# Electric Potential

# Contents

# Chapter 1

# Electric potential

Not to be confused with Electric potential energy.

An **electric potential** (also called the *electric field potential* or the *electrostatic potential*) is the amount of electric potential energy that a unitary point electric charge would have if located at any point in space, and is equal to the work done by an electric field in carrying a unit positive charge from infinity to that point.

According to theoretical electromagnetics, electric potential is a scalar quantity denoted by Φ(Phi), Φ***E*** or *V*, equal to the electric potential energy of any charged particle at any location (measured in joules) divided by the charge of that particle (measured in coulombs). By dividing out the charge on the particle a remainder is obtained that is a property of the electric field itself.

This value can be calculated in either a static (time-invariant) or a dynamic (varying with time) electric field at a specific time in units of joules per coulomb ($J\ C^{-1}$), or volts (V). The electric potential at infinity is assumed to be zero.

A generalized electric scalar potential is also used in electrodynamics when time-varying electromagnetic fields are present, but this can not be so simply calculated. The electric potential and the magnetic vector potential together form a four vector, so that the two kinds of potential are mixed under Lorentz transformations.

## 1.1 Introduction

Classical mechanics explores concepts such as force, energy, potential etc. Force and potential energy are directly related. A net force acting on any object will cause it to accelerate. As an object moves in the direction in which the force accelerates it, its potential energy decreases: the gravitational potential energy of a cannonball at the top of a hill is greater than at the base of the hill. As it rolls downhill its potential energy decreases, being translated to motion, inertial (kinetic) energy.

It is possible to define the potential of certain force fields so that the potential energy of an object in that field depends only on the position of the object with respect to the field. Two such force fields are the gravitational field and an electric field (in the absence of time-varying magnetic fields). Such fields must affect objects due to the intrinsic properties of the object (e.g., mass or charge) and the position of the object.

Objects may possess a property known as electric charge and an electric field exerts a force on charged objects. If the charged object has a positive charge the force will be in the direction of the electric field vector at that point while if the charge is negative the force will be in the opposite direction. The magnitude of the force is given by the quantity of the charge multiplied by the magnitude of the electric field vector.

## 1.2 Electrostatics

Main article: Electrostatics

The electric potential at a point **r** in a static electric field **E** is given by the line integral

where *C* is an arbitrary path connecting the point with zero potential to **r**. When the curl $\nabla \times \mathbf{E}$ is zero, the line integral above does not depend on the specific path *C* chosen but only on its endpoints. In this case, the electric field is conservative and determined by the gradient of the potential:

Then, by Gauss's law, the potential satisfies Poisson's equation:

$$\nabla \cdot \mathbf{E} = \nabla \cdot (-\nabla V_{\mathbf{E}}) = -\nabla^2 V_{\mathbf{E}} = \rho/\varepsilon_0,$$

where $\varrho$ is the total charge density (including bound charge) and $\nabla\cdot$ denotes the divergence.

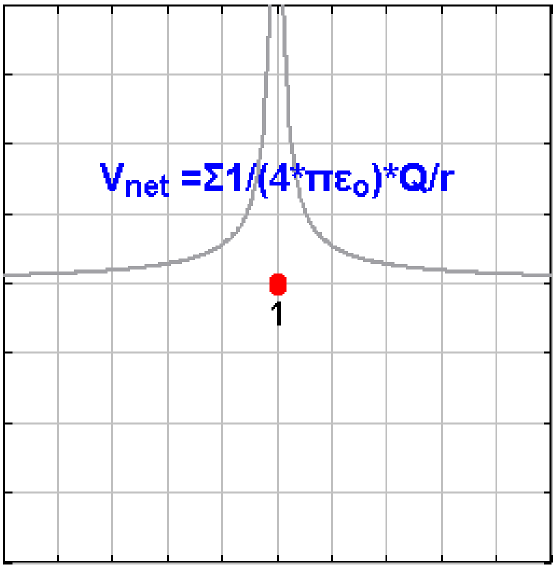
The concept of electric potential is closely linked with potential energy. A test charge *q* has an electric potential energy $U$**E** given by

$$U_\mathbf{E} = q\,V.$$

The potential energy and hence also the electric potential is only defined up to an additive constant: one must arbitrarily choose a position where the potential energy and the electric potential are zero.

These equations cannot be used if the curl $\nabla \times \mathbf{E} \neq 0$, i.e., in the case of a *nonconservative electric field* (caused by a changing magnetic field; see Maxwell's equations). The generalization of electric potential to this case is described below.

### 1.2.1 Electric potential due to a point charge




*The electric potential created by a charge* Q *is* V=Q/(4$\pi\varepsilon$or). *Different values of* Q *will make different values of electric potential* V *(shown in the image).*

The electric potential created by a point charge $Q$, at a distance $r$ from the charge (relative to the potential at infinity), can be shown to be

$$V_\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r},$$

where $\varepsilon_0$ is the electric constant (permittivity of vacuum). This is known as the Coulomb potential.

The electric potential due to a system of point charges is equal to the sum of the point charges' individual potentials. This fact simplifies calculations significantly, since addition of potential (scalar) fields is much easier than addition of the electric (vector) fields.

The equation given above for the electric potential (and all the equations used here) are in the forms required by SI units. In some other (less common) systems of units, such as CGS-Gaussian, many of these equations would be altered.

## 1.3 Generalization to electrodynamics

When time-varying magnetic fields are present (which is true whenever there are time-varying electric fields and vice versa), it is not possible to describe the electric field simply in terms of a scalar potential $V$ because the electric field is no longer conservative: $\int_C \mathbf{E}\cdot \mathrm{d}\boldsymbol{\ell}$ is path-dependent because $\nabla \times \mathbf{E} \neq \mathbf{0}$ (Faraday's law of induction).

Instead, one can still define a scalar potential by also including the magnetic vector potential $\mathbf{A}$. In particular, $\mathbf{A}$ is defined to satisfy:

$$\mathbf{B} = \nabla \times \mathbf{A},$$

where $\mathbf{B}$ is the magnetic field. Because the divergence of the magnetic field is always zero due to the absence of magnetic monopoles, such an $\mathbf{A}$ can always be found. Given this, the quantity

$$\mathbf{F} = \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}$$

*is* a conservative field by Faraday's law and one can therefore write

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t},$$

where $V$ is the scalar potential defined by the conservative field $\mathbf{F}$.

The electrostatic potential is simply the special case of this definition where $\mathbf{A}$ is time-invariant. On the other hand, for time-varying fields,

$$-\int_a^b \mathbf{E}\cdot \mathrm{d}\boldsymbol{\ell} \neq V_{(b)} - V_{(a)},$$

unlike electrostatics.

## 1.4 Units

The SI unit of electric potential is the volt (in honor of Alessandro Volta), which is why a difference in electric potential between two points is known as voltage. Older units are rarely used today. Variants of the centimeter gram second system of units included a number of different units for electric potential, including the abvolt and the statvolt.

## 1.5 Galvani potential versus electrochemical potential

Main articles: Galvani potential, Electrochemical potential and Fermi level

Inside metals (and other solids and liquids), the energy of an electron is affected not only by the electric potential, but also by the specific atomic environment that it is in. When a voltmeter is connected between two different types of metal, it measures not the electric potential difference, but instead the potential difference corrected for the different atomic environments.[1] The quantity measured by a voltmeter is called electrochemical potential or fermi level, while the pure unadjusted electric potential is sometimes called Galvani potential. The terms "voltage" and "electric potential" are a bit ambiguous in that, in practice, they can refer to *either* of these in different contexts.

## 1.6 See also

- Absolute electrode potential
- Electrochemical potential
- Electrode potential
- Gluon field
- Liénard–Wiechert potential
- Mathematical descriptions of the electromagnetic field
- Voltage, or (electric) potential difference

## 1.7 References

[1] Bagotskii, Vladimir Sergeevich (2006). *Fundamentals of electrochemistry*. p. 22. ISBN 978-0-471-70058-6.

- Griffiths, David J. (1998). *Introduction to Electrodynamics* (3rd. ed.). Prentice Hall. ISBN 0-13-805326-X.
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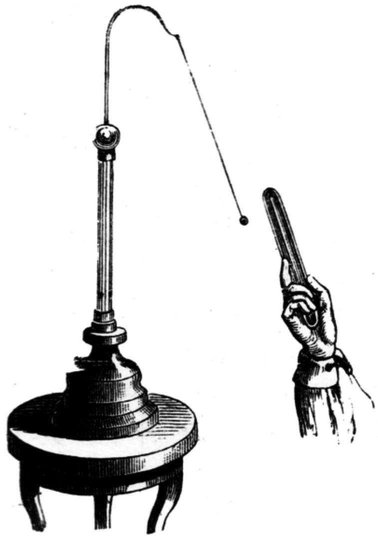
# Chapter 2

# Voltage

"Potential difference" redirects here. For other uses, see Potential.

**Voltage**, **electric potential difference**, **electric pressure** or **electric tension** (denoted $\Delta V$ or $\Delta U$) is the difference in electric potential energy between two points per unit electric charge. The voltage between two points is equal to the work done per unit of charge against a static electric field to move the charge between two points and is measured in units of *volts* (a joule per coulomb).

Voltage can be caused by static electric fields, by electric current through a magnetic field, by time-varying magnetic fields, or some combination of these three.[1][2] A voltmeter can be used to measure the voltage (or potential difference) between two points in a system; often a common reference potential such as the ground of the system is used as one of the points. A voltage may represent either a source of energy (electromotive force), or lost, used, or stored energy (potential drop).



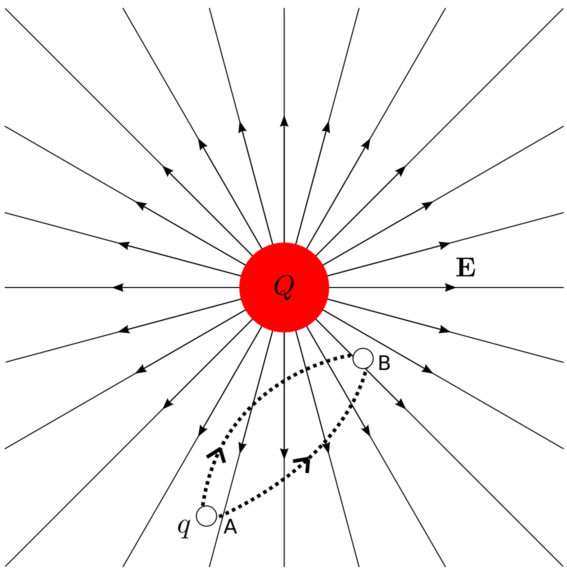
*The electric field around the rod exerts a force on the charged pith ball, in an electroscope*

## 2.1 Definition

Given two points in space, A and B, voltage is the difference in electric potential between those two points. From the definition of electric potential it follows that:

$$\Delta V_{BA} = V_B - V_A = -\int_{r_0}^{B} \vec{E} \cdot d\vec{l} - \left(-\int_{r_0}^{A} \vec{E} \cdot d\vec{l}\right)$$

$$= \int_{B}^{r_0} \vec{E} \cdot d\vec{l} + \int_{r_0}^{A} \vec{E} \cdot d\vec{l} = \int_{B}^{A} \vec{E} \cdot d\vec{l}$$




*In a static field, the work is independent of the path*

Voltage is electric potential energy per unit charge, measured in joules per coulomb ( = volts). It is often referred to as "electric potential", which then must be distinguished from electric potential energy by noting that the "potential" is a "per-unit-charge" quantity. Like mechanical potential energy, the zero of potential can be chosen at any point, so the difference in voltage is the quantity which is physically meaningful. The difference in voltage measured when moving from point A to point B is equal to the work which would have to be done, per unit charge, against the electric field to move the charge from A to B. The voltage between the two ends of a path is the total energy required to move a small electric charge along that path, divided by the magnitude of the charge. Mathematically this is expressed as the line integral of the electric field and the time rate of change of magnetic field along that path. In the general case, both a static (unchanging) electric field and a dynamic (time-varying) electromagnetic field must be included in determining the voltage between two points.

