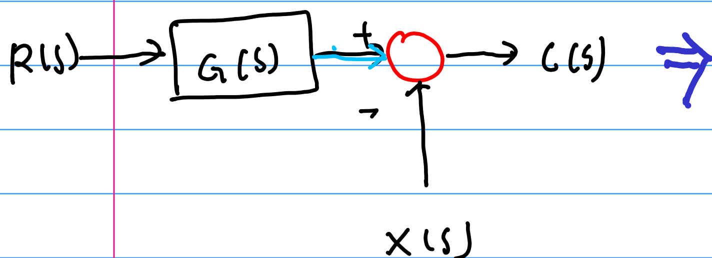
② Moving $\boxed{G(s)}$ forward

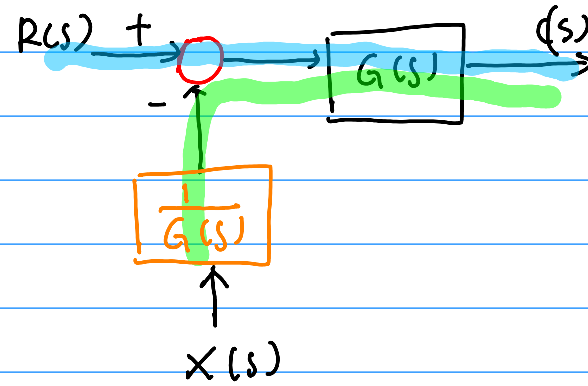




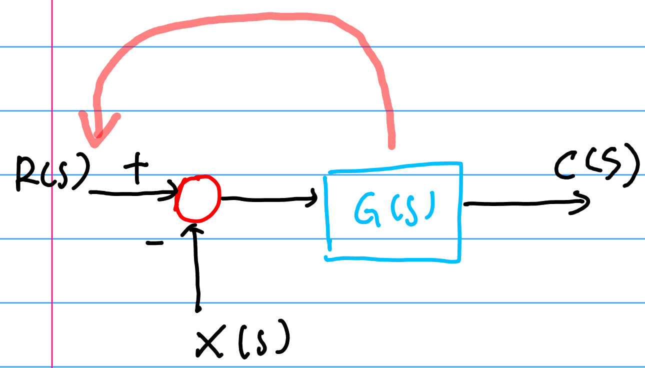
$$C(s) = G(s)R(s) - X(s)$$

$$= G(s)\left(R(s) - \frac{X(s)}{G(s)}\right)$$

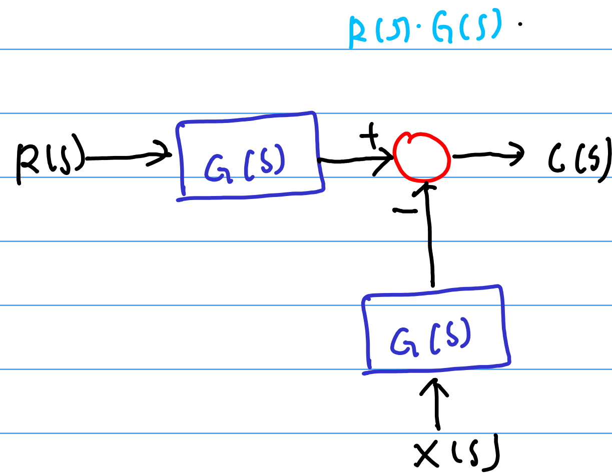


