Set Operations (1A)

Young Won Lim
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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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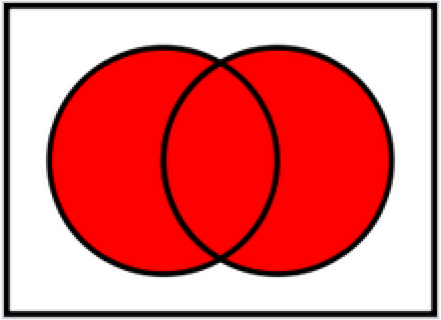
Unions

Two sets can be "added" together. The *union* of *A* and *B*, denoted by *A* ∪ *B*, is the set of all things that are members of either *A* or *B*.

Examples:

- {1, 2} ∪ {1, 2} = {1, 2}.
- {1, 2} ∪ {2, 3} = {1, 2, 3}.
- {1, 2, 3} ∪ {3, 4, 5} = {1, 2, 3, 4, 5}



The **union** of *A* and *B*, denoted *A* ∪ *B*

https://en.wikipedia.org/wiki/Set_(mathematics)

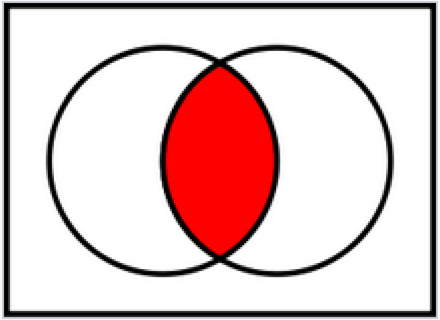
Properties of Unions

- $A \cup B = B \cup A$.
- $A \cup (B \cup C) = (A \cup B) \cup C$.
- $A \subseteq (A \cup B)$.
- $A \cup A = A$.
- $A \cup U = U$.
- $A \cup \emptyset = A$.
- $A \subseteq B$ if and only if $A \cup B = B$.

https://en.wikipedia.org/wiki/Set_(mathematics)

Intersections

A new set can also be constructed by determining which members two sets have "in common". The *intersection* of A and B, denoted by $A \cap B$, is the set of all things that are members of both A and B. If $A \cap B = \varnothing$, then A and B are said to be *disjoint*.



The **intersection** of A and B, denoted $A \cap B$.

https://en.wikipedia.org/wiki/Set_(mathematics)

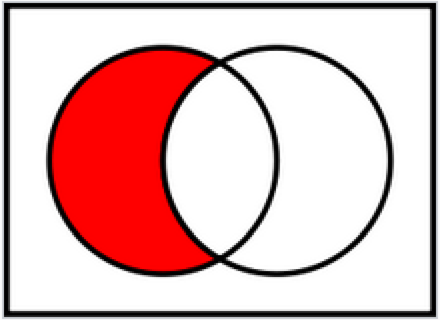
Properties of Intersections

- $A \cap B = B \cap A$.
- $A \cap (B \cap C) = (A \cap B) \cap C$.
- $A \cap B \subseteq A$.
- $A \cap A = A$.
- $A \cap U = A$.
- $A \cap \varnothing = \varnothing$.
- $A \subseteq B$ if and only if $A \cap B = A$.

https://en.wikipedia.org/wiki/Set_(mathematics)

Complements

Two sets can also be "subtracted". The *relative complement* of B in A (also called the *set-theoretic difference* of A and B), denoted by $A \setminus B$ (or $A - B$), is the set of all elements that are members of A but not members of B. Note that it is valid to "subtract" members of a set that are not in the set, such as removing the element *green* from the set {1, 2, 3}; doing so has no effect.