Historically this quantity has also been called “tension”[3] and “pressure”. Pressure is now obsolete but tension is still used, for example within the phrase "high tension" (HT) which is commonly used in thermionic valve (vacuum tube) based electronics.

Voltage is defined so that negatively charged objects are pulled towards higher voltages, while positively charged objects are pulled towards lower voltages. Therefore, the conventional current in a wire or resistor always flows from higher voltage to lower voltage. Current can flow from lower voltage to higher voltage, but only when a source of energy is present to “push” it against the opposing electric field. For example, inside a battery, chemical reactions provide the energy needed for current to flow from the negative to the positive terminal.

The electric field is not the only factor determining charge flow in a material, and different materials naturally develop electric potential differences at equilibrium (Galvani potentials). The electric potential of a material is not even a well defined quantity, since it varies on the subatomic scale. A more convenient definition of 'voltage' can be found instead in the concept of Fermi level. In this case the voltage between two bodies is the thermodynamic work required to move a unit of charge between them. This definition is practical since a real voltmeter actually measures this work, not differences in electric potential.

## 2.2 Hydraulic analogy

Main article: Hydraulic analogy

A simple analogy for an electric circuit is water flowing in a closed circuit of pipework, driven by a mechanical pump. This can be called a “water circuit”. Potential difference between two points corresponds to the pressure difference between two points. If the pump creates a pressure difference between two points, then water flowing from one point to the other will be able to do work, such as driving a turbine. Similarly, work can be done by an electric current driven by the potential difference provided by a battery. For example, the voltage provided by a sufficiently-charged automobile battery can “push” a large current through the windings of an automobile’s starter motor. If the pump isn't working, it produces no pressure difference, and the turbine will not rotate. Likewise, if the automobile’s battery is very weak or “dead” (or “flat”), then it will not turn the starter motor.

The hydraulic analogy is a useful way of understanding many electrical concepts. In such a system, the work done to move water is equal to the pressure multiplied by the volume of water moved. Similarly, in an electrical circuit, the work done to move electrons or other charge-carriers is equal to “electrical pressure” multiplied by the quantity of electrical charges moved. In relation to “flow”, the larger the “pressure difference” between two points (potential difference or water pressure difference), the greater the flow between them (electric current or water flow). (See "Electric power".)

## 2.3 Applications


*Working on high voltage power lines*

Specifying a voltage measurement requires explicit or implicit specification of the points across which the voltage is measured. When using a voltmeter to measure potential difference, one electrical lead of the voltmeter must be connected to the first point, one to the second point.

A common use of the term “voltage” is in describing the voltage dropped across an electrical device (such as a resistor). The voltage drop across the device can be understood as the difference between measurements at each terminal of the device with respect to a common reference point (or ground). The voltage drop is the difference between the two readings. Two points in an electric circuit that are connected by an ideal conductor without resistance and not within a changing magnetic field have a voltage of zero. Any two points with the same potential may be connected by a conductor and no current will flow between them.

### 2.3.1 Addition of voltages

The voltage between $A$ and $C$ is the sum of the voltage between $A$ and $B$ and the voltage between $B$ and $C$. The various voltages in a circuit can be computed using Kirchhoff's circuit laws.

When talking about alternating current (AC) there is a difference between instantaneous voltage and average voltage. Instantaneous voltages can be added for direct current (DC) and AC, but average voltages can be meaningfully added only when they apply to signals that all have the same frequency and phase.

## 2.4 Measuring instruments


*Multimeter set to measure voltage*

Instruments for measuring voltages include the voltmeter, the potentiometer, and the oscilloscope. The voltmeter works by measuring the current through a fixed resistor, which, according to Ohm's Law, is proportional to the voltage across the resistor. The potentiometer works by balancing the unknown voltage against a known voltage in a bridge circuit. The cathode-ray oscilloscope works by amplifying the voltage and using it to deflect an electron beam from a straight path, so that the deflection of the beam is proportional to the voltage.

## 2.5 Typical voltages

Main article: Mains electricity § Choice of voltage

A common voltage for flashlight batteries is 1.5 volts (DC). A common voltage for automobile batteries is 12 volts (DC).

Common voltages supplied by power companies to consumers are 110 to 120 volts (AC) and 220 to 240 volts (AC). The voltage in electric power transmission lines used to distribute electricity from power stations can be several hundred times greater than consumer voltages, typically 110 to 1200 kV (AC).

The voltage used in overhead lines to power railway locomotives is between 12 kV and 50 kV (AC).

## 2.6 Galvani potential vs. electrochemical potential

Main articles: Galvani potential, Electrochemical potential and Fermi level

Inside a conductive material, the energy of an electron is affected not only by the average electric potential, but also by the specific thermal and atomic environment that it is in. When a voltmeter is connected between two different types of metal, it measures not the electrostatic potential difference, but instead something else that is affected by thermodynamics.[4] The quantity measured by a voltmeter is the negative of difference of electrochemical potential of electrons (Fermi level) divided by electron charge, while the pure unadjusted electrostatic potential (not measurable with voltmeter) is sometimes called Galvani potential. The terms "voltage" and "electric potential" are a bit ambiguous in that, in practice, they can refer to *either* of these in different contexts.

## 2.7 See also

- Alternating current (AC)
- Direct current (DC)
- Electric potential
- Electric shock
- Electrical measurements
- Electrochemical potential
- Fermi level
- High voltage

- Mains electricity (an article about domestic power supply voltages)
- Mains electricity by country (list of countries with mains voltage and frequency)
- Ohm's law
- Ohm
- Open-circuit voltage
- Phantom voltage

## 2.8 References

[1] Demetrius T. Paris and F. Kenneth Hurd, *Basic Electromagnetic Theory*, McGraw-Hill, New York 1969, ISBN 0-07-048470-8, pp. 512, 546

[2] P. Hammond, *Electromagnetism for Engineers*, p. 135, Pergamon Press 1969 OCLC 854336.

[3] "Tension". *CollinsLanguage*.

[4] Bagotskii, Vladimir Sergeevich (2006). *Fundamentals of electrochemistry*. p. 22. ISBN 978-0-471-70058-6.

## 2.9 External links

- Electrical voltage *V*, amperage *I*, resistivity *R*, impedance *Z*, wattage *P*
- Elementary explanation of voltage at NDT Resource Center

# Chapter 3

# Electric potential energy

Not to be confused with Electric potential or Electric power.
This article is about the physical magnitude Electric Potential Energy. For electrical energy, see Electrical energy. For energy sources, see Energy development. For electricity generation, see Electricity generation.

**Electric potential energy**, or **electrostatic potential energy**, is a potential energy (measured in joules) that results from conservative Coulomb forces and is associated with the configuration of a particular set of point charges within a defined system. An *object* may have electric potential energy by virtue of two key elements: its own electric charge and its relative position to other electrically charged *objects*.

The term "electric potential energy" is used to describe the potential energy in systems with time-variant electric fields, while the term "electrostatic potential energy" is used to describe the potential energy in systems with time-invariant electric fields.

## 3.1 Definition

We define the electric potential energy of a system of point charges as the work required assembling this system of charges by bringing them close together, as in the system from an infinite distance.

> The electrostatic potential energy, *UE*, of one point charge $q$ at position $\mathbf{r}$ in the presence of an electric field $\mathbf{E}$ is defined as the negative of the work $W$ done by the electrostatic force to bring it from the reference position $\mathbf{r}_{\text{ref}}$[note 1] to that position $\mathbf{r}$.[1][2]:§25-1[note 2]

> where $\mathbf{E}$ is the electrostatic field and d$\mathbf{s}$ is the displacement vector in a curve from the reference position $\mathbf{r}_{\text{ref}}$ to the final position $\mathbf{r}$.

The electrostatic potential energy can also be defined from the electric potential as follows:

> The electrostatic potential energy, *UE*, of one point charge $q$ at position $\mathbf{r}$ in the presence of an electric potential $\Phi$ is defined as the product of the charge and the electric potential.

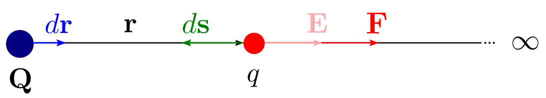
> where $\Phi$ is the electric potential generated by the charges, which is a function of position $\mathbf{r}$.

## 3.2 Units

The SI unit of electric potential energy is the joule (named after the English physicist James Prescott Joule). In the CGS system the erg is the unit of energy, being equal to $10^{-7}$ J. Also electronvolts may be used, 1 eV = $1.602{\times}10^{-19}$ J.

## 3.3 Electrostatic potential energy of one point charge

### 3.3.1 One point charge *q* in the presence of one point charge *Q*

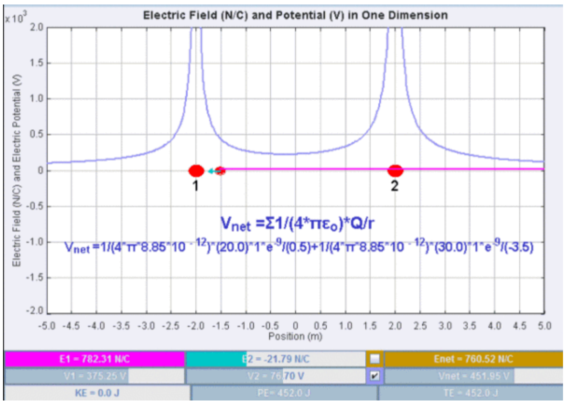



*A point charge q in the electric field of another charge Q.*

The electrostatic potential energy, *UE*, of one point charge $q$ at position $\mathbf{r}$ in the presence of a point charge $Q$, taking an infinite separation between the charges as the reference position, is:

where $k_e = \frac{1}{4\pi\varepsilon_0}$ is Coulomb's constant, $r$ is the distance between the point charges $q$ & $Q$, and $q$ & $Q$ are the signed values of the charges (not the modules of the charges. For example, an electron would have a negative value of charge when placed in the formula). The following outline of proof states the derivation from the definition of electric potential energy and Coulomb's law to this formula.

### 3.3.2 One point charge *q* in the presence of *n* point charges *Qi*




*Electrostatic potential energy of* q *due to* $Q_1$ *and* $Q_2$ *charge system:* $U_E = q\frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right)$

The electrostatic potential energy, *UE*, of one point charge *q* in the presence of *n* point charges *Qi*, taking an infinite separation between the charges as the reference position, is:

where $k_e = \frac{1}{4\pi\varepsilon_0}$ is Coulomb's constant, *ri* is the distance between the point charges *q* & *Qi*, and *q* & *Qi* are the signed values of the charges.

## 3.4 Electrostatic potential energy stored in a system of point charges

The electrostatic potential energy *U*E stored in a system of *N* charges $q_1$, $q_2$, ..., *qN* at positions $\mathbf{r}_1$, $\mathbf{r}_2$, ..., **r***N* respectively, is:

where, for each *i* value, Φ(**r***i*) is the electrostatic potential due to all point charges except the one at **r***i*,[note 3] and is equal to:

$\Phi(\mathbf{r}_i) = \sum_{j=1}^{N(j\neq i)} k_e \frac{q_j}{\mathbf{r}_{ij}}$ ,

where $\mathbf{r}_{ij}$ is the distance between $q_j$ and $\mathbf{r}_i$.