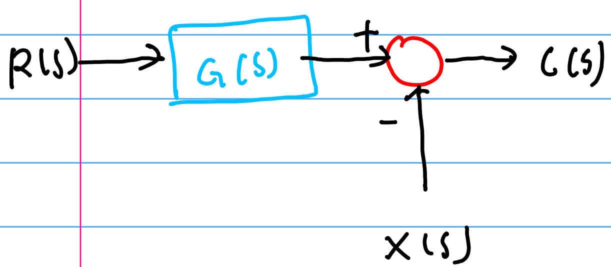


* moving backward

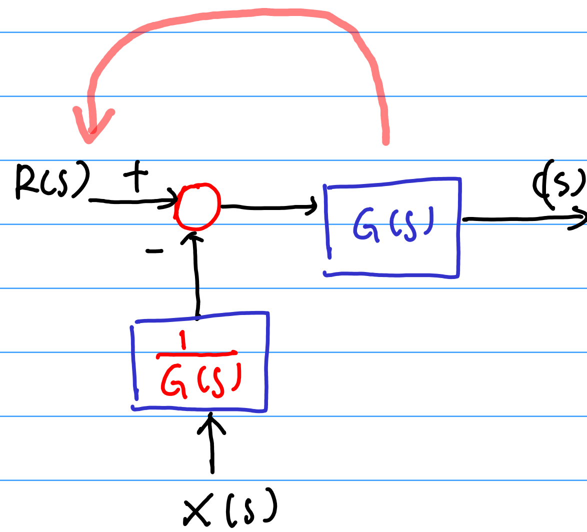


$\Rightarrow$

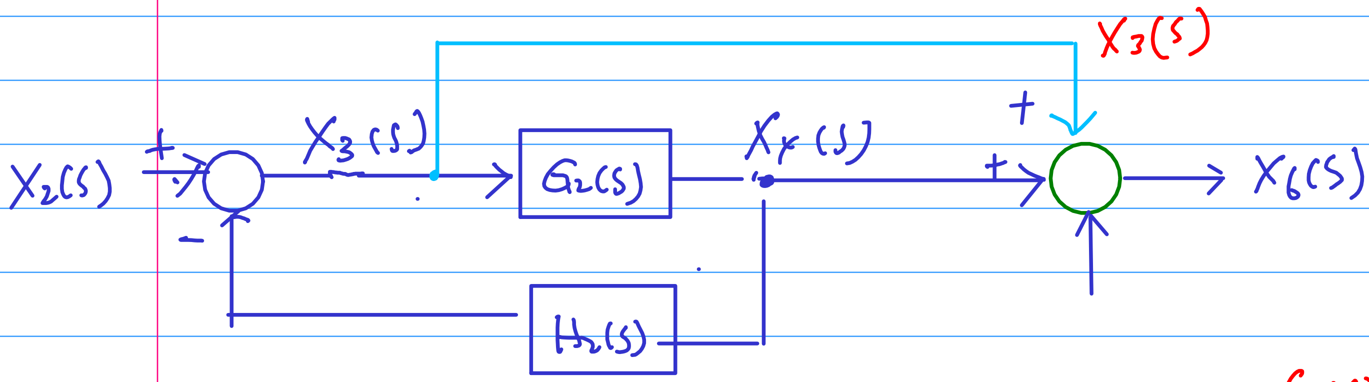




$\Leftarrow$

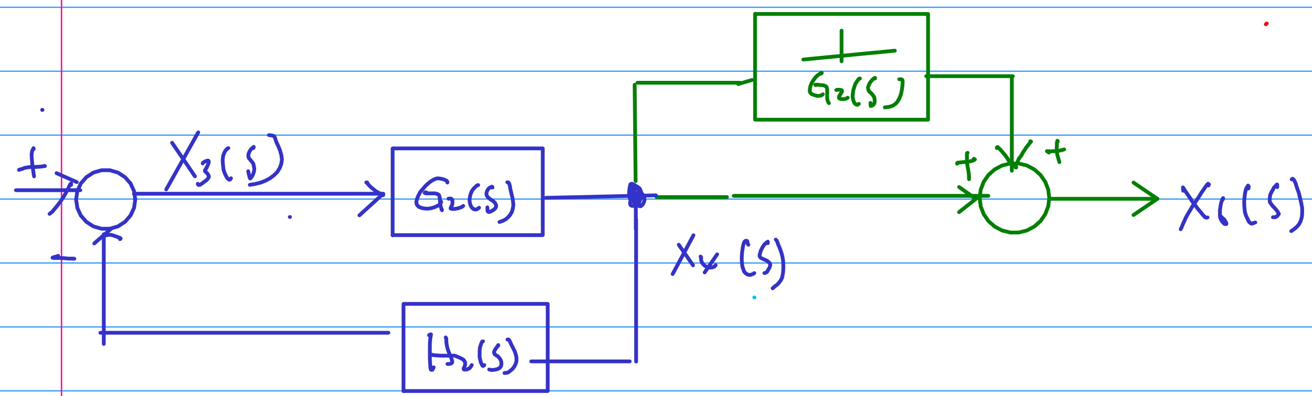


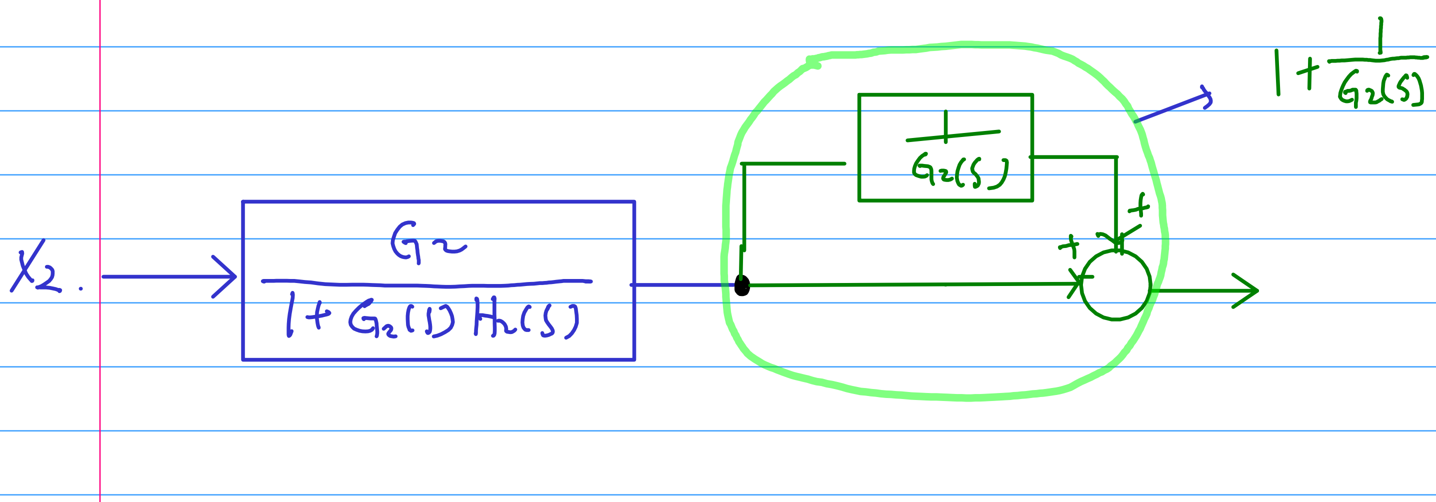
Ex 1)



