Algorithms – Insertion Sort (1C)

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Insertion Sort Algorithm

```
procedure insertion sort($a_1$, … , $a_n$ : real numbers with n ≥ 2)
for j := 2 to n
        i := 1
        while $a_j > a_i$
                i := i + 1
        m := $a_j$
        for k := 0 to j– i–1
                $a_{j-k} = a_{j-k-1}$
        $a_i$ := m
{$a_1$, …, $a_n$ is in increasing order}
```

Nested loop k – constraints

```
for k := 0 to j– i–1
        a_{j-k} = a_{j-k-1}
```

$j-i-1 \geq 0$ $\quad$ $j \geq i+1$ $\quad$ $i \leq j-1$ $\quad$ $i < j$

$$a_{j-k} = a_{j-k-1}$$

$(k=0)$	$a_{j-0} = a_{j-0-1}$	➡	$a_j = a_{j-1}$
$(k=1)$	$a_{j-1} = a_{j-1-1}$	➡	$a_{j-1} = a_{j-2}$
$(k=2)$	$a_{j-2} = a_{j-2-1}$	➡	$a_{j-2} = a_{j-3}$
	$\vdots = \vdots$		$\vdots = \vdots$
$(k=j-i-1)$	$a_{j-(j-i-1)} = a_{j-(j-i-1)-1}$	➡	$a_{i+1} = a_i$

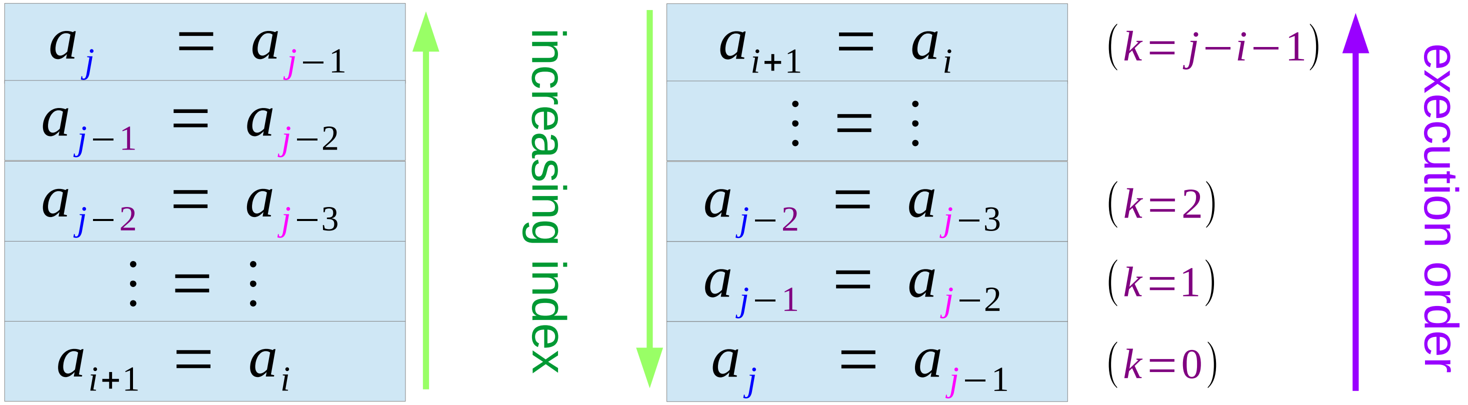
↑ increasing index

Nested loop k – rearranging for understanding

```
for $k$ := 0 to $j – i–1$
        $a_{j-k} = a_{j-k-1}$
```