### 3.4.1 Energy stored in a system of one point charge

The electrostatic potential energy of a system containing only one point charge is zero, as there are no other sources of electrostatic potential against which an external agent must do work in moving the point charge from infinity to its final location.

### 3.4.2 Energy stored in a system of two point charges

Consider bringing a point charge, *q*, into its final position in the vicinity of a point charge, $Q_1$. The electrostatic potential Φ(**r**) due to $Q_1$ is

$$\Phi(r) = k_e \frac{Q_1}{r}$$

Hence we obtain, the electric potential energy of *q* in the potential of $Q_1$ as

$$U_E = \frac{1}{4\pi\varepsilon_0}\frac{qQ_1}{r_1}$$

where $r_1$is the separation between the two point charges.

### 3.4.3 Energy stored in a system of three point charges

The electrostatic potential energy of a system of three charges should not be confused with the electrostatic potential energy of $Q_1$ due to two charges $Q_2$ and $Q_3$, because the latter doesn't include the electrostatic potential energy of the system of the two charges $Q_2$ and $Q_3$.

The electrostatic potential energy stored in the system of three charges is:

$$U_\mathrm{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right)$$

## 3.5 Energy stored in an electrostatic field distribution

The energy density, or energy per unit volume, $\frac{dU}{dV}$ , of the electrostatic field of a continuous charge distribution is:

$$u_e = \frac{dU}{dV} = \frac{1}{2}\varepsilon_0 \left|\mathbf{E}\right|^2 .$$

## 3.6 Energy in electronic elements


*The electric potential energy stored in a* capacitor *is $UE=\tfrac{1}{2}CV^2$*

Some elements in a circuit can convert energy from one form to another. For example, a resistor converts electrical energy to heat, this is known as the Joule effect. A capacitor stores it in its electric field. The total electric potential energy stored in a capacitor is given by

$$U_E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

where *C* is the capacitance, *V* is the electric potential difference, and *Q* the charge stored in the capacitor.

## 3.7 Notes

[1] The reference zero is usually taken to be a state in which the individual point charges are very well separated ("are at infinite separation") and are at rest.

[2] Alternatively, it can also be defined as the work *W* done by the an external force to bring it from the reference position $\mathbf{r}_{\text{ref}}$ to some position $\mathbf{r}$. Nonetheless, both definitions yield the same results.

[3] The factor of one half accounts for the 'double counting' of charge pairs. For example, consider the case of just two charges.

## 3.8 References

[1] Electromagnetism (2nd edition), I.S. Grant, W.R. Phillips, Manchester Physics Series, 2008 ISBN 0-471-92712-0

[2] Halliday, David; Resnick, Robert; Walker, Jearl (1997). "Electric Potential". *Fundamentals of Physics* (5th ed.). John Wiley & Sons. ISBN 0-471-10559-7.

# Chapter 4

# Work (physics)

"Mechanical work" redirects here. For other uses of "Work" in physics, see Work (electrical) and Work (thermodynamics).

In physics, a force is said to do **work** if, when acting on a body, there is a displacement of the point of application in the direction of the force. For example, when a ball is held above the ground and then dropped, the work done on the ball as it falls is equal to the weight of the ball (a force) multiplied by the distance to the ground (a displacement).

The term *work* was introduced in 1826 by the French mathematician Gaspard-Gustave Coriolis[1][2] as "weight *lifted* through a height", which is based on the use of early steam engines to lift buckets of water out of flooded ore mines. The SI unit of work is the newton-metre or joule (J).

## 4.1 Units

The SI unit of work is the joule (J), which is defined as the work expended by a force of one newton through a distance of one metre.

The dimensionally equivalent newton-metre (N·m) is sometimes used as the measuring unit for work, but this can be confused with the unit newton-metre, which is the measurement unit of torque. Usage of N·m is discouraged by the SI authority, since it can lead to confusion as to whether the quantity expressed in newton metres is a torque measurement, or a measurement of energy.[3]

Non-SI units of work include the erg, the foot-pound, the foot-poundal, the kilowatt hour, the litre-atmosphere, and the horsepower-hour. Due to work having the same physical dimension as heat, occasionally measurement units typically reserved for heat or energy content, such as therm, BTU and Calorie, are utilized as a measuring unit.

## 4.2 Work and energy

The work done by a constant force of magnitude $F$ on a point that moves a displacement (not distance) $s$ in the direction of the force is the product,

$$W = Fs.$$

For example, if a force of 10 newtons ($F$ = 10 N) acts along a point that travels 2 metres ($s$ = 2 m), then it does the work $W$ = (10 N)(2 m) = 20 N m = 20 J. This is approximately the work done lifting a 1 kg weight from ground to over a person's head against the force of gravity. Notice that the work is doubled either by lifting twice the weight the same distance or by lifting the same weight twice the distance.

Work is closely related to energy. The law of conservation of energy states that the change in total internal energy of a system equals the added heat, minus the work performed by the system (see the first law of thermodynamics),

$$dE = \delta Q - \delta W,$$

where the symbol $\delta$ indicates that heat ($Q$) and work ($W$) are inexact differentials.

From Newton's second law, it can be shown that work on a free (no fields), rigid (no internal degrees of freedom) body, is equal to the change in kinetic energy of the velocity and rotation of that body,

$$W = \Delta KE.$$

The work of forces generated by a potential function is known as potential energy and the forces are said to be conservative. Therefore, work on an object that is merely displaced in a conservative force field, without change in velocity or rotation, is equal to *minus* the change of potential energy of the object,

$$W = -\Delta PE.$$

These formulas demonstrate that work is the energy associated with the action of a force, so work subsequently possesses the physical dimensions, and units, of energy. The work/energy principles discussed here are identical to Electric work/energy principles.

## 4.3 Constraint forces

Constraint forces determine the movement of components in a system, constraining the object within a boundary (in the case of a slope plus gravity, the object is *stuck to* the slope, when attached to a taut string it cannot move in an outwards direction to make the string any 'tauter'). Constraint forces ensure the velocity in the direction of the constraint is zero, which means the constraint forces do not perform work on the system.

If the system doesn't change in time,[4] they eliminate all movement in the direction of the constraint, thus constraint forces do not perform work on the system, as the velocity of that object is constrained to be 0 parallel to this force, due to this force. This only applies for a single particle system. For example, in an Atwood machine, the rope does work on each body, but keeping always the net virtual work null. There are, however, cases where this is not true.[4]

For example, the centripetal force exerted *inwards* by a string on a ball in uniform circular motion *sideways* constrains the ball to circular motion restricting its movement away from the center of the circle. This force does zero work because it is perpendicular to the velocity of the ball.

Another example is a book on a table. If external forces are applied to the book so that it slides on the table, then the force exerted by the table constrains the book from moving downwards. The force exerted by the table supports the book and is perpendicular to its movement which means that this constraint force does not perform work.

The magnetic force on a charged particle is F = $q\mathbf{v} \times \mathbf{B}$, where $q$ is the charge, $\mathbf{v}$ is the velocity of the particle, and $\mathbf{B}$ is the magnetic field. The result of a cross product is always perpendicular to both of the original vectors, so $\mathbf{F} \perp \mathbf{v}$. The dot product of two perpendicular vectors is always zero, so the work $W = \mathbf{F} \cdot \mathbf{v} = 0$, and the magnetic force does not do work. It can change the direction of motion but never change the speed.

## 4.4 Mathematical calculation

For moving objects, the quantity of work/time (power) is calculated. Thus, at any instant, the rate of the work done by a force (measured in joules/second, or **watts**) is the scalar product of the force (a vector), and the velocity vector of the point of application. This scalar product of force and velocity is classified as instantaneous power. Just as velocities may be integrated over time to obtain a total distance, by the fundamental theorem of calculus, the total work along a path is similarly the time-integral of instantaneous power applied along the trajectory of the point of application.[5]

Work is the result of a force on a point that moves through a distance. As the point moves, it follows a curve $\mathbf{X}$, with a velocity $\mathbf{v}$, at each instant. The small amount of work $\delta W$ that occurs over an instant of time $dt$ is calculated as

$$\delta W = \mathbf{F} \cdot d\mathbf{s} = \mathbf{F} \cdot \mathbf{v} dt$$

where the $\mathbf{F} \cdot \mathbf{v}$ is the power over the instant $dt$. The sum of these small amounts of work over the trajectory of the point yields the work,

$$W = \int_{t_1}^{t_2} \mathbf{F} \cdot \mathbf{v} dt = \int_{t_1}^{t_2} \mathbf{F} \cdot \tfrac{d\mathbf{s}}{dt} dt = \int_C \mathbf{F} \cdot d\mathbf{s},$$

where $C$ is the trajectory from $\mathbf{x}(t_1)$ to $\mathbf{x}(t_2)$. This integral is computed along the trajectory of the particle, and is therefore said to be *path dependent*.

If the force is always directed along this line, and the magnitude of the force is $F$, then this integral simplifies to

$$W = \int_C F\, ds$$

where $s$ is distance along the line. If $\mathbf{F}$ is constant, in addition to being directed along the line, then the integral simplifies further to

$$W = \int_C F\, ds = F \int_C ds = Fs$$

where $s$ is the distance travelled by the point along the line.

This calculation can be generalized for a constant force that is not directed along the line, followed by the particle. In this case the dot product $\mathbf{F} \cdot d\mathbf{s} = F \cos \theta \, ds$, where $\theta$ is the angle between the force vector and the direction of movement,[5] that is

$$W = \int_C \mathbf{F} \cdot d\mathbf{s} = Fs\cos\theta.$$

In the notable case of a force applied to a body always at an angle of 90° from the velocity vector (as when a body moves in a circle under a central force), no work is done at all, since the cosine of 90 degrees is zero. Thus, no work can be performed by gravity on a planet with a circular orbit (this is ideal, as all orbits are slightly elliptical). Also, no work is done on a body moving circularly at a constant speed while constrained by mechanical force, such as moving at constant speed in a frictionless ideal centrifuge.

Calculating the work as "force times straight path segment" would only apply in the most simple of circumstances, as noted above. If force is changing, or if the body is moving along a curved path, possibly rotating and not necessarily rigid, then only the path of the application point of the force is relevant for the work done, and only the component of the force parallel to the application point velocity is doing work (positive work when in the same direction, and negative when in the opposite direction of the velocity). This component of force can be described by the scalar quantity called *scalar tangential component* ( $F\cos\theta$ , where $\theta$ is the angle between the force and the velocity). And then the most general definition of work can be formulated as follows:

> *Work of a force is the line integral of its scalar tangential component along the path of its application point.*

### 4.4.1 Torque and rotation

A force couple results from equal and opposite forces, acting on two different points of a rigid body. The sum (resultant) of these forces may cancel, but their effect on the body is the couple or torque **T**. The work of the torque is calculated as

$$\delta W = \mathbf{T}\cdot\vec{\omega}\delta t,$$

where the $\mathbf{T}\cdot\boldsymbol{\omega}$ is the power over the instant $\delta t$. The sum of these small amounts of work over the trajectory of the rigid body yields the work,

$$W = \int_{t_1}^{t_2}\mathbf{T}\cdot\vec{\omega}dt.$$

This integral is computed along the trajectory of the rigid body with an angular velocity $\omega$ that varies with time, and is therefore said to be *path dependent*.