$$\frac{X_4(S)}{G_2(S)} = \frac{G_2(S)\,X_3(S)}{G_2(S)}$$

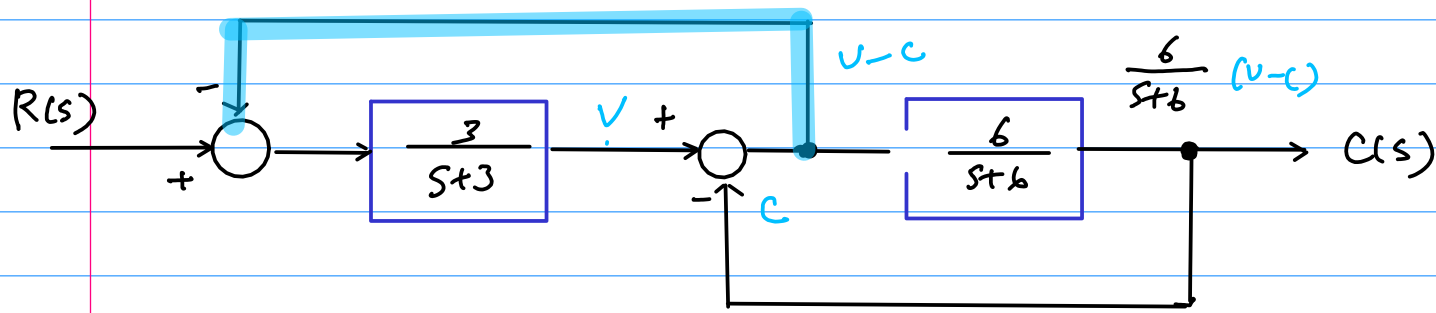
$$= X_3(S)$$



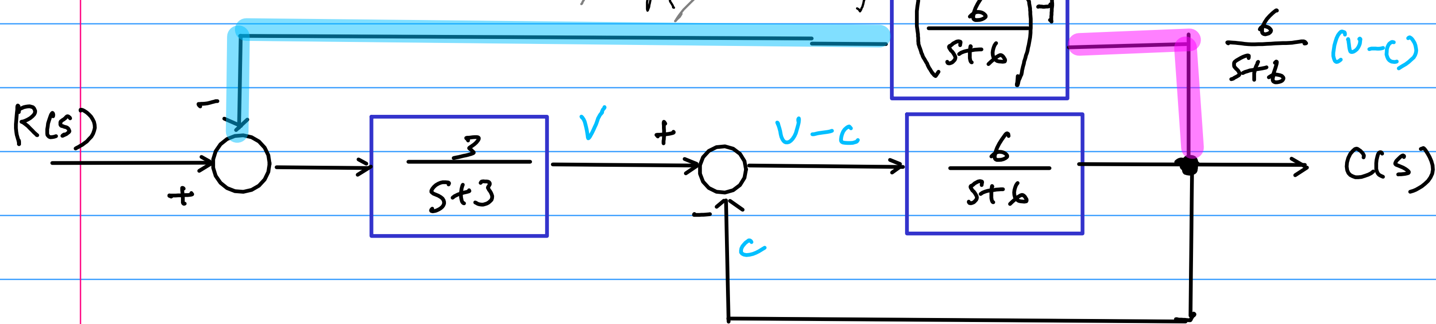


Ex 2)

make the same pickoff point → parallel subsystems



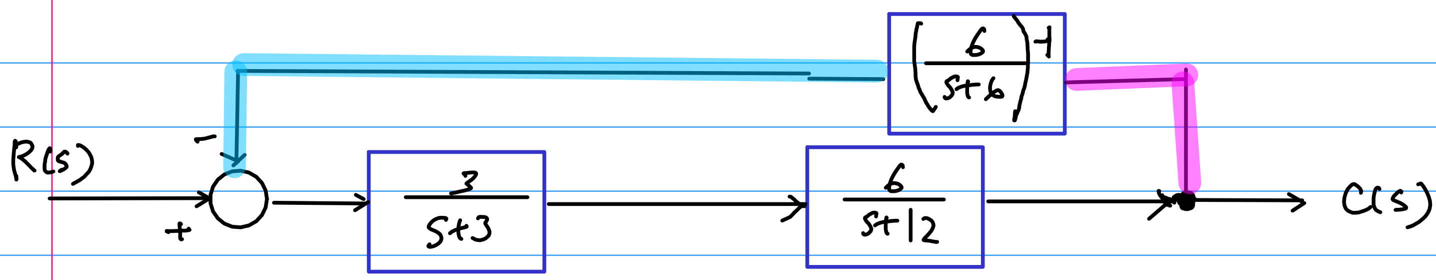
$$v-c = \left(\frac{\cancel{s+6}}{\cancel{6}}\right)\left(\frac{\cancel{6}}{\cancel{s+6}}(v-c)\right)$$

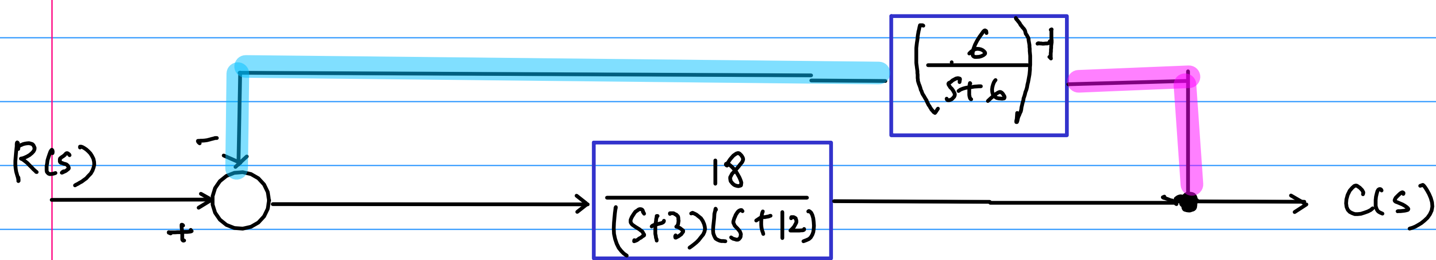


$$\frac{\frac{6}{s+6}}{1+\frac{6}{s+6}} = \frac{6}{s+12}$$

Find equivalent for the inner feedback system

2





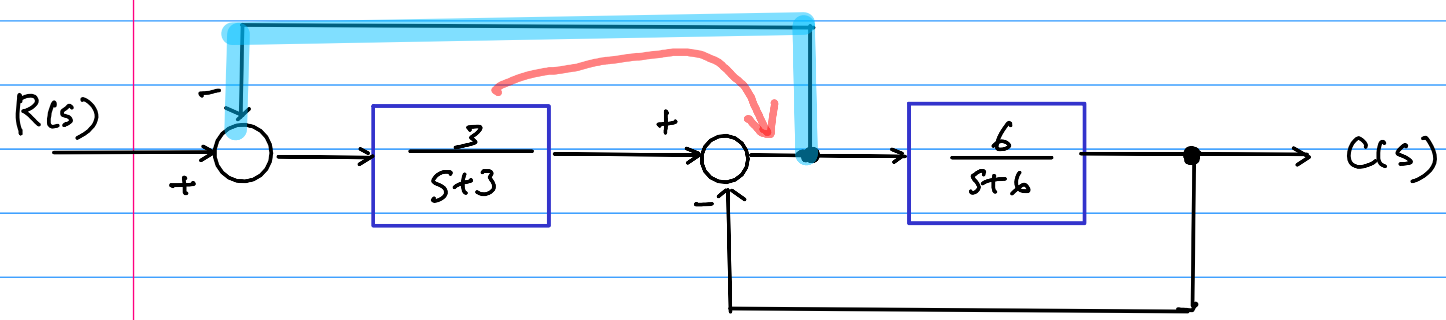
$$\frac{\frac{18}{(s+3)(s+12)}}{1+\frac{\cancel{18}\;3}{(s+3)(s+12)}\cdot\frac{s+6}{\cancel{6}}} = \frac{18}{(s+3)(s+12)+3(s+6)}$$