$j-i-1 \geq 0$ $j \geq i+1$ $i \leq j-1$ $i < j$

$a_{j-k} = a_{j-k-1}$



$a_j = a_{j-1}$

$a_{j-1} = a_{j-2}$

$a_{j-2} = a_{j-3}$

$\vdots = \vdots$

$a_{i+1} = a_i$

increasing index

$a_{i+1} = a_i$ $(k=j-i-1)$

$\vdots = \vdots$

$a_{j-2} = a_{j-3}$ $(k=2)$

$a_{j-1} = a_{j-2}$ $(k=1)$

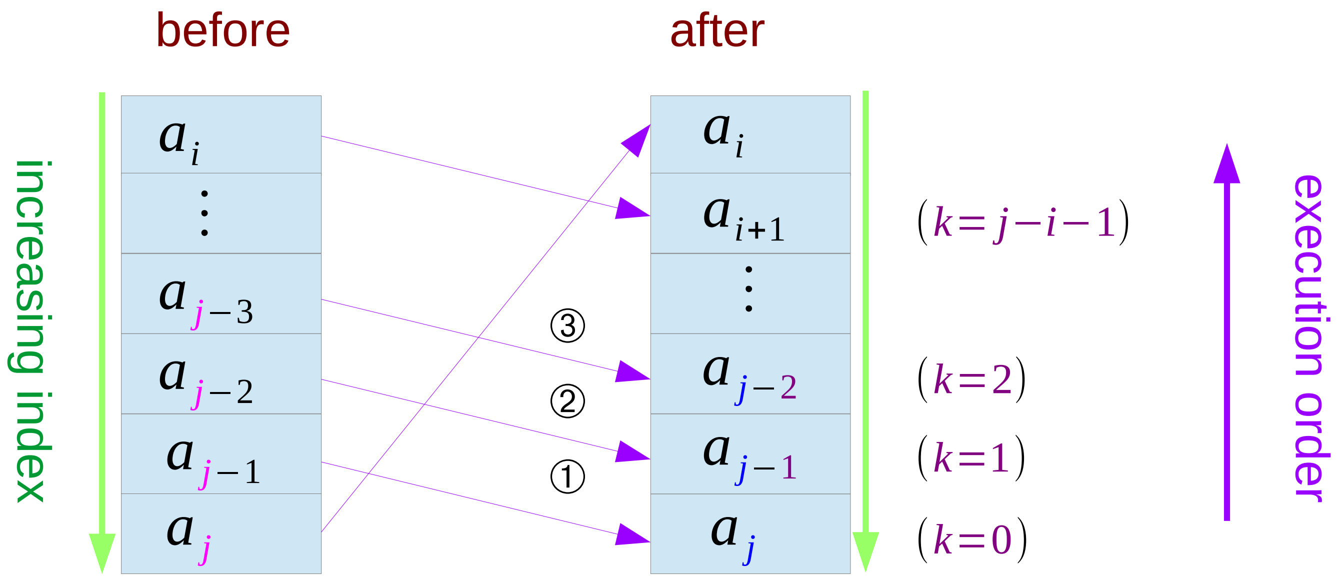
$a_j = a_{j-1}$ $(k=0)$

execution order

Nested loop k – data movement

```
m := $a_j$
for k := 0 to j– i–1
        $a_{j-k}$ = $a_{j-k-1}$
$a_i$ := m
```

$i < j$



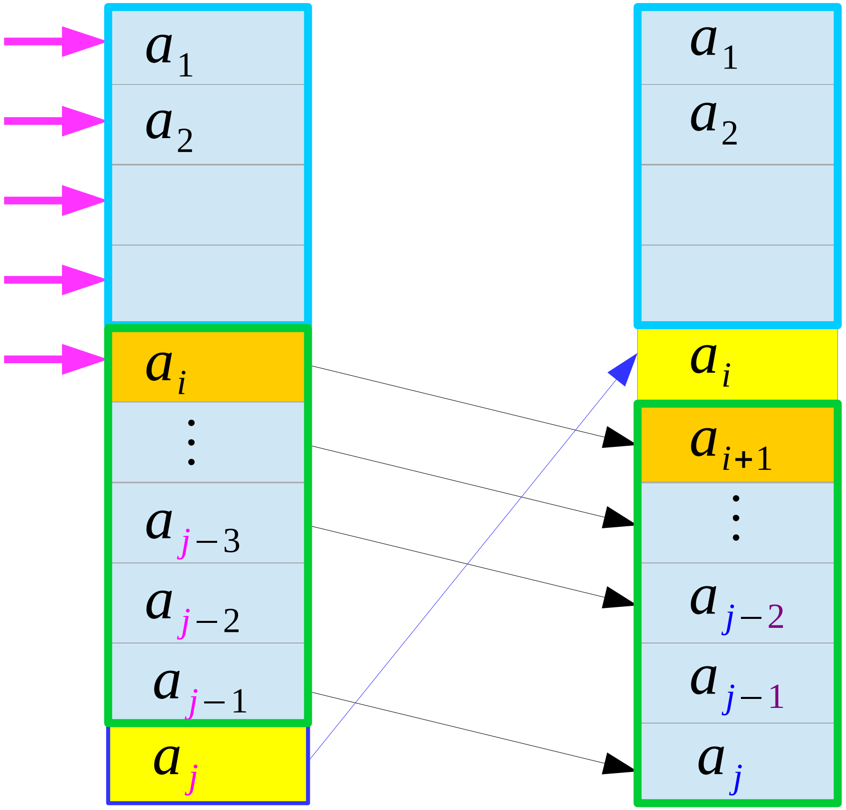
Nested loop i – finding out of order a_i

```
i := 1
while $a_j > a_i$
        i := i + 1
```