If the angular velocity vector maintains a constant direction, then it takes the form,

$$\vec{\omega} = \dot{\phi}\mathbf{S},$$

where $\varphi$ is the angle of rotation about the constant unit vector **S**. In this case, the work of the torque becomes,

$$W = \int_{t_1}^{t_2}\mathbf{T}\cdot\vec{\omega}dt = \int_{t_1}^{t_2}\mathbf{T}\cdot\mathbf{S}\frac{d\phi}{dt}dt = \int_C \mathbf{T}\cdot\mathbf{S}d\phi,$$

where $C$ is the trajectory from $\varphi(t_1)$ to $\varphi(t_2)$. This integral depends on the rotational trajectory $\varphi(t)$, and is therefore path-dependent.

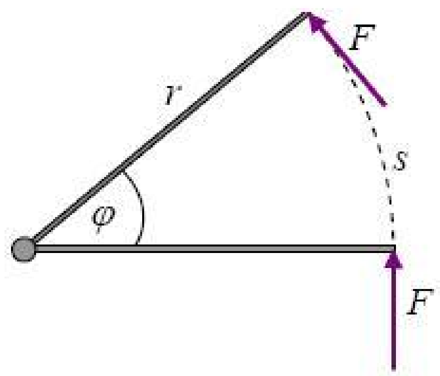
If the torque **T** is aligned with the angular velocity vector so that,

$$\mathbf{T} = \tau\mathbf{S},$$

and both the torque and angular velocity are constant, then the work takes the form,[6]

$$W = \int_{t_1}^{t_2}\tau\dot{\phi}dt = \tau(\phi_2 - \phi_1).$$

This result can be understood more simply by considering the torque as arising from a force of constant magnitude $F$, being applied perpendicularly to a lever arm at a distance $r$, as shown in the figure. This force will act through the distance along the circular arc $s = r\varphi$, so the work done is



*A force of constant magnitude and perpendicular to the lever arm*

$$W = Fs = Fr\phi.$$

Introduce the torque $\tau = Fr$, to obtain

$$W = Fr\phi = \tau\phi,$$

as presented above.

Notice that only the component of torque in the direction of the angular velocity vector contributes to the work.

## 4.5 Work and potential energy

The scalar product of a force **F** and the velocity **v** of its point of application defines the power input to a system at an instant of time. Integration of this power over the trajectory of the point of application, $C = \mathbf{x}(t)$, defines the work input to the system by the force.

### 4.5.1 Path dependence

Therefore, the work done by a force **F** on an object that travels along a curve $C$ is given by the line integral:

$$W = \int_C \mathbf{F} \cdot d\mathbf{x} = \int_{t_1}^{t_2} \mathbf{F} \cdot \mathbf{v} dt,$$

where $dx(t)$ defines the trajectory $C$ and **v** is the velocity along this trajectory. In general this integral requires the path along which the velocity is defined, so the evaluation of work is said to be path dependent.

The time derivative of the integral for work yields the instantaneous power,

$$\frac{dW}{dt} = P(t) = \mathbf{F} \cdot \mathbf{v}.$$

### 4.5.2 Path independence

If the work for an applied force is independent of the path, then the work done by the force is, by the gradient theorem, the potential function evaluated at the start and end of the trajectory of the point of application. Such a force is said to be conservative. This means that there is a potential function $U(\mathbf{x})$, that can be evaluated at the two points $\mathbf{x}(t_1)$ and $\mathbf{x}(t_2)$ to obtain the work over any trajectory between these two points. It is tradition to define this function with a negative sign so that positive work is a reduction in the potential, that is

$$W = \int_C \mathbf{F} \cdot \mathrm{d}\mathbf{x} = \int_{\mathbf{x}(t_1)}^{\mathbf{x}(t_2)} \mathbf{F} \cdot \mathrm{d}\mathbf{x} = U(\mathbf{x}(t_1)) - U(\mathbf{x}(t_2)).$$

The function $U(\mathbf{x})$ is called the potential energy associated with the applied force. Examples of forces that have potential energies are gravity and spring forces.

In this case, the gradient of work yields

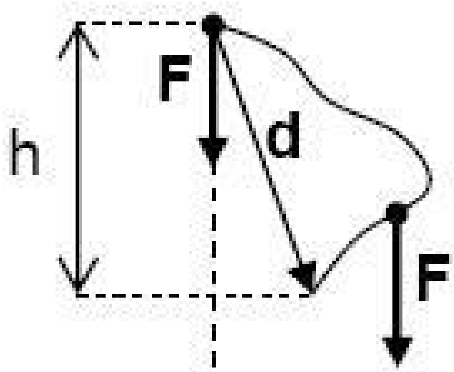
$$\frac{\partial W}{\partial \mathbf{x}} = -\frac{\partial U}{\partial \mathbf{x}} = -(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}) = \mathbf{F},$$

and the force **F** is said to be “derivable from a potential.”[7]

Because the potential $U$ defines a force **F** at every point **x** in space, the set of forces is called a force field. The power applied to a body by a force field is obtained from the gradient of the work, or potential, in the direction of the velocity **V** of the body, that is

$$P(t) = -\frac{\partial U}{\partial \mathbf{x}} \cdot \mathbf{v} = \mathbf{F} \cdot \mathbf{v}.$$

### 4.5.3 Work by gravity




*Gravity* F = mg *does work* W = mgh *along any descending path*

Gravity exerts a constant downward force on every object. Near Earth’s surface the acceleration due to gravity is $g$ = 9.8 m·s$^{-2}$ and the gravitational force on an object of mass $m$ is $\mathbf{F}_g = mg$. It is convenient to imagine this gravitational force concentrated at the center of mass of the object.

If an object is displaced upwards or downwards a vertical distance $y_2 - y_1$, the work $W$ done on the object by its weight mg is:

$$W = F_g(y_2 - y_1) = F_g \Delta y = mg\Delta y$$

where $Fg$ is weight (pounds in imperial units, and newtons in SI units), and $\Delta y$ is the change in height $y$. Notice that the work done by gravity depends only on the vertical movement of the object. The presence of friction does not affect the work done on the object by its weight.

### 4.5.4 Work by gravity in space

The force of gravity exerted by a mass $M$ on another mass $m$ is given by

$$\mathbf{F} = -\frac{GMm}{r^3}\mathbf{r},$$

where **r** is the position vector from $M$ to $m$.

Let the mass $m$ move at the velocity **v** then the work of gravity on this mass as it moves from position $\mathbf{r}(t_1)$ to $\mathbf{r}(t_2)$ is given by

$$W = -\int_{\mathbf{r}(t_1)}^{\mathbf{r}(t_2)} \frac{GMm}{r^3}\mathbf{r}\cdot d\mathbf{r} = -\int_{t_1}^{t_2} \frac{GMm}{r^3}\mathbf{r}\cdot \mathbf{v}dt.$$

Notice that the position and velocity of the mass *m* are given by

$$\mathbf{r} = r\mathbf{e}_r, \qquad \mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_t,$$

where **e***r* and **e***t* are the radial and tangential unit vectors directed relative to the vector from *M* to *m*. Use this to simplify the formula for work of gravity to,

$$W = -\int_{t_1}^{t_2} \frac{GmM}{r^3}(r\mathbf{e}_r)\cdot(\dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_t)dt = -\int_{t_1}^{t_2} \frac{GmM}{r^3}r\dot{r}dt = \frac{GMm}{r(t_2)} - \frac{GMm}{r(t_1)}.$$

This calculation uses the fact that

$$\frac{d}{dt}r^{-1} = -r^{-2}\dot{r} = -\frac{\dot{r}}{r^2}.$$

The function

$$U = -\frac{GMm}{r},$$

is the gravitational potential function, also known as gravitational potential energy. The negative sign follows the convention that work is gained from a loss of potential energy.

### 4.5.5 Work by a spring

Consider a spring that exerts a horizontal force **F** = (−*kx*, 0, 0) that is proportional to its deflection in the *x* direction independent of how a body moves. The work of this spring on a body moving along the space curve **X**(*t*) = (*x*(*t*), *y*(*t*), *z*(*t*)), is calculated using its velocity, **v** = ($v_x$, $v_y$, $v_z$), to obtain

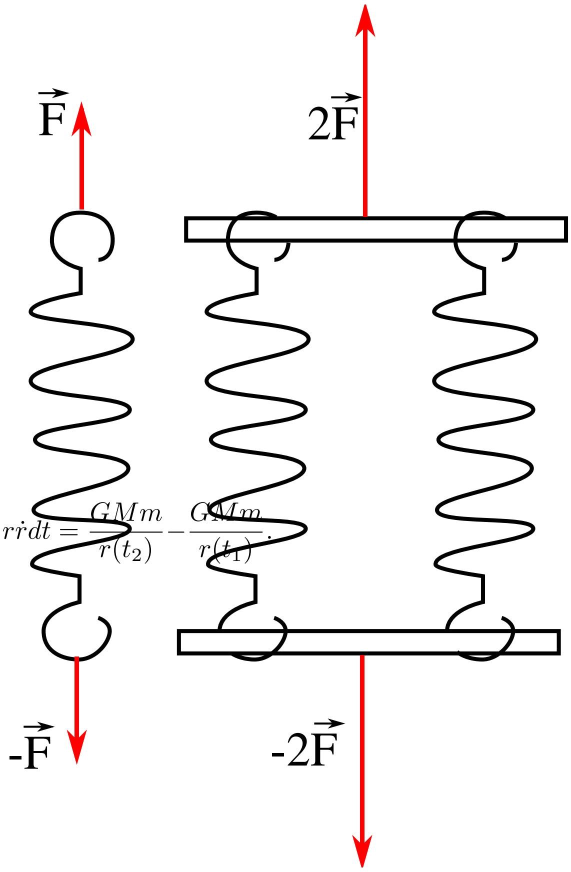
$$W = \int_0^t \mathbf{F}\cdot\mathbf{v}dt = -\int_0^t kxv_x dt = -\frac{1}{2}kx^2.$$

For convenience, consider contact with the spring occurs at *t* = 0, then the integral of the product of the distance *x* and the x-velocity, $xv_x$, is (1/2)$x^2$.

### 4.5.6 Work by a gas

$$W = \int_a^b PdV$$

Where *P* is pressure, *V* is volume, and *a* and *b* are initial and final volumes.




*Forces in springs assembled in parallel*

## 4.6 Work–energy principle

The principle of work and kinetic energy (also known as the **work–energy principle**) states that *the work done by all forces acting on a particle (the work of the resultant force) equals the change in the kinetic energy of the particle.*[8] That is, the work *W* done by the resultant force on a particle equals the change in the particle's kinetic energy $E_k$ ,[6]

$$W = \Delta E_k = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

where $v_1$ and $v_2$ are the speeds of the particle before and after the work is done and *m* is its mass.

The derivation of the *work–energy principle* begins with Newton's second law and the resultant force on a particle which includes forces applied to the particle and constraint forces imposed on its movement. Computation of the scalar product of the forces with the velocity of the particle evaluates the instantaneous power added to the system.[9]

Constraints define the direction of movement of the particle by ensuring there is no component of velocity in the direction of the constraint force. This also means the constraint forces do not add to the instantaneous power. The time integral of this scalar equation yields work from the instantaneous power, and kinetic energy from the scalar product of velocity and acceleration. The fact the work–energy principle eliminates the constraint forces underlies Lagrangian mechanics.[10]

This section focuses on the work–energy principle as it applies to particle dynamics. In more general systems work can change the potential energy of a mechanical device, the heat energy in a thermal system, or the electrical energy in an electrical device. Work transfers energy from one place to another or one form to another.