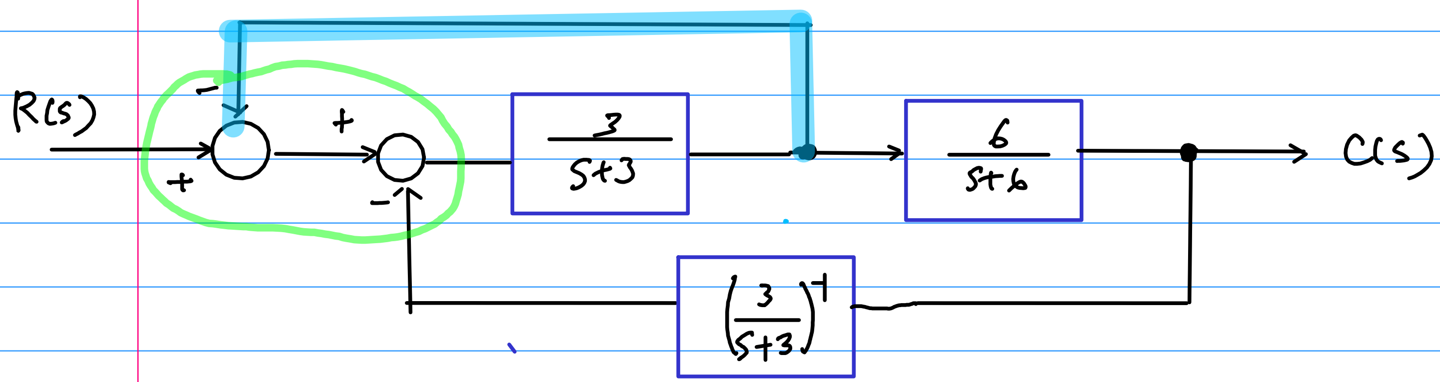
$$= \frac{18}{s^2+15s+36+3s+18}$$

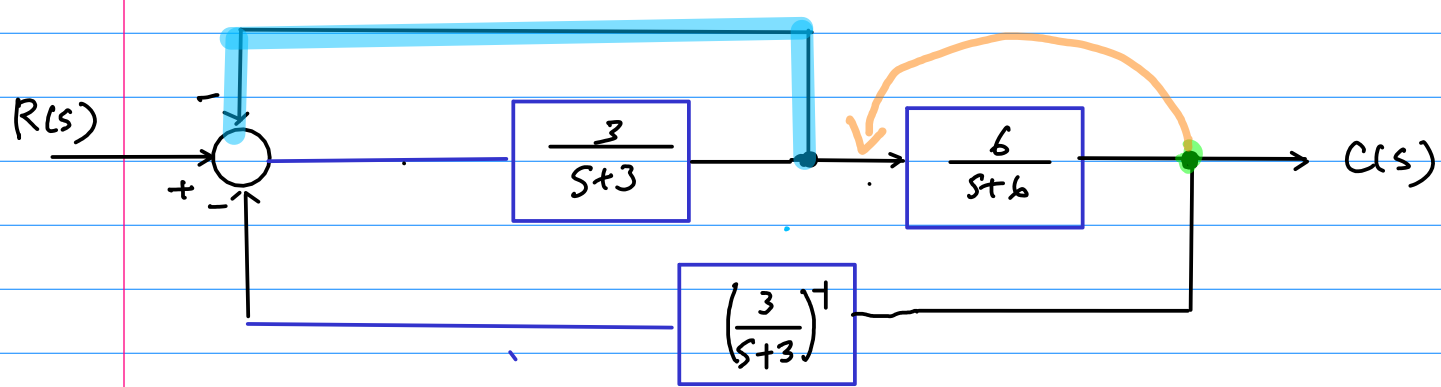
$$= \frac{18}{s^2+18s+54}$$

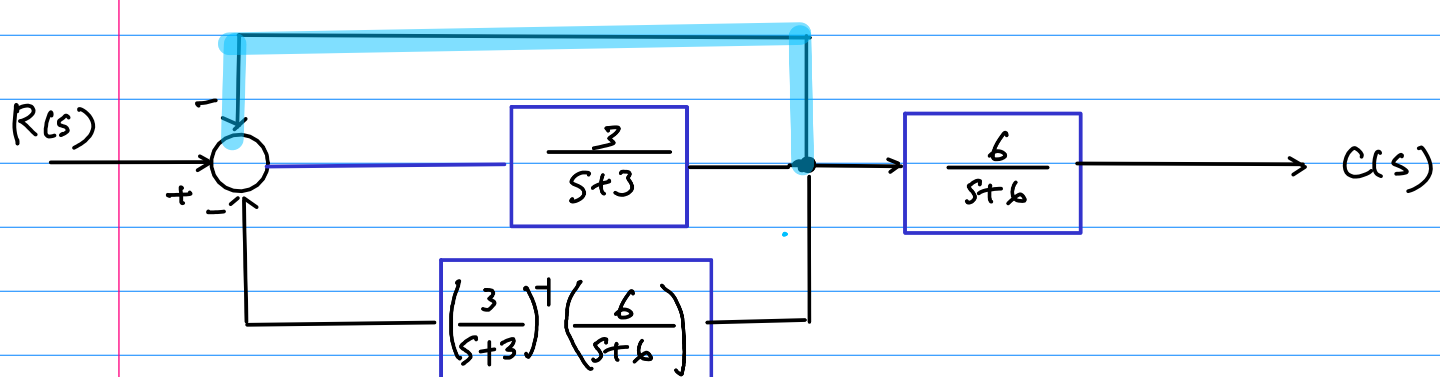
other approach

③



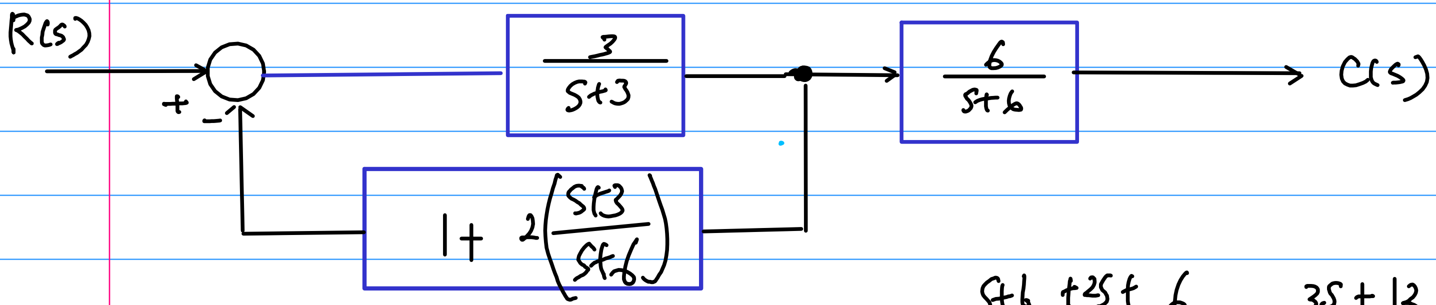






$$\frac{s+3}{\cancel{3}}\,\frac{\cancel{6}}{s+6} = 2\left(\frac{s+3}{s+6}\right)$$