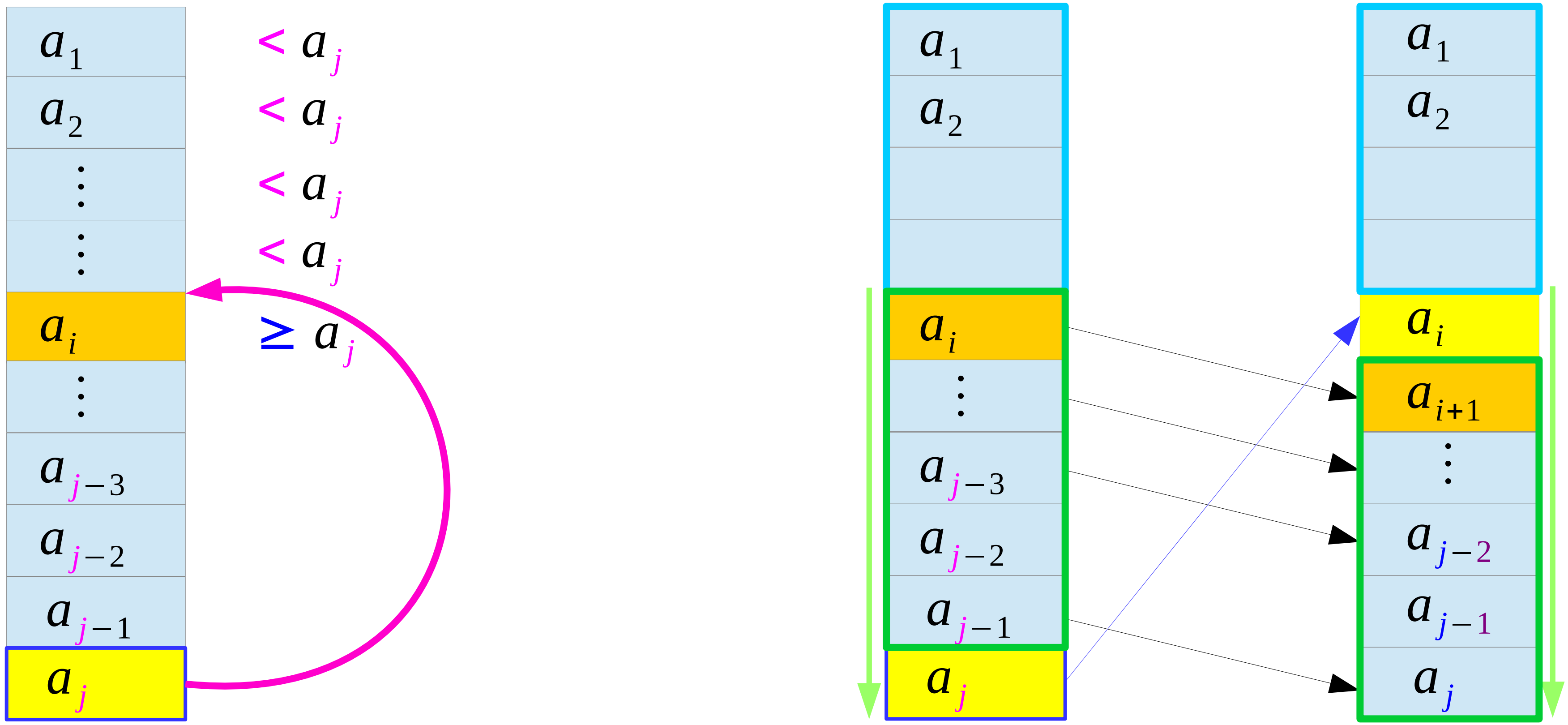
If $a_i < a_j$ increment I

If $a_i >= a_j$ break the loop

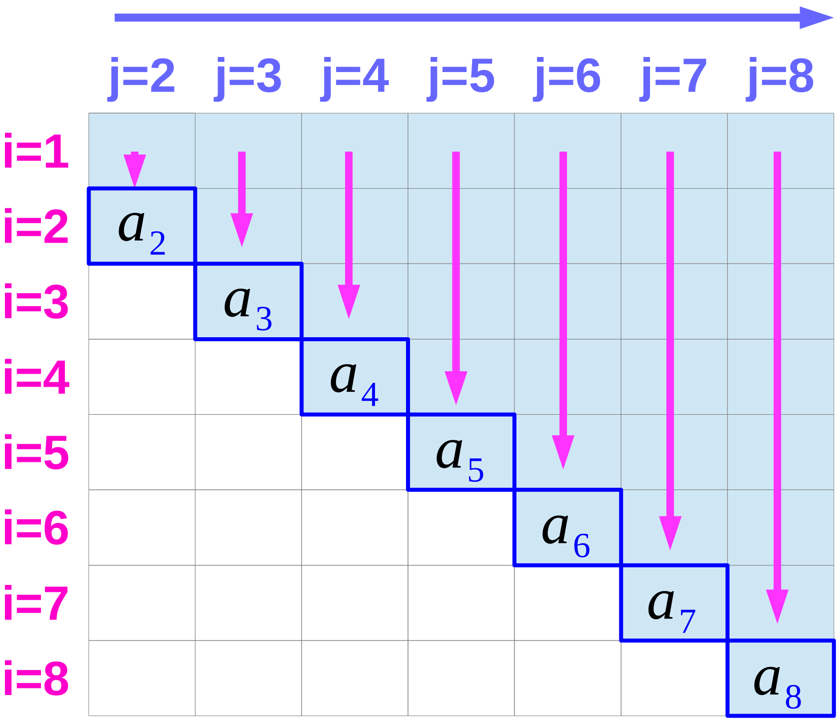
a_i **is the 1st one that is greater than** a_j