### 4.6.1 Derivation for a particle moving along a straight line

In the case the resultant force **F** is constant in both magnitude and direction, and parallel to the velocity of the particle, the particle is moving with constant acceleration *a* along a straight line.[11] The relation between the net force and the acceleration is given by the equation $F = ma$ (Newton's second law), and the particle displacement *s* can be expressed by the equation

$$s = \frac{v_2^2 - v_1^2}{2a}$$

which follows from $v_2^2 = v_1^2 + 2as$ (see Equations of motion).

The work of the net force is calculated as the product of its magnitude and the particle displacement. Substituting the above equations, one obtains:

$$W = Fs = mas = ma\left(\frac{v_2^2 - v_1^2}{2a}\right) = \frac{mv_2^2}{2} - \frac{mv_1^2}{2} = \Delta E_k$$

Other derivation:

$$W = Fs = mas = m\left(\frac{v_2^2 - u_1^2}{2s}\right)s$$

$$W = m\left(\frac{0_2^2 - v_1^2}{2s}\right)s$$

$$W = -\frac{1}{2}mv^2$$

Vertical displacement derivation

$W = F \times S = mg \times h$

In the general case of rectilinear motion, when the net force **F** is not constant in magnitude, but is constant in direction, and parallel to the velocity of the particle, the work must be integrated along the path of the particle:

$$W = \int_{t_1}^{t_2} \mathbf{F}\cdot\mathbf{v}dt = \int_{t_1}^{t_2} F\,vdt = \int_{t_1}^{t_2} ma\,vdt = m\int_{t_1}^{t_2} v\frac{dv}{dt}\,dt = m\int_{v_1}^{v_2} v\,dv = \tfrac{1}{2}m(v_2^2 - v_1^2).$$

### 4.6.2 General derivation of the work–energy theorem for a particle

For any net force acting on a particle moving along any curvilinear path, it can be demonstrated that its work equals the change in the kinetic energy of the particle by a simple derivation analogous to the equation above. Some authors call this result *work–energy principle*, but it is more widely known as **the work–energy theorem**:

$$W = \int_{t_1}^{t_2} \mathbf{F}\cdot\mathbf{v}dt = m\int_{t_1}^{t_2} \mathbf{a}\cdot\mathbf{v}dt = \frac{m}{2}\int_{t_1}^{t_2} \frac{dv^2}{dt}\,dt = \frac{m}{2}\int_{v_1^2}^{v_2^2} dv^2 = \frac{mv_2^2}{2}$$

The identity $\mathbf{a}\cdot\mathbf{v} = \frac{1}{2}\frac{dv^2}{dt}$ requires some algebra. From the identity $v^2 = \mathbf{v}\cdot\mathbf{v}$ and definition $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ it follows

$$\frac{dv^2}{dt} = \frac{d(\mathbf{v}\cdot\mathbf{v})}{dt} = \frac{d\mathbf{v}}{dt}\cdot\mathbf{v} + \mathbf{v}\cdot\frac{d\mathbf{v}}{dt} = 2\frac{d\mathbf{v}}{dt}\cdot\mathbf{v} = 2\mathbf{a}\cdot\mathbf{v}$$

The remaining part of the above derivation is just simple calculus, same as in the preceding rectilinear case.

### 4.6.3 Derivation for a particle in constrained movement

In particle dynamics, a formula equating work applied to a system to its change in kinetic energy is obtained as a first integral of Newton's second law of motion. It is useful to notice that the resultant force used in Newton's laws can be separated into forces that are applied to the particle and forces imposed by constraints on the movement of the particle. Remarkably, the work of a constraint force is zero, therefore only the work of the applied forces need be considered in the work–energy principle.

To see this, consider a particle P that follows the trajectory $\mathbf{X}(t)$ with a force **F** acting on it. Isolate the particle from its environment to expose constraint forces **R**, then Newton's Law takes the form

$$\mathbf{F} + \mathbf{R} = m\ddot{\mathbf{X}},$$

where *m* is the mass of the particle.

#### Vector formulation

Note that n dots above a vector indicates its nth time derivative. The scalar product of each side of Newton's law with the velocity vector yields

$$\mathbf{F}\cdot\dot{\mathbf{X}} = m\ddot{\mathbf{X}}\cdot\dot{\mathbf{X}},$$

because the constraint forces are perpendicular to the particle velocity. Integrate this equation along its trajectory from the point $\mathbf{X}(t_1)$ to the point $\mathbf{X}(t_2)$ to obtain

$$\int_{t_1}^{t_2} \mathbf{F}\cdot\dot{\mathbf{X}}dt = m\int_{t_1}^{t_2} \ddot{\mathbf{X}}\cdot\dot{\mathbf{X}}dt.$$

The left side of this equation is the work of the applied force as it acts on the particle along the trajectory from time $t_1$ to time $t_2$. This can also be written as

$$W = \int_{t_1}^{t_2} \mathbf{F}\cdot\dot{\mathbf{X}}dt = \int_{\mathbf{X}(t_1)}^{\mathbf{X}(t_2)} \mathbf{F}\cdot d\mathbf{X}.$$

This integral is computed along the trajectory $\mathbf{X}(t)$ of the particle and is therefore path dependent.

The right side of the first integral of Newton's equations can be simplified using the following identity

$$\frac{1}{2}\frac{d}{dt}(\dot{\mathbf{X}}\cdot\dot{\mathbf{X}}) = \ddot{\mathbf{X}}\cdot\dot{\mathbf{X}},$$

(see product rule for derivation). Now it is integrated explicitly to obtain the change in kinetic energy,

$$\Delta K = m\int_{t_1}^{t_2} \ddot{\mathbf{X}}\cdot\dot{\mathbf{X}}dt = \frac{m}{2}\int_{t_1}^{t_2} \frac{d}{dt}(\dot{\mathbf{X}}\cdot\dot{\mathbf{X}})dt = \frac{m}{2}\dot{\mathbf{X}}\cdot\dot{\mathbf{X}}(t_2) - \frac{m}{2}\dot{\mathbf{X}}\cdot\dot{\mathbf{X}}(t_1) = \frac{1}{2}m\Delta\mathbf{v}^2,$$

where the kinetic energy of the particle is defined by the scalar quantity,

$$K = \frac{m}{2}\dot{\mathbf{X}}\cdot\dot{\mathbf{X}} = \frac{1}{2}m\mathbf{v}^2$$

**Tangential and normal components**

It is useful to resolve the velocity and acceleration vectors into tangential and normal components along the trajectory $\mathbf{X}(t)$, such that

$$\dot{\mathbf{X}} = v\mathbf{T}, \quad \text{and} \quad \ddot{\mathbf{X}} = \dot{v}\mathbf{T} + v^2\kappa\mathbf{N}.$$

where

$$v = |\dot{\mathbf{X}}| = \sqrt{\dot{\mathbf{X}}\cdot\dot{\mathbf{X}}}.$$

Then, the scalar product of velocity with acceleration in Newton's second law takes the form

$$\Delta K = m\int_{t_1}^{t_2} \dot{v}v dt = \frac{m}{2}\int_{t_1}^{t_2} \frac{d}{dt}v^2 dt = \frac{m}{2}v^2(t_2) - \frac{m}{2}v^2(t_1),$$

where the kinetic energy of the particle is defined by the scalar quantity,

$$K = \frac{m}{2}v^2 = \frac{m}{2}\dot{\mathbf{X}}\cdot\dot{\mathbf{X}}.$$

The result is the work–energy principle for particle dynamics,

$$W = \Delta K.$$

This derivation can be generalized to arbitrary rigid body systems.

### 4.6.4 Moving in a straight line (skid to a stop)

Consider the case of a vehicle moving along a straight horizontal trajectory under the action of a driving force and gravity that sum to **F**. The constraint forces between the vehicle and the road define **R**, and we have

$$\mathbf{F} + \mathbf{R} = m\ddot{\mathbf{X}}.$$

For convenience let the trajectory be along the X-axis, so $\mathbf{X} = (d, 0)$ and the velocity is $\mathbf{V} = (v, 0)$, then $\mathbf{R}\cdot\mathbf{V} = 0$, and $\mathbf{F}\cdot\mathbf{V} = F_x v$, where $F_x$ is the component of **F** along the X-axis, so

$$F_x v = m\dot{v}v.$$

Integration of both sides yields

$$\int_{t_1}^{t_2} F_x v dt = \frac{m}{2}v^2(t_2) - \frac{m}{2}v^2(t_1).$$

If $F_x$ is constant along the trajectory, then the integral of velocity is distance, so

$$F_x(d(t_2) - d(t_1)) = \frac{m}{2}v^2(t_2) - \frac{m}{2}v^2(t_1).$$

As an example consider a car skidding to a stop, where $k$ is the coefficient of friction and $W$ is the weight of the car. Then the force along the trajectory is $F_x = -kW$. The velocity $v$ of the car can be determined from the length $s$ of the skid using the work–energy principle,

$$kWs = \frac{W}{2g}v^2, \quad \text{or} \quad v = \sqrt{2ksg}.$$

Notice that this formula uses the fact that the mass of the vehicle is $m = W/g$.


*Lotus type 119B gravity racer at Lotus 60th celebration.*


*Gravity racing championship in Campos Novos, Santa Catarina, Brazil, 8 September 2010.*

### 4.6.5 Coasting down a mountain road (gravity racing)

Consider the case of a vehicle that starts at rest and coasts down a mountain road, the work-energy principle helps compute the minimum distance that the vehicle travels to reach a velocity $V$, of say 60 mph (88 fps). Rolling resistance and air drag will slow the vehicle down so the actual distance will be greater than if these forces are neglected.

Let the trajectory of the vehicle following the road be $\mathbf{X}(t)$ which is a curve in three-dimensional space. The force acting on the vehicle that pushes it down the road is the constant force of gravity $\mathbf{F} = (0, 0, W)$, while the force of the road on the vehicle is the constraint force $\mathbf{R}$. Newton's second law yields,

$$\mathbf{F} + \mathbf{R} = m\ddot{\mathbf{X}}.$$

The scalar product of this equation with the velocity, $\mathbf{V} = (v_x, v_y, v_z)$, yields

$$Wv_z = m\dot{V}V,$$

where $V$ is the magnitude of $\mathbf{V}$. The constraint forces between the vehicle and the road cancel from this equation because $\mathbf{R} \cdot \mathbf{V} = 0$, which means they do no work. Integrate both sides to obtain

$$\int_{t_1}^{t_2} Wv_z dt = \frac{m}{2}V^2(t_2) - \frac{m}{2}V^2(t_1).$$

The weight force $W$ is constant along the trajectory and the integral of the vertical velocity is the vertical distance, therefore,

$$W\Delta z = \frac{m}{2}V^2.$$

Recall that $V(t_1)=0$. Notice that this result does not depend on the shape of the road followed by the vehicle.

In order to determine the distance along the road assume the downgrade is 6%, which is a steep road. This means the altitude decreases 6 feet for every 100 feet traveled—for angles this small the sin and tan functions are approximately equal. Therefore, the distance $s$ in feet down a 6% grade to reach the velocity $V$ is at least

$$s = \frac{\Delta z}{0.06} = 8.3\frac{V^2}{g}, \quad \text{or} \quad s = 8.3\frac{88^2}{32.2} \approx 2000\text{ft}.$$

This formula uses the fact that the weight of the vehicle is $W = mg$.