④



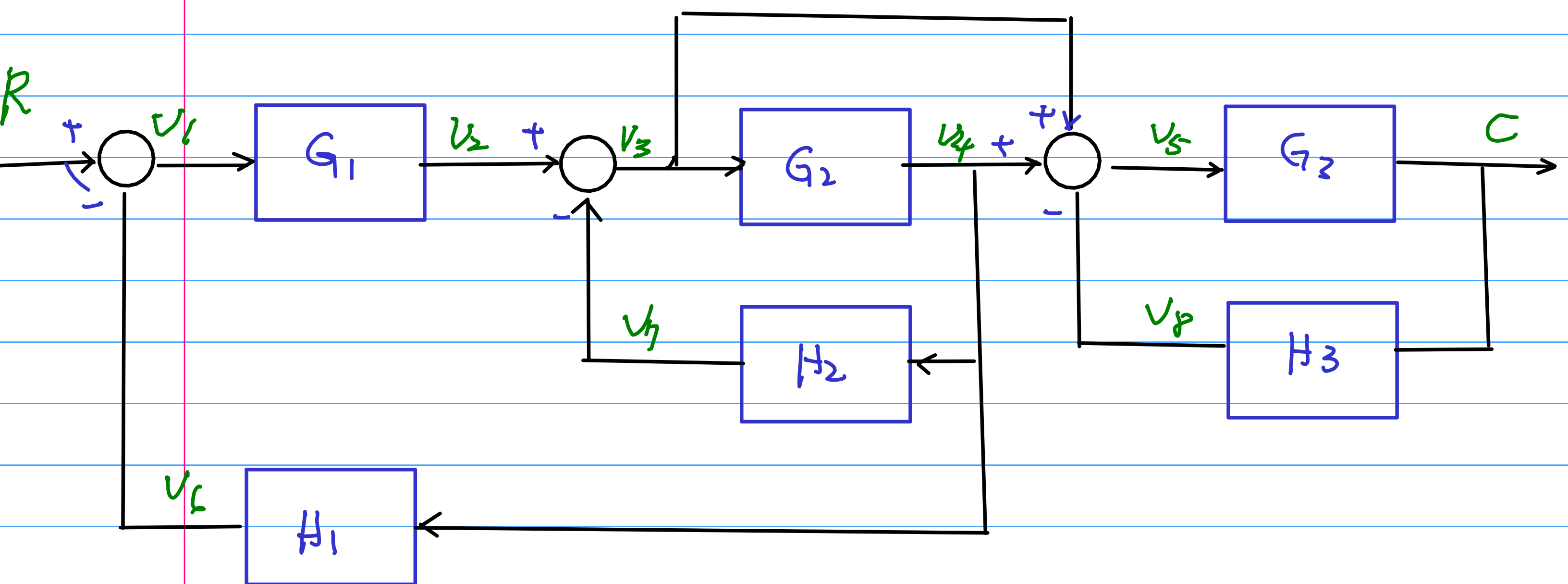
$$\frac{s+6+2s+6}{s+6} = \frac{3s+12}{s+6}$$

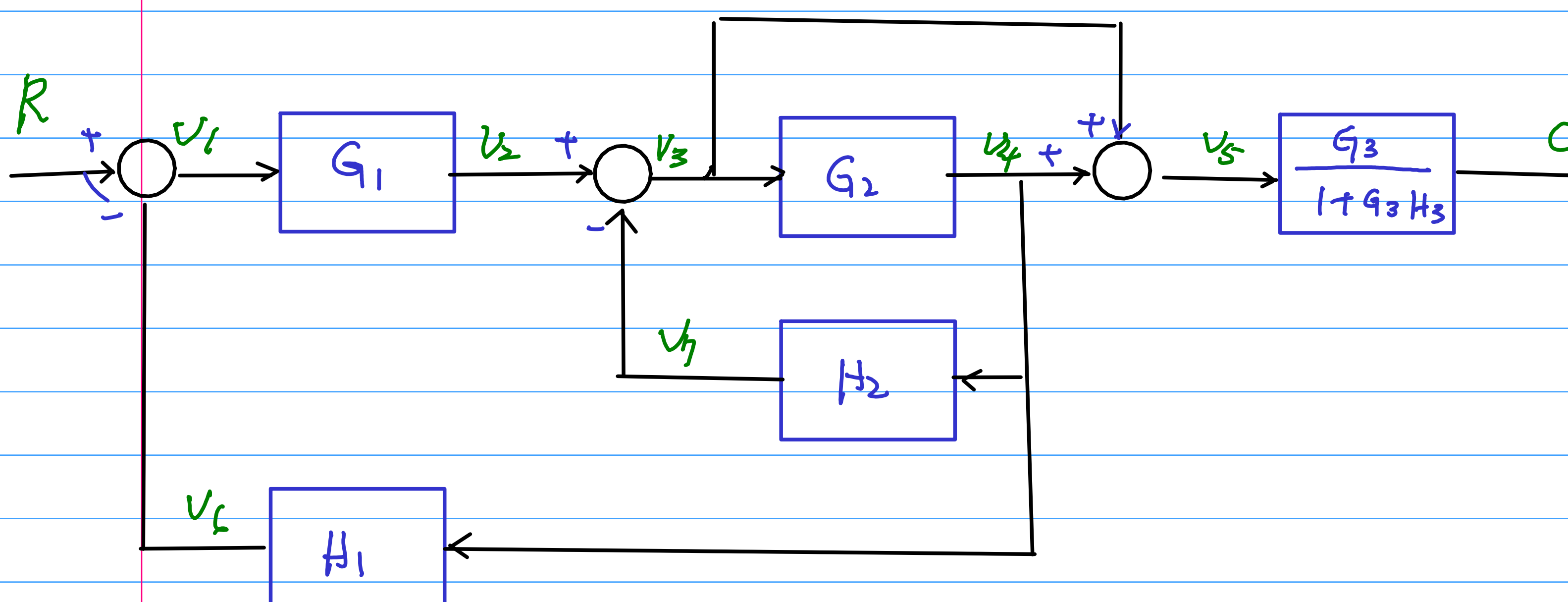
$$\frac{\frac{3}{s+3}}{1+\frac{3}{(s+3)}\frac{3(s+4)}{s+6}}\ \frac{6}{s+6} = \frac{3}{(s+3)+9\frac{(s+4)}{(s+6)}}\ \frac{6}{s+6}$$