Nested loop i – inserting **a_i** at the correct position

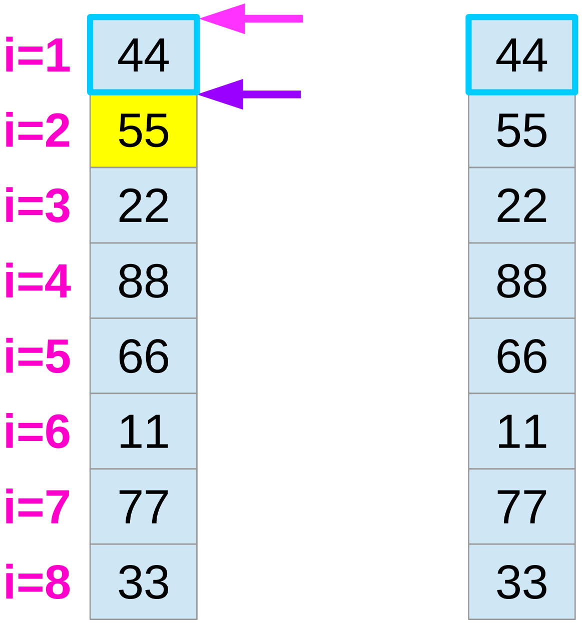


Nested loop iterations

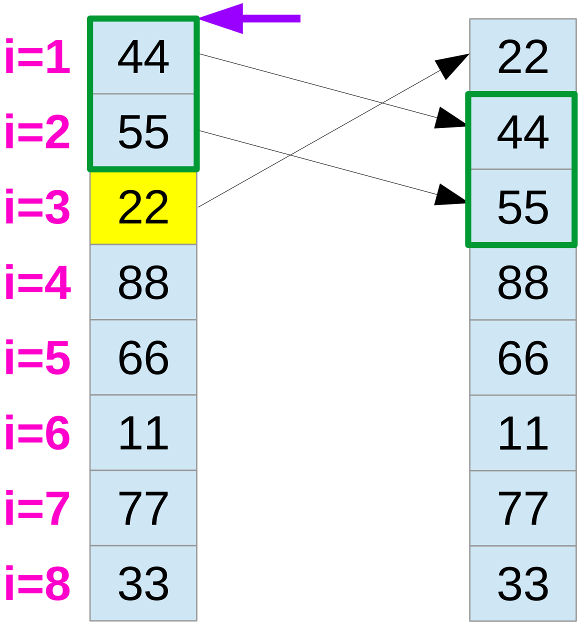


```
for j := 2 to n
    i := 1
    while $a_j > a_i$
        i := i + 1
    m := $a_j$
    for k := 0 to j– i–1
        $a_{j-k} = a_{j-k-1}$
    $a_i$ := m
```

