Partial Oder Relations (5A)

Young Won Lim
4/14/18

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Partial Order Relation

A (non-strict) **partial order** is a binary relation **≤** over a set P satisfying particular axioms.
When **a ≤ b**, we say that **a** is related to **b**.
(This does not imply that **b** is also related to **a**, because the relation need not be symmetric.)

That is, for all **a**, **b**, and **c** in P, it must satisfy:

a ≤ a (reflexivity)
if **a ≤ b** and **b ≤ a**, then **a = b** (antisymmetry)
if **a ≤ b** and **b ≤ c**, then **a ≤ c** (transitivity)

https://en.wikipedia.org/wiki/Hasse_diagram

Partial Order Relation

The axioms for a non-strict partial order state that the relation **≤** is

reflexive: every element is related to itself.

antisymmetric: two distinct elements cannot be related in both directions

transitive: if a first element is related to a second element,
and, in turn, that element is related to a third element,
then the first element is related to the third element

https://en.wikipedia.org/wiki/Hasse_diagram

Relation Examples (1)

$$x \geq y$$

	1	2	3	4	5
1	(1,1)				
2	(2,1)	(2,2)			
3	(3,1)	(3,2)	(3,3)		
4	(4,1)	(4,2)	(4,3)	(4,4)	
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)

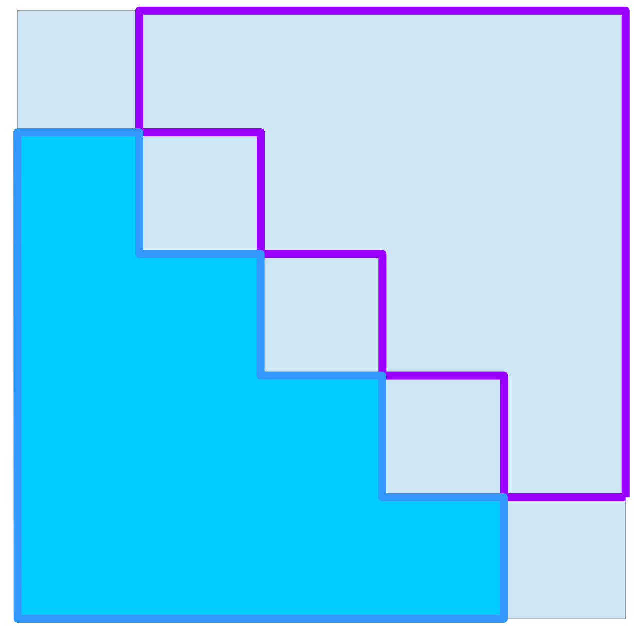
Reflexive Relation &
Anti-Symmetric Relation &
Transitive Relation

Partial Order Relation

Anti-symmetric Relation

$$x \geq y$$

	1	2	3	4	5
1	(1,1)				
2	(2,1)	(2,2)			
3	(3,1)	(3,2)	(3,3)		
4	(4,1)	(4,2)	(4,3)	(4,4)	
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)



Transitive Relation

```
(%i2) A:matrix(
       [0,0,0,0,0],
       [1,0,0,0,0],
       [1,1,0,0,0],
       [1,1,1,0,0],
       [1,1,1,1,0]
      );
```

$$(\%o2)\quad \begin{bmatrix} 0&0&0&0&0\\ 1&0&0&0&0\\ 1&1&0&0&0\\ 1&1&1&0&0\\ 1&1&1&1&0 \end{bmatrix}$$

```
(%i4) A2 : A.A;
```

$$(\%o4)\quad \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 1&0&0&0&0\\ 2&1&0&0&0\\ 3&2&1&0&0 \end{bmatrix}$$

```
(%i5) A3 : A.A.A;
```

$$(\%o5)\quad \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 1&0&0&0&0\\ 3&1&0&0&0 \end{bmatrix}$$

```
(%i6) A4: A.A.A.A;
```

$$(\%o6)\quad \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 1&0&0&0&0 \end{bmatrix}$$

```
(%i7) A5: A.A.A.A.A;
```

$$(\%o7)\quad \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix}$$

```
(%i8) A6: A.A.A.A.A.A;
```

$$(\%o8)\quad \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix}$$

```
(%i9) A7 : A6.A;
```

$$(\%o9)\quad \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix}$$

```
(%i10) A8 : A7.A;
```

$$(\%o10)\quad \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix}$$

```
(%i11) A+A2+A3+A4+A5;
```

$$(\%o11)\quad \begin{bmatrix} 0&0&0&0&0\\ 1&0&0&0&0\\ 2&1&0&0&0\\ 4&2&1&0&0\\ 8&4&2&1&0 \end{bmatrix}$$

Relation Examples (1)

$x > y$

~~Reflexive Relation~~ &
Anti-Symmetric Relation &
Transitive Relation

Not Partial Order Relation

	1	2	3	4	5
1					
2	(2,1)				
3	(3,1)	(3,2)			
4	(4,1)	(4,2)	(4,3)		
5	(5,1)	(5,2)	(5,3)	(5,4)	

https://en.wikipedia.org/wiki/Cartesian_product

Equivalence Relation

Partial Order Relation ⟺ Reflexive Relation & Anti-Symmetric Relation & Transitive Relation

References

[1] http://en.wikipedia.org/
[2]