## 4.7 Work of forces acting on a rigid body

The work of forces acting at various points on a single rigid body can be calculated from the work of a resultant force and torque. To see this, let the forces $\mathbf{F}_1$, $\mathbf{F}_2$ ... $\mathbf{F}_n$ act on the points $\mathbf{X}_1$, $\mathbf{X}_2$ ... $\mathbf{X}n$ in a rigid body.

The trajectories of $\mathbf{X}i$, $i = 1, ..., n$ are defined by the movement of the rigid body. This movement is given by the set of rotations $[A(t)]$ and the trajectory $\mathbf{d}(t)$ of a reference point in the body. Let the coordinates $\mathbf{x}i$ $i = 1, ..., n$ define these points in the moving rigid body's reference frame $M$, so that the trajectories traced in the fixed frame $F$ are given by

$$\mathbf{X}_i(t) = [A(t)]\mathbf{x}_i + \mathbf{d}(t) \quad i = 1, \ldots, n.$$

The velocity of the points $\mathbf{X}i$ along their trajectories are

$$\mathbf{V}_i = \vec{\omega} \times (\mathbf{X}_i - \mathbf{d}) + \dot{\mathbf{d}},$$

where $\boldsymbol{\omega}$ is the angular velocity vector obtained from the skew symmetric matrix

$$[\Omega] = \dot{A}A^{\mathrm{T}},$$

known as the angular velocity matrix.

The small amount of work by the forces over the small displacements $\delta \mathbf{r}i$ can be determined by approximating the displacement by $\delta \mathbf{r} = \mathbf{v}\delta t$ so

$$\delta W = \mathbf{F}_1 \cdot \mathbf{V}_1 \delta t + \mathbf{F}_2 \cdot \mathbf{V}_2 \delta t + \ldots + \mathbf{F}_n \cdot \mathbf{V}_n \delta t$$

or

$$\delta W = \sum_{i=1}^{n} \mathbf{F}_i \cdot (\vec{\omega} \times (\mathbf{X}_i - \mathbf{d}) + \dot{\mathbf{d}})\delta t.$$

This formula can be rewritten to obtain

$$\delta W = (\sum_{i=1}^{n} \mathbf{F}_i) \cdot \dot{\mathbf{d}} \delta t + (\sum_{i=1}^{n} (\mathbf{X}_i - \mathbf{d}) \times \mathbf{F}_i) \cdot \vec{\omega} \delta t = (\mathbf{F} \cdot \dot{\mathbf{d}} + \mathbf{T} \cdot \vec{\omega})\delta t,$$

where **F** and **T** are the resultant force and torque applied at the reference point **d** of the moving frame *M* in the rigid body.

## 4.8 References

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## 4.10 External links

- Work – a chapter from an online textbook
- Work (in negative direction) – a chapter to explain the energy expended LOWERING an OBJECT (a crane lowering a heavy item)
- Work–energy principle

# Chapter 5

# Work (electrical)

For other examples of “work” in physics, see Work (physics).

**Electrical work** is the work done on a charged particle by an electric field. The equation for 'electrical' work is equivalent to that of 'mechanical' work:

$$W = Q\int_a^b \mathbf{E}\cdot d\mathbf{r} = Q\int_a^b \frac{\mathbf{F_E}}{q}\cdot d\mathbf{r} = \int_a^b \mathbf{F_E}\cdot d\mathbf{r}$$

where

*Q* is the charge of the particle, q, the unit charge

*E* is the electric field, which at a location is the force at that location divided by a unit ('test') charge

*FE* is the Coulomb (electric) force

*r* is the displacement

$\cdot$ is the dot product

The electrical work per unit of charge, when moving a negligible test charge between two points, is defined as the voltage between those points.

## 5.1 Overview

### 5.1.1 Qualitative overview

Particles that are free to move, if positively charged, normally tend towards regions of lower voltage (net negative charge), while if negatively charged they tend to shift towards regions of higher voltage (net positive charge).

However, any movement of a positive charge into a region of higher voltage requires external work to be done against the field of the electric force, work equal to that electric field would do in moving that positive charge the same distance in the opposite direction. Similarly, it requires positive external work to transfer a negatively charged particle from a region of higher voltage to a region of lower voltage.

The electric force is a conservative force: work done by a static electric field is independent of the path taken by the charge. There is no change in the voltage (electric potential) around any closed path; when returning to the starting point in a closed path, the net of the external work done is zero. The same holds for electric fields.

This is the basis of Kirchhoff’s voltage law, one of the most fundamental laws governing electrical and electronic circuits, according to which the voltage gains and the drops in any electrical circuit always sum to zero.

### 5.1.2 Mathematical overview

Main article: Work (physics)

Given a charged object in empty space, Q+. To move q+ (with the same charge) *closer* to Q+ (starting from infinity, where the potential energy=0, for convenience), positive work would be performed. Mathematically:

$$-\frac{\partial U}{\partial \mathbf{r}} = \mathbf{F}$$

In this case, U is the potential energy of q+. So, integrating and using Coulomb’s Law for the force:

$$U = -\int_{r_0}^{r} \mathbf{F}\cdot d\mathbf{r} = -\int_{r_0}^{r} \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\mathbf{r^2}}\cdot d\mathbf{r} = \frac{q_1q_2}{4\pi\varepsilon_0}(\frac{1}{r_0}-\frac{1}{r})+c$$

c is usually set to 0 and r(0) to infinity (making the 1/r(0) term=0) Now, use the relationship

$$W = -\Delta U$$

To show that in this case if we start at infinity and move the charge to r,

$$W = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{1}{r}$$

This could have been obtained equally by using the definition of W and integrating F with respect to r, which will *prove* the above relationship.

In the example both charges are positive; this equation is applicable to any charge configuration (as the product of the charges will be either positive or negative according to their (dis)similarity). If one of the charges were to be negative in the earlier example, the work taken to wrench that charge away to infinity would be exactly the same as the work needed in the earlier example to push that charge back to that same position. This is easy to see mathematically, as reversing the boundaries of integration reverses the sign.

#### Uniform electric field

Where the electric field is constant (i.e. *not* a function of displacement, r), the work equation simplifies to:

$$W = Q(\mathbf{E} \cdot \mathbf{r}) = \mathbf{F_E} \cdot \mathbf{r}$$

or 'force times distance' (times the cosine of the angle between them).

## 5.2 Electric power

Main article: Electric power

$$P = \frac{\partial W}{\partial t} = \frac{\partial QV}{\partial t}$$

V is the voltage. Work is defined by:

$$\delta W = \mathbf{F} \cdot \mathbf{v} \delta t,$$

Therefore

$$\frac{\partial W}{\partial t} = \mathbf{F_E} \cdot \mathbf{v}$$

si unit of current ampere=A

# Chapter 6

# Potential energy

In physics, **potential energy** is the energy that an object has due to its position in a force field or that a system has due to the configuration of its parts.[1][2] Common types include the gravitational potential energy of an object that depends on its vertical position and mass, the elastic potential energy of an extended spring, and the electric potential energy of a charge in an electric field. The SI unit for energy is the joule (symbol J).

The term *potential energy* was introduced by the 19th century Scottish engineer and physicist William Rankine,[3][4] although it has links to Greek philosopher Aristotle's concept of potentiality. Potential energy is associated with forces that act on a body in a way that depends only on the body's position in space. These forces can be represented by a vector at every point in space forming what is known as a vector field of forces, or a force field.

If the work of a force field acting on a body that moves from a start to an end position is determined only by these two positions, and does not depend on the trajectory of the body, then there is a function known as *potential energy* that can be evaluated at the two positions to determine this work. Furthermore, the force field is determined by this potential energy and is described as derivable from a potential.

## 6.1 Overview

**Potential energy** is the stored or pent-up energy of an object. Potential energy is often associated with restoring forces such as a spring or the force of gravity. The action of stretching the spring or lifting the mass is performed by an external force that works against the force field of the potential. This work is stored in the force field, which is said to be stored as potential energy. If the external force is removed the force field acts on the body to perform the work as it moves the body back to the initial position, reducing the stretch of the spring or causing a body to fall.

The more formal definition is that potential energy is the energy difference between the energy of an object in a given position and its energy at a reference position.

There are various types of potential energy, each associated with a particular type of force. For example, the work of an elastic force is called elastic potential energy; work of the gravitational force is called gravitational potential energy; work of the Coulomb force is called electric potential energy; work of the strong nuclear force or weak nuclear force acting on the baryon charge is called nuclear potential energy; work of intermolecular forces is called intermolecular potential energy. Chemical potential energy, such as the energy stored in fossil fuels, is the work of the Coulomb force during rearrangement of mutual positions of electrons and nuclei in atoms and molecules. Thermal energy usually has two components: the kinetic energy of random motions of particles and the potential energy of their mutual positions.

Forces derivable from a potential are also called conservative forces. The work done by a conservative force is

$$W = -\Delta U$$

where $\Delta U$ is the change in the potential energy associated with the force. The negative sign provides the convention that work done against a force field increases potential energy, while work done by the force field decreases potential energy. Common notations for potential energy are *U*, *V*, and *Ep*.

## 6.2 Work and potential energy

Potential energy is closely linked with forces. If the work done by a force on a body that moves from *A* to *B* does not depend on the path between these points, then the work of this force measured from *A* assigns a scalar value to every other point in space and defines a scalar potential field. In this case, the force can be defined as the negative of the vector gradient of the potential field.

If the work for an applied force is independent of the path, then the work done by the force is evaluated at the start and end of the trajectory of the point of application. This means that there is a function *U* (**x**), called a "potential," that can be evaluated at the two points **x**A and **x**B to obtain the work over any trajectory between these two points. It is tradition to define this function with a negative sign so that positive work is a reduction in the potential, that is

$$W = \int_C \mathbf{F} \cdot \mathrm{d}\mathbf{x} = U(\mathbf{x}_A) - U(\mathbf{x}_B)$$

where $C$ is the trajectory taken from A to B. Because the work done is independent of the path taken, then this expression is true for any trajectory, $C$, from A to B.

The function $U(\mathbf{x})$ is called the potential energy associated with the applied force. Examples of forces that have potential energies are gravity and spring forces.

### 6.2.1 Derivable from a potential

In this section the relationship between work and potential energy is presented in more detail. The line integral that defines work along curve $C$ takes a special form if the force $\mathbf{F}$ is related to a scalar field $\varphi(\mathbf{x})$ so that

$$\mathbf{F} = \nabla\varphi = \left(\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z}\right).$$

In this case, work along the curve is given by

$$W = \int_C \mathbf{F} \cdot \mathrm{d}\mathbf{x} = \int_C \nabla\varphi \cdot \mathrm{d}\mathbf{x},$$

which can be evaluated using the gradient theorem to obtain

$$W = \varphi(\mathbf{x}_B) - \varphi(\mathbf{x}_A).$$

This shows that when forces are derivable from a scalar field, the work of those forces along a curve $C$ is computed by evaluating the scalar field at the start point $A$ and the end point $B$ of the curve. This means the work integral does not depend on the path between $A$ and $B$ and is said to be independent of the path.

Potential energy $U$=-$\varphi(\mathbf{x})$ is traditionally defined as the negative of this scalar field so that work by the force field decreases potential energy, that is

$$W = U(\mathbf{x}_A) - U(\mathbf{x}_B).$$

In this case, the application of the del operator to the work function yields,

$$\nabla W = -\nabla U = -\left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right) = \mathbf{F},$$

and the force $\mathbf{F}$ is said to be "derivable from a potential."[5] This also necessarily implies that $\mathbf{F}$ must be a conservative vector field. The potential $U$ defines a force $\mathbf{F}$ at every point $\mathbf{x}$ in space, so the set of forces is called a force field.