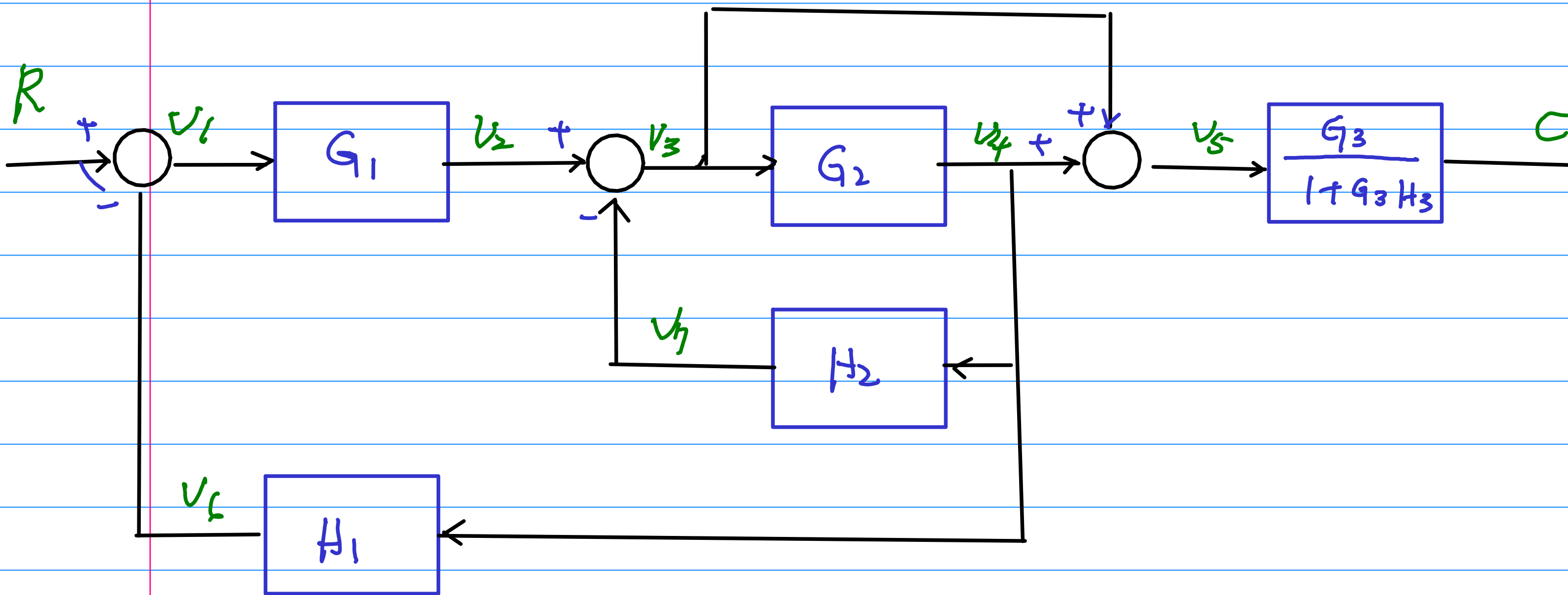
$$= \frac{18}{(s+3)(s+6)+9(s+4)}$$

$$s^2+9s+18+9s+36$$

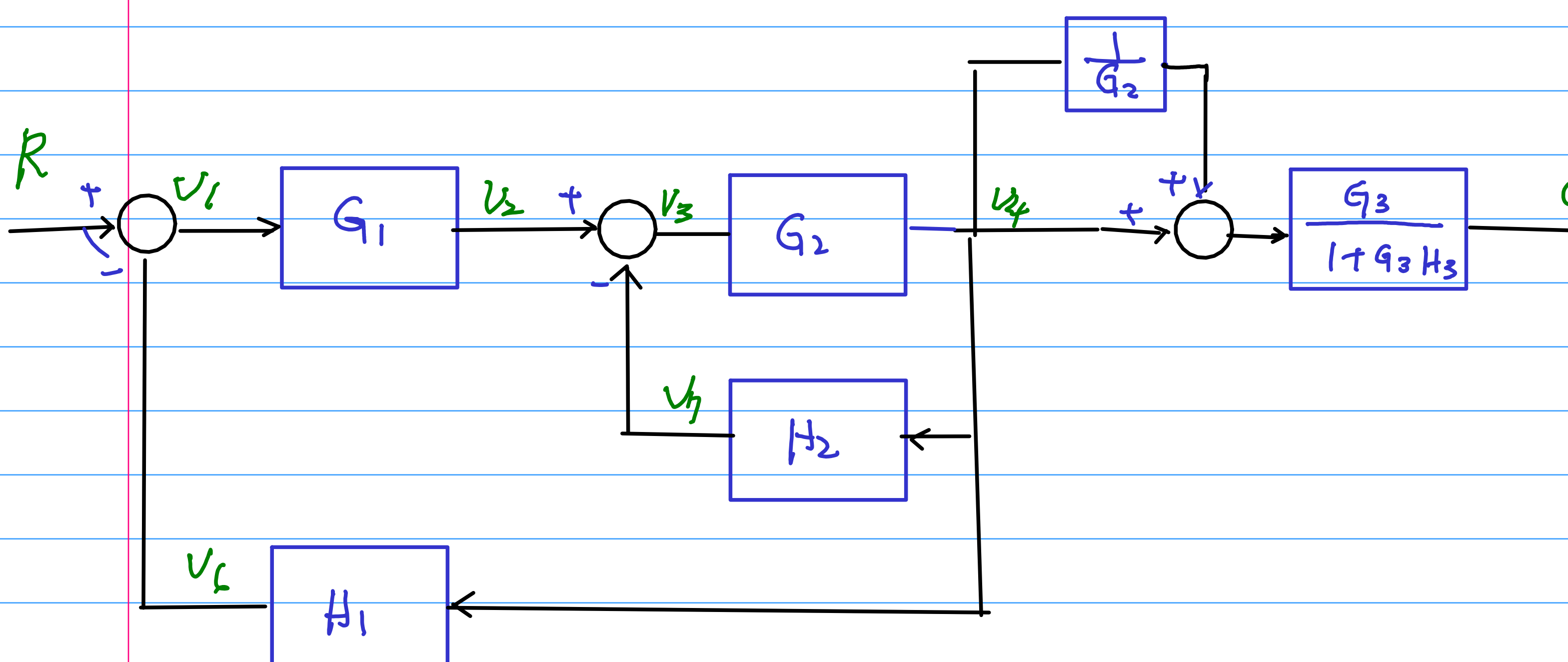
$$\frac{18}{s^2+18s+54}$$


R
+
-
v_1
G_1
v_2
+
v_3
-
G_2
v_4
+
+
-
v_5
G_3
C
v_7
H_2
v_8