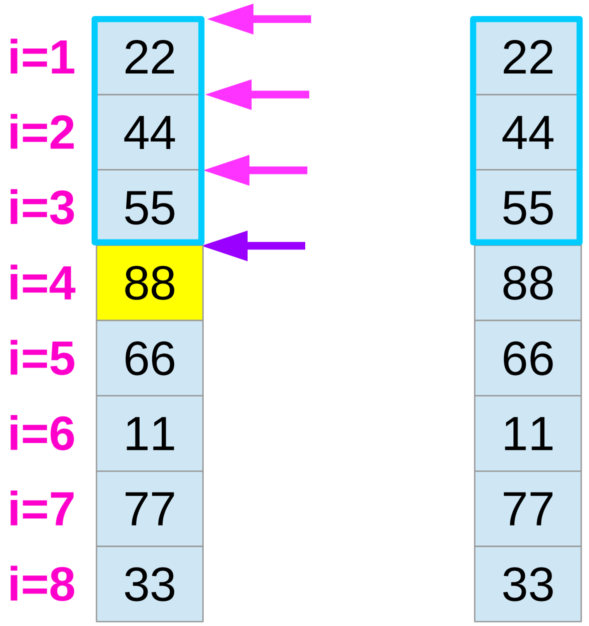
Step j=2



Step j=3

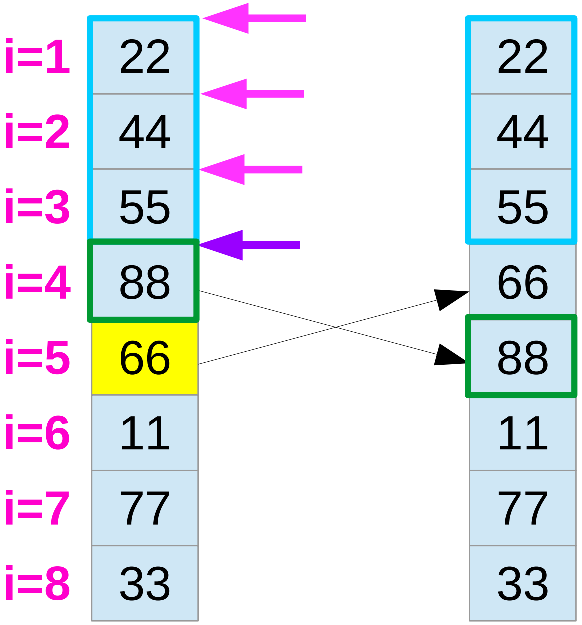


Step j=4

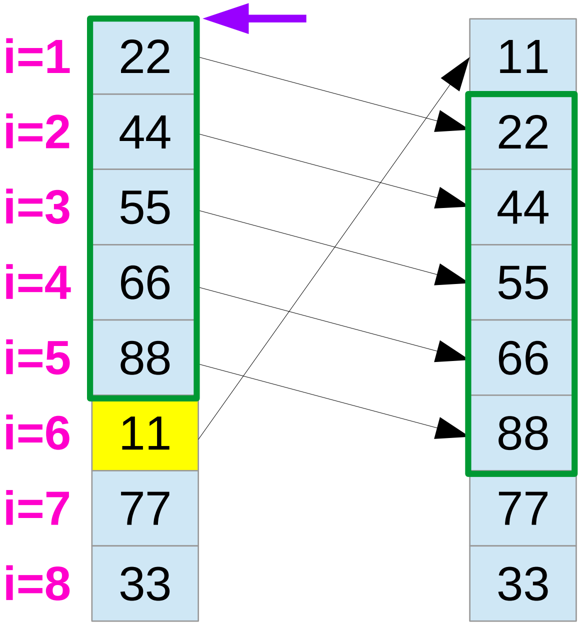


i=1	22	22
i=2	44	44
i=3	55	55
i=4	88	88
i=5	66	66
i=6	11	11