### 6.2.2 Computing potential energy

Given a force field $\mathbf{F}(\mathbf{x})$, evaluation of the work integral using the gradient theorem can be used to find the scalar function associated with potential energy. This is done by introducing a parameterized curve γ(t)=**r**(t) from γ(a)=A to γ(b)=B, and computing,

$$\int_\gamma \nabla\varphi(\mathbf{r}) \cdot d\mathbf{r} = \int_a^b \nabla\varphi(\mathbf{r}(t)) \cdot \mathbf{r}'(t)dt,$$
$$= \int_a^b \frac{d}{dt}\varphi(\mathbf{r}(t))dt = \varphi(\mathbf{r}(b)) - \varphi(\mathbf{r}(a)) = \varphi\left(\mathbf{x}_B\right) - \varphi\,(\mathbf{x}$$

For the force field $\mathbf{F}$, let $\mathbf{v}$= d$\mathbf{r}$/dt, then the gradient theorem yields,

$$\int_\gamma \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F} \cdot \mathbf{v}dt,$$
$$= -\int_a^b \frac{d}{dt}U(\mathbf{r}(t))dt = U\left(\mathbf{x}_A\right) - U\left(\mathbf{x}_B\right).$$

The power applied to a body by a force field is obtained from the gradient of the work, or potential, in the direction of the velocity $\mathbf{v}$ of the point of application, that is

$$P(t) = -\nabla U \cdot \mathbf{v} = \mathbf{F} \cdot \mathbf{v}.$$

Examples of work that can be computed from potential functions are gravity and spring forces.[6]

## 6.3 Potential energy for near Earth gravity

In classical physics, gravity exerts a constant downward force $\boldsymbol{F}$=(0, 0, $F_z$) on the center of mass of a body moving near the surface of the Earth. The work of gravity on a body moving along a trajectory $\boldsymbol{r}$(t) = ($x$(t), $y$(t), $z$(t)), such as the track of a roller coaster is calculated using its velocity, $\boldsymbol{v}$=($v_x$, $v_y$, $v_z$), to obtain

$$W = \int_{t_1}^{t_2} \boldsymbol{F} \cdot \boldsymbol{v}\mathrm{d}t = \int_{t_1}^{t_2} F_z v_z \mathrm{d}t = F_z \Delta z.$$

where the integral of the vertical component of velocity is the vertical distance. Notice that the work of gravity depends only on the vertical movement of the curve $\boldsymbol{r}$(t).

*A trebuchet uses the gravitational potential energy of the counterweight to throw projectiles over two hundred meters*

The function

$$U(\mathbf{r}) = mg\,h(\mathbf{r}),$$

is called the potential energy of a near earth gravity field.

## 6.4 Potential energy for a linear spring

Main article: Elastic potential energy

A horizontal spring exerts a force $\mathbf{F} = (-kx, 0, 0)$ that is

*Springs are used for storing elastic potential energy*

proportional to its deflection in the $x$ direction. The work of this spring on a body moving along the space curve $\mathbf{s}(t) = (x(t), y(t), z(t))$, is calculated using its velocity, $\mathbf{v} = (v_x, v_y, v_z)$, to obtain

$$W = \int_0^t \mathbf{F}\cdot\mathbf{v}\,dt = -\int_0^t kxv_x\,dt = -\frac{1}{2}kx^2.$$

For convenience, consider contact with the spring occurs at $t = 0$, then the integral of the product of the distance $x$ and the $x$-velocity, $xvx$, is $x^2/2$.

*Archery is one of humankind's oldest applications of elastic potential energy*

The function

$$U(x) = \frac{1}{2}kx^2,$$

is called the potential energy of a linear spring.

Elastic potential energy is the potential energy of an elastic object (for example a bow or a catapult) that is deformed under tension or compression (or stressed in formal terminology). It arises as a consequence of a force that tries to restore the object to its original shape, which is most often the electromagnetic force between the atoms and molecules that constitute the object. If the stretch is released, the energy is transformed into kinetic energy.

## 6.5 Potential energy for gravitational forces between two bodies

Gravitational potential energy between two bodies in space is obtained from the force exerted by a mass $M$ on another mass $m$ is given by

$$\mathbf{F} = -\frac{GMm}{r^3}\mathbf{r},$$

where $\mathbf{r}$ is the position vector from $M$ to $m$.

This can also be expressed as

$$\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}},$$

where $\hat{\mathbf{r}}$ is a vector of length 1 pointing from $M$ to $m$.

Let the mass $m$ move at the velocity $\mathbf{v}$ then the work of gravity on this mass as it moves from position $\mathbf{r}(t_1)$ to $\mathbf{r}(t_2)$ is given by

$$W = -\int_{\mathbf{r}(t_1)}^{\mathbf{r}(t_2)} \frac{GMm}{r^3}\mathbf{r}\cdot d\mathbf{r} = -\int_{t_1}^{t_2} \frac{GMm}{r^3}\mathbf{r}\cdot \mathbf{v}dt.$$

Notice that the position and velocity of the mass $m$ are given by

$$\mathbf{r} = r\mathbf{e}_r, \qquad \mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_t,$$

where **e***r* and **e***t* are the radial and tangential unit vectors directed relative to the vector from $M$ to $m$. Use this to simplify the formula for work of gravity to,

$$W = -\int_{t_1}^{t_2} \frac{GmM}{r^3}(r\mathbf{e}_r)\cdot(\dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_t)\mathrm{d}t = -\int_{t_1}^{t_2} \frac{GmM}{r^3} r\dot{r}\mathrm{d}t = \frac{GMm}{r(t_2)} - \frac{GMm}{r(t_1)}.$$

This calculation uses the fact that

$$\frac{\mathrm{d}}{\mathrm{d}t}r^{-1} = -r^{-2}\dot{r} = -\frac{\dot{r}}{r^2}.$$

The function

$$U = -\frac{GMm}{r},$$

is the gravitational potential function, also known as gravitational potential energy. The negative sign follows the convention that work is gained from a loss of potential energy.

## 6.6 Potential energy for electrostatic forces between two bodies

The electrostatic force exerted by a charge $Q$ on another charge $q$ is given by

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^3}\mathbf{r},$$

where $\mathbf{r}$ is the position vector from $Q$ to $q$ and $\varepsilon_0$ is the vacuum permittivity. This may also be written using Coulomb's constant $k_e = 1/4\pi\varepsilon_0$.

The work $W$ required to move $q$ from $A$ to any point $B$ in the electrostatic force field is given by the potential function

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}.$$

## 6.7 Reference level

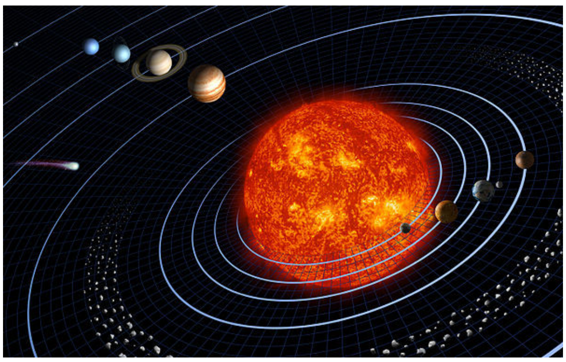
The potential energy is a function of the state a system is in, and is defined relative to that for a particular state. This reference state is not always a real state, it may also be a limit, such as with the distances between all bodies tending to infinity, provided that the energy involved in tending to that limit is finite, such as in the case of inverse-square law forces. Any arbitrary reference state could be used, therefore it can be chosen based on convenience.

Typically the potential energy of a system depends on the *relative* positions of its components only, so the reference state can also be expressed in terms of relative positions.

## 6.8 Gravitational potential energy

Main articles: Gravitational potential, Gravitational energy and Gravity field

Gravitational energy is the potential energy associated with gravitational force, as work is required to elevate objects against Earth's gravity. The potential energy due to elevated positions is called gravitational potential energy, and is evidenced by water in an elevated reservoir or kept behind a dam. If an object falls from one point to another point inside a gravitational field, the force of gravity will do positive work on the object, and the gravitational potential energy will decrease by the same amount.



*Gravitational force keeps the planets in orbit around the Sun*

Consider a book placed on top of a table. As the book is raised from the floor, to the table, some external force works against the gravitational force. If the book falls back to the floor, the "falling" energy the book receives is provided by the gravitational force. Thus, if the book falls off the table, this potential energy goes to accelerate the mass of the book and is converted into kinetic energy. When the book hits the floor this kinetic energy is converted into heat, deformation and sound by the impact.

The factors that affect an object's gravitational potential energy are its height relative to some reference point, its mass, and the strength of the gravitational field it is in. Thus, a book lying on a table has less gravitational potential energy than the same book on top of a taller cupboard, and less gravitational potential energy than a heavier book lying on the same table. An object at a certain height above the Moon's surface has less gravitational potential energy than at the same height above the Earth's surface because the Moon's gravity is weaker. Note that "height" in the common sense of the term cannot be used for gravitational potential energy calculations when gravity is not assumed to be a constant. The following sections provide more detail.

### 6.8.1 Local approximation

The strength of a gravitational field varies with location. However, when the change of distance is small in relation to the distances from the center of the source of the gravitational field, this variation in field strength is negligible and we can assume that the force of gravity on a particular object is constant. Near the surface of the Earth, for example, we assume that the acceleration due to gravity is a constant $g = 9.8$ m/s$^2$ ("standard gravity"). In this case, a simple expression for gravitational potential energy can be derived using the $W = Fd$ equation for work, and the equation

$$W_F = -\Delta U_F.$$

The amount of gravitational potential energy possessed by an elevated object is equal to the work done against gravity in lifting it. The work done equals the force required to move it upward multiplied with the vertical distance it is moved (remember $W = Fd$). The upward force required while moving at a constant velocity is equal to the weight, $mg$, of an object, so the work done in lifting it through a height $h$ is the product $mgh$. Thus, when accounting only for mass, gravity, and altitude, the equation is:[7]

$$U = mgh$$

where $U$ is the potential energy of the object relative to its being on the Earth's surface, $m$ is the mass of the object, $g$ is the acceleration due to gravity, and $h$ is the altitude of the object.[8] If $m$ is expressed in kilograms, $g$ in m/s$^2$ and $h$ in metres then $U$ will be calculated in joules.

Hence, the potential difference is

$$\Delta U = mg\Delta h.$$

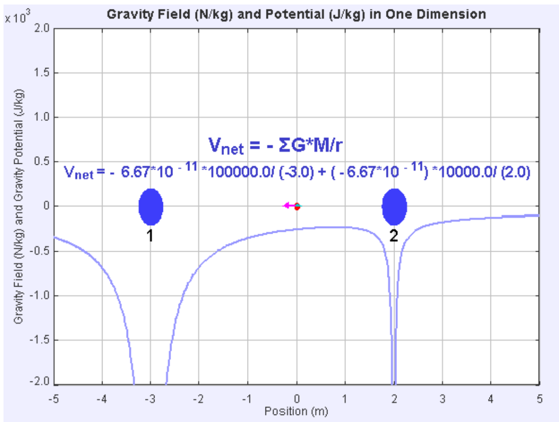
### 6.8.2 General formula

However, over large variations in distance, the approximation that $g$ is constant is no longer valid, and we have to use calculus and the general mathematical definition of work to determine gravitational potential energy. For the computation of the potential energy we can integrate the gravitational force, whose magnitude is given by Newton's law of gravitation, with respect to the distance $r$ between the two bodies. Using that definition, the gravitational potential energy of a system of masses $m_1$ and $M_2$ at a distance $r$ using gravitational constant $G$ is

$$U = -G\frac{m_1 M_2}{r} + K$$

where $K$ is an arbitrary constant dependent on the choice of datum from which potential is measured. Choosing the convention that $K$=0 (i.e. in relation to a point at infinity) makes calculations simpler, albeit at the cost of making $U$ negative; for why this is physically reasonable, see below.