H_3
v_6
H_1

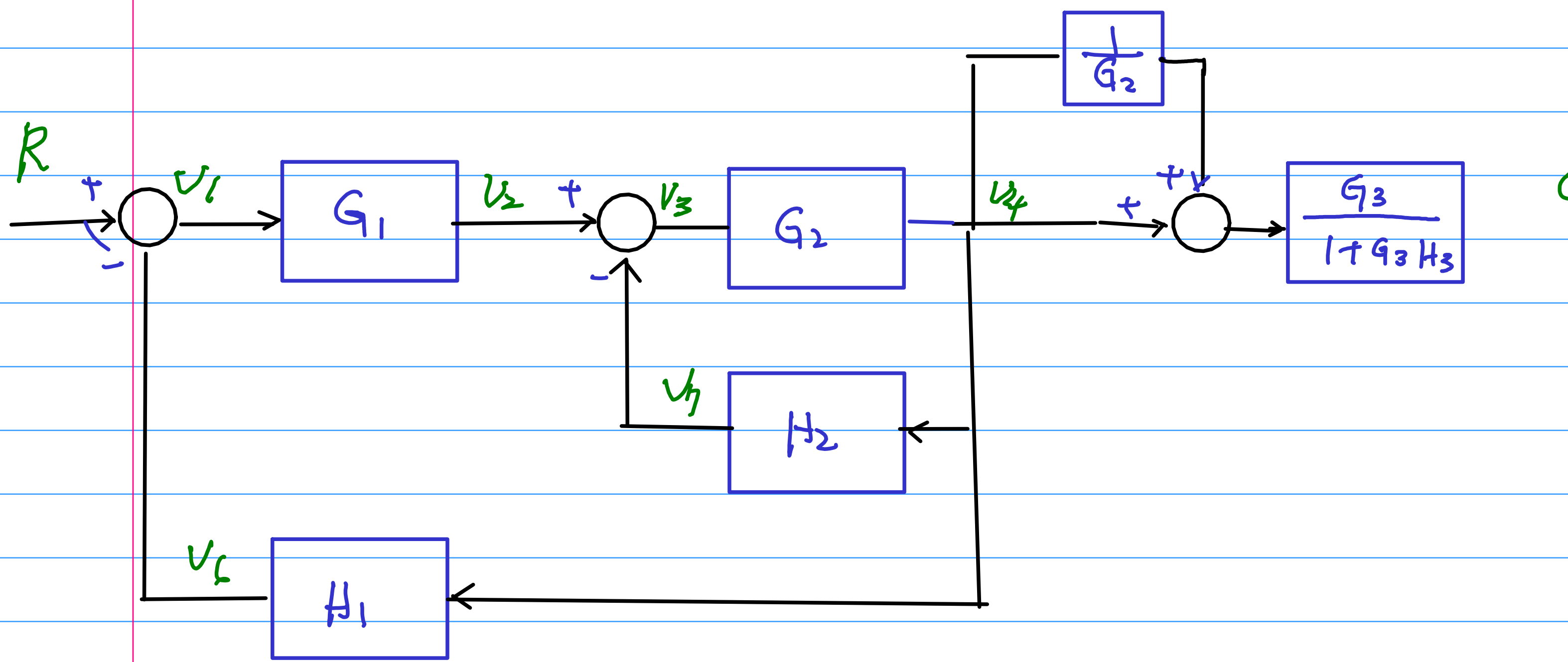

R
+
-
v_1
G_1
v_2
+
v_3
-
G_2
v_4
+
+
v_5
$\frac{G_3}{1+G_3H_3}$
C
v_7
H_2
v_6
H_1



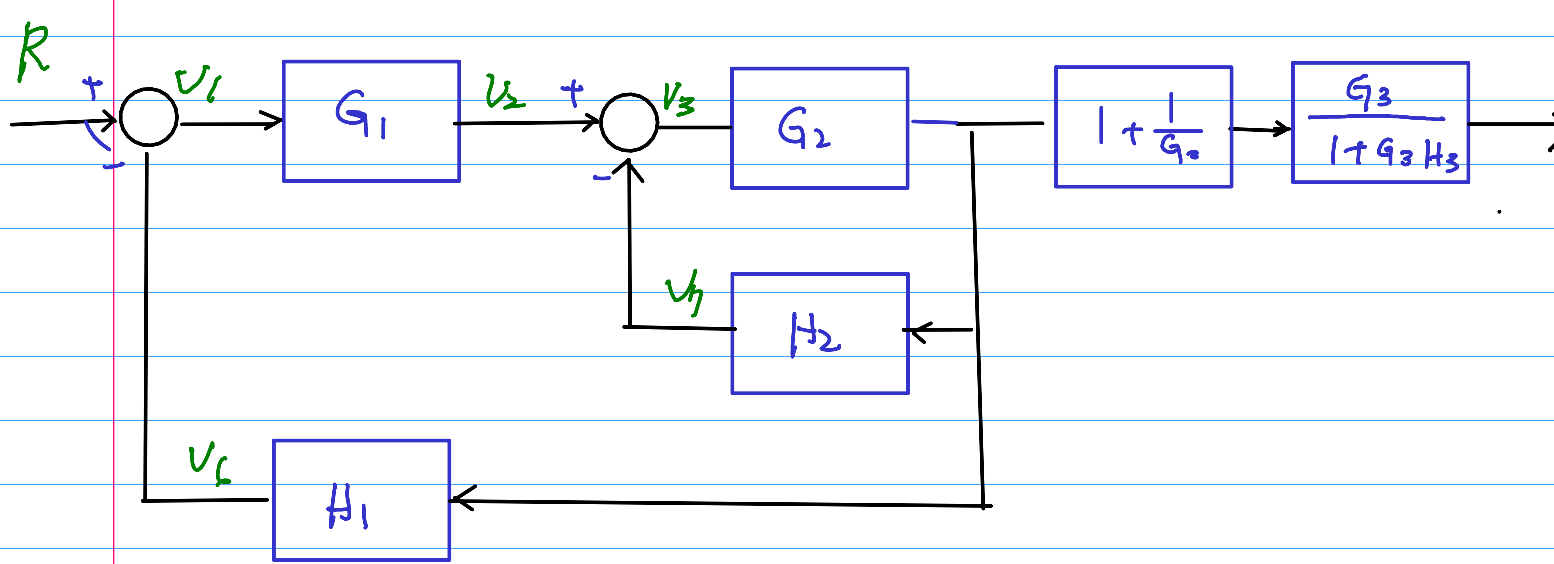
R
V1
G1
V2
V3
G2
V4
V5
G3
1+G3H3
C
V7
H2
V6
H1