i=7	77	77
i=8	33	33

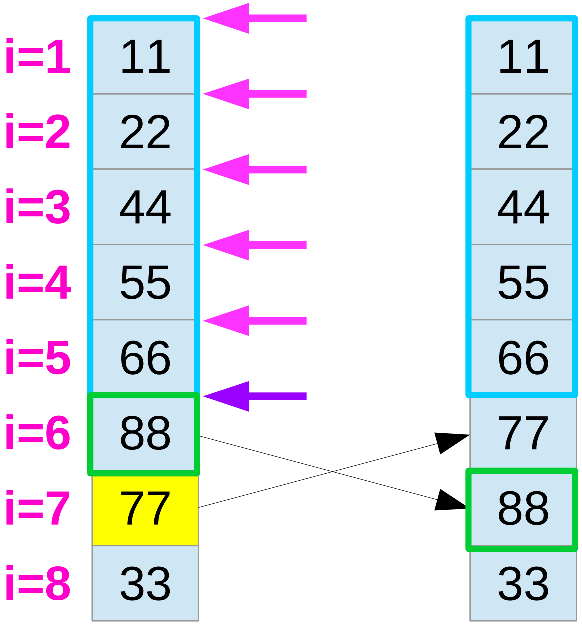
Step j=5



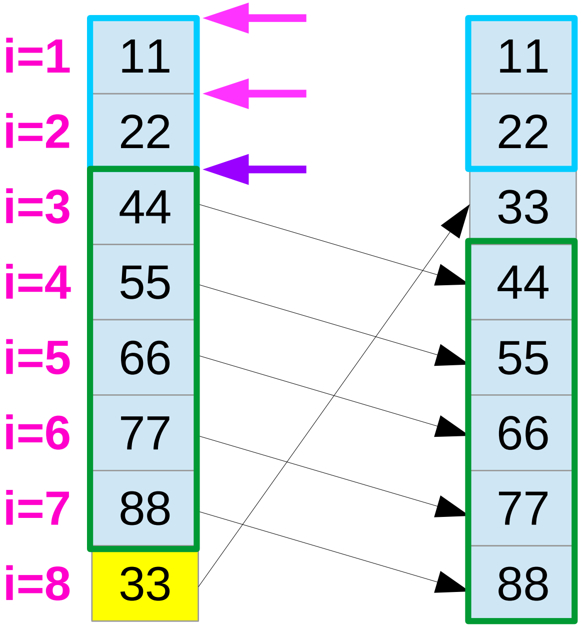
Step j=6



Step j=7



Step j=8



Nested loop iterations

References

[1] http://en.wikipedia.org/
[2]