Given this formula for $U$, the total potential energy of a system of $n$ bodies is found by summing, for all $\frac{n(n-1)}{2}$ pairs of two bodies, the potential energy of the system of those two bodies.




*Gravitational potential summation* $U = -m(G\frac{M_1}{r_1} + G\frac{M_2}{r_2})$

Considering the system of bodies as the combined set of small particles the bodies consist of, and applying the previous on the particle level we get the negative gravitational binding energy. This potential energy is more strongly negative than the total potential energy of the system of bodies as such since it also includes the negative gravitational binding energy of each body. The potential energy of the system of bodies as such is the negative of the energy needed to separate the bodies from each other to infinity, while the gravitational binding energy is the energy needed to separate all particles from each other to infinity.

$$U = -m\left(G\frac{M_1}{r_1} + G\frac{M_2}{r_2}\right)$$

therefore,

$$U = -m\sum G\frac{M}{r}$$

### 6.8.3 Why choose a convention where gravitational energy is negative?

As with all potential energies, only differences in gravitational potential energy matter for most physical purposes, and the choice of zero point is arbitrary. Given that there is no reasonable criterion for preferring one particular finite $r$ over another, there seem to be only two reasonable choices for the distance at which $U$ becomes zero: $r = 0$ and $r = \infty$ . The choice of $U = 0$ at infinity may seem peculiar, and the consequence that gravitational energy is always negative may seem counterintuitive, but this choice allows gravitational potential energy values to be finite, albeit negative.

The singularity at $r = 0$ in the formula for gravitational potential energy means that the only other apparently reasonable alternative choice of convention, with $U = 0$ for $r = 0$ , would result in potential energy being positive, but infinitely large for all nonzero values of $r$, and would make calculations involving sums or differences of potential energies beyond what is possible with the real number system. Since physicists abhor infinities in their calculations, and $r$ is always non-zero in practice, the choice of $U = 0$ at infinity is by far the more preferable choice, even if the idea of negative energy in a gravity well appears to be peculiar at first.

The negative value for gravitational energy also has deeper implications that make it seem more reasonable in cosmological calculations where the total energy of the universe can meaningfully be considered; see inflation theory for more on this.

### 6.8.4 Uses

Gravitational potential energy has a number of practical uses, notably the generation of pumped-storage hydroelectricity. For example, in Dinorwig, Wales, there are two lakes, one at a higher elevation than the other. At times when surplus electricity is not required (and so is comparatively cheap), water is pumped up to the higher lake, thus converting the electrical energy (running the pump) to gravitational potential energy. At times of peak demand for electricity, the water flows back down through electrical generator turbines, converting the potential energy into kinetic energy and then back into electricity. The process is not completely efficient and some of the original energy from the surplus electricity is in fact lost to friction.[9][10][11][12][13]

Gravitational potential energy is also used to power clocks in which falling weights operate the mechanism.

It's also used by counterweights for lifting up an elevator, crane, or sash window.

Roller coasters are an entertaining way to utilize potential energy - chains are used to move a car up an incline (building up gravitational potential energy), to then have that energy converted into kinetic energy as it falls.

Another practical use is utilizing gravitational potential energy to descend (perhaps coast) downhill in transportation such as the descent of an automobile, truck, railroad train, bicycle, airplane, or fluid in a pipeline. In some cases the kinetic energy obtained from potential energy of descent may be used to start ascending the next grade such as what happens when a road is undulating and has frequent dips. The commercialization of stored energy (in the form of rail cars raised to higher elevations) that is then converted to electrical energy when needed by an electrical grid, is being undertaken in the United States in a system called Advanced Rail Energy Storage (ARES).[14][15][16]

*Further information: Gravitational potential energy storage*

## 6.9 Chemical potential energy

Main article: Chemical energy

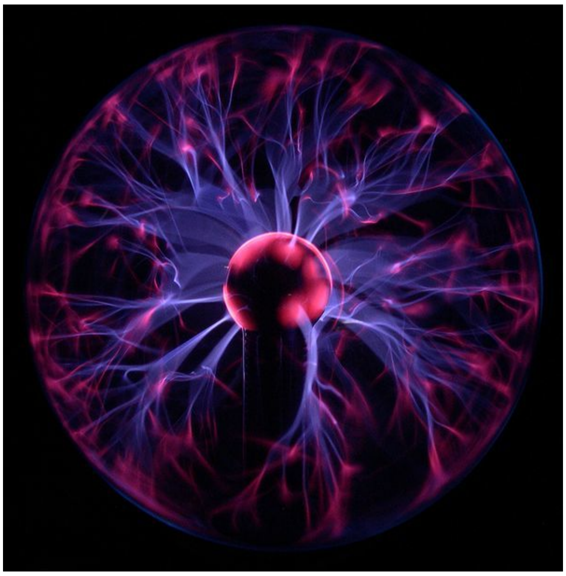
Chemical potential energy is a form of potential energy related to the structural arrangement of atoms or molecules. This arrangement may be the result of chemical bonds within a molecule or otherwise. Chemical energy of a chemical substance can be transformed to other forms of energy by a chemical reaction. As an example, when a fuel is burned the chemical energy is converted to heat, same is the case with digestion of food metabolized in a biological organism. Green plants transform solar energy to chemical energy through the process known as photosynthesis, and electrical energy can be converted to chemical energy through electrochemical reactions.

The similar term chemical potential is used to indicate the potential of a substance to undergo a change of configuration, be it in the form of a chemical reaction, spatial transport, particle exchange with a reservoir, etc.

## 6.10 Electric potential energy

Main article: Electric potential energy

An object can have potential energy by virtue of its electric charge and several forces related to their presence. There are two main types of this kind of potential energy: electrostatic potential energy, electrodynamic potential energy (also sometimes called magnetic potential energy).



*Plasma formed inside a gas filled sphere*

### 6.10.1 Electrostatic potential energy

Electrostatic potential energy between two bodies in space is obtained from the force exerted by a charge $Q$ on another charge $q$ which is given by

$$\mathbf{F} = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^3}\mathbf{r},$$

where $\mathbf{r}$ is the position vector from $Q$ to $q$ and $\varepsilon_0$ is the vacuum permittivity. This may also be written using Coulomb's constant $k_e = 1/4\pi\varepsilon_0$.

If the electric charge of an object can be assumed to be at rest, then it has potential energy due to its position relative to other charged objects. The electrostatic potential energy is the energy of an electrically charged particle (at rest) in an electric field. It is defined as the work that must be done to move it from an infinite distance away to its present location, adjusted for non-electrical forces on the object. This energy will generally be non-zero if there is another electrically charged object nearby.

The work $W$ required to move $q$ from $A$ to any point $B$ in the electrostatic force field is given by the potential function

$$U(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}.$$

A related quantity called *electric potential* (commonly denoted with a $V$ for voltage) is equal to the electric potential energy per unit charge.

### 6.10.2 Magnetic potential energy

The energy of a magnetic moment $\mathbf{m}$ in an externally produced magnetic B-field $\mathbf{B}$ has potential energy[17]

$$U = -\mathbf{m} \cdot \mathbf{B}.$$

The magnetization $\mathbf{M}$ in a field is

$$U = -\frac{1}{2}\int \mathbf{M} \cdot \mathbf{B} dV,$$

where the integral can be over all space or, equivalently, where $\mathbf{M}$ is nonzero.[18] Magnetic potential energy is the form of energy related not only to the distance between magnetic materials, but also to the orientation, or alignment, of those materials within the field. For example, the needle of a compass has the lowest magnetic potential energy when it is aligned with the north and south poles of the Earth's magnetic field. If the needle is moved by an outside force, torque is exerted on the magnetic dipole of the needle by the Earth's magnetic field, causing it to move back into alignment. The magnetic potential energy of the needle is highest when its field is in the same direction as the Earth's magnetic field. Two magnets will have potential energy in relation to each other and the distance between them, but this also depends on their orientation. If the opposite poles are held apart, the potential energy will be the highest when they are near the edge of their attraction, and the lowest when they pull together. Conversely, like poles will have the highest potential energy when forced together, and the lowest when they spring apart.[19][20]

## 6.11 Nuclear potential energy

Nuclear potential energy is the potential energy of the particles inside an atomic nucleus. The nuclear particles are bound together by the strong nuclear force. Weak nuclear forces provide the potential energy for certain kinds of radioactive decay, such as beta decay.

Nuclear particles like protons and neutrons are not destroyed in fission and fusion processes, but collections of them have less mass than if they were individually free, and this mass difference is liberated as heat and radiation in nuclear reactions (the heat and radiation have the missing mass, but it often escapes from the system, where it is not measured). The energy from the Sun is an example of this form of energy conversion. In the Sun, the process of hydrogen fusion converts about 4 million tonnes of solar matter per second into electromagnetic energy, which is radiated into space.

## 6.12 Forces, potential and potential energy

Potential energy is closely linked with forces. If the work done by a force on a body that moves from *A* to *B* does not depend on the path between these points, then the work of this force measured from *A* assigns a scalar value to every other point in space and defines a scalar potential field. In this case, the force can be defined as the negative of the vector gradient of the potential field.

For example, gravity is a conservative force. The associated potential is the gravitational potential, often denoted by $\phi$ or $V$ , corresponding to the energy per unit mass as a function of position. The gravitational potential energy of two particles of mass *M* and *m* separated by a distance *r* is

$$U = -\frac{GMm}{r},$$

The gravitational potential (specific energy) of the two bodies is

$$\phi = -\left(\frac{GM}{r} + \frac{Gm}{r}\right) = -\frac{G(M+m)}{r} = -\frac{GMm}{\mu r} = \frac{U}{\mu}.$$

where $\mu$ is the reduced mass.

The work done against gravity by moving an infinitesimal mass from point A with $U = a$ to point B with $U = b$ is $(b - a)$ and the work done going back the other way is $(a - b)$ so that the total work done in moving from A to B and returning to A is

$$U_{A\to B\to A} = (b - a) + (a - b) = 0.$$

If the potential is redefined at A to be $a + c$ and the potential at B to be $b + c$ , where $c$ is a constant (i.e. $c$ can be any number, positive or negative, but it must be the same at A as it is at B) then the work done going from A to B is

$$U_{A\to B} = (b + c) - (a + c) = b - a$$

as before.

In practical terms, this means that one can set the zero of $U$ and $\phi$ anywhere one likes. One may set it to be zero at the surface of the Earth, or may find it more convenient to set zero at infinity (as in the expressions given earlier in this section).

A conservative force can be expressed in the language of differential geometry as a closed form. As Euclidean space is contractible, its de Rham cohomology vanishes, so every closed form is also an exact form, and can be expressed as the gradient of a scalar field. This gives a mathematical justification of the fact that all conservative forces are gradients of a potential field.

## 6.13 Notes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## 6.15 External links

- What is potential energy?

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