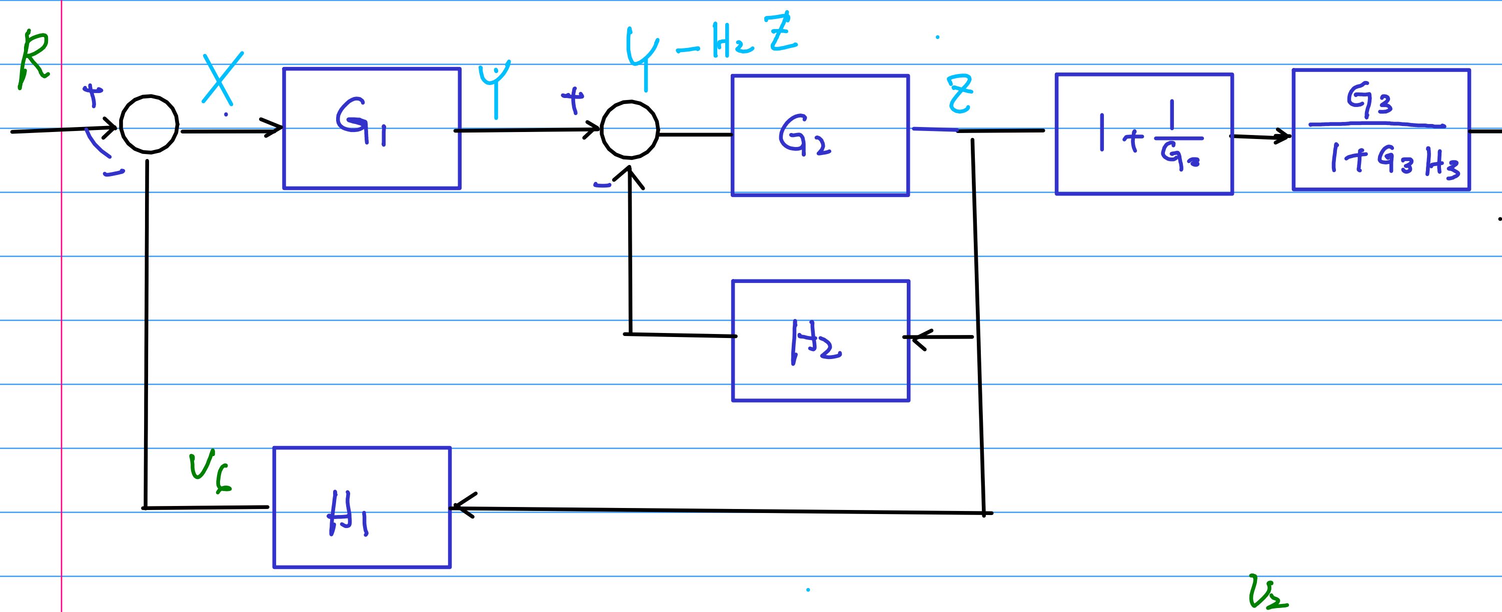
R
V1
G1
V2
V3
G2
1
G2
V4
G3
1+G3H3
C
V7
H2
V6
H1

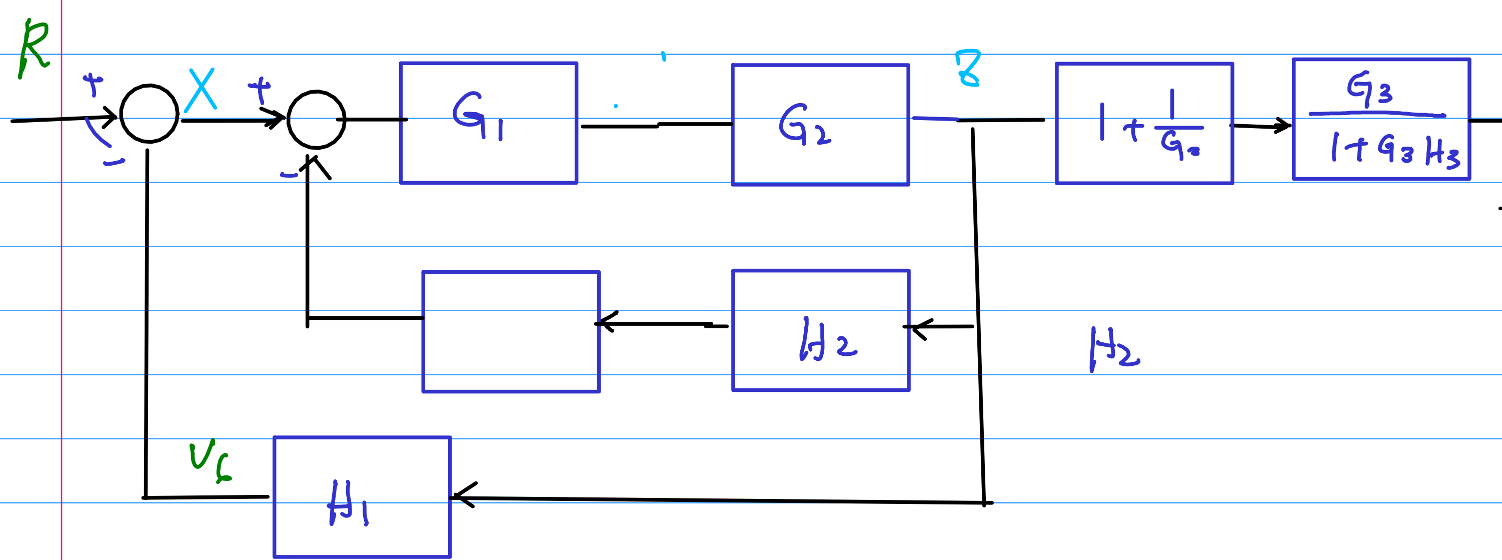


R
+
-
V1
G1
V2
+
-
V3
G2
V4
1/G2
+
+
G3/(1+G3H3)
V7
H2
V6
H1



R
+
-
V1
G1
V2
+
-
V3
G2
1 + 1/G2
G3/(1+G3H3)
V7
H2
V6
H1


R
+
-
X
G1
Y
+
Y - H2 Z
-
G2
Z
1 + 1/G0
G3 / (1+G3H3)
H2
Vf
H1
Uz


R
+
-
X
+
-
G1
G2
Z
1 + 1/G0
G3 / (1+G3H3)
H2
H2
Vf
H1