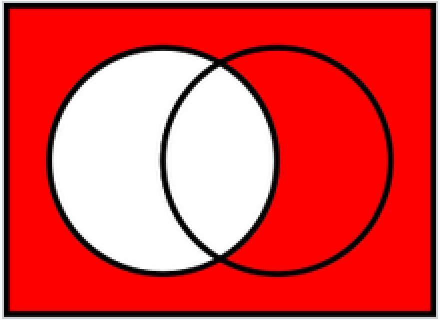
The **relative complement** of B in A

https://en.wikipedia.org/wiki/Set_(mathematics)

Complements

In certain settings all sets under discussion are considered to be subsets of a given universal set U. In such cases, $U \setminus A$ is called the *absolute complement* or simply *complement* of A, and is denoted by A'.

- $A' = U \setminus A$



The **complement** of A in U

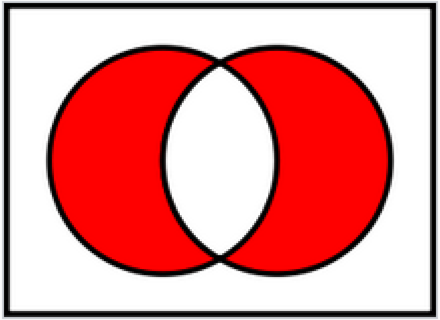
https://en.wikipedia.org/wiki/Set_(mathematics)

Complements

An extension of the complement is the symmetric difference, defined for sets *A*, *B* as

$$A \,\Delta\, B = (A \setminus B) \cup (B \setminus A).$$

For example, the symmetric difference of {7,8,9,10} and {9,10,11,12} is the set {7,8,11,12}. The power set of any set becomes a Boolean ring with symmetric difference as the addition of the ring (with the empty set as neutral element) and intersection as the multiplication of the ring.



The **symmetric difference** of *A* and *B*

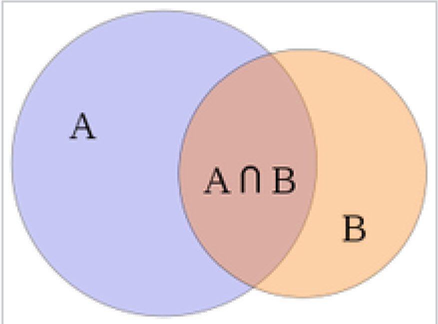
https://en.wikipedia.org/wiki/Set_(mathematics)

Properties of Complements

- $A \setminus B \neq B \setminus A$ for $A \neq B$.
- $A \cup A' = U$.
- $A \cap A' = \varnothing$.
- $(A')' = A$.
- $\varnothing \setminus A = \varnothing$.
- $A \setminus \varnothing = A$.
- $A \setminus A = \varnothing$.
- $A \setminus U = \varnothing$.
- $A \setminus A' = A$ and $A' \setminus A = A'$.
- $U' = \varnothing$ and $\varnothing' = U$.
- $A \setminus B = A \cap B'$.
- if $A \subseteq B$ then $A \setminus B = \varnothing$.

https://en.wikipedia.org/wiki/Set_(mathematics)

Inclusion and Exclusion



The inclusion-exclusion principle can be used to calculate the size of the union of sets: the size of the union is the size of the two sets, minus the size of their intersection.

The inclusion–exclusion principle is a counting technique that can be used to count the number of elements in a union of two sets, if the size of each set and the size of their intersection are known. It can be expressed symbolically as

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

A more general form of the principle can be used to find the cardinality of any finite union of sets:

https://en.wikipedia.org/wiki/Set_(mathematics)

Inclusion and Exclusion

$$
\begin{aligned}
|A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n| = & (|A_1| + |A_2| + |A_3| + \ldots |A_n|) \\
& - (|A_1 \cap A_2| + |A_1 \cap A_3| + \ldots |A_{n-1} \cap A_n|) \\
& + \ldots \\
& + (-1)^{n-1} (|A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n|) \, .
\end{aligned}
$$

https://en.wikipedia.org/wiki/Set_(mathematics)

De Morgan's Law

If A and B are any two sets then,

- **$(A \cup B)' = A' \cap B'$**

The complement of A union B equals the complement of A intersected with the complement of B.

- **$(A \cap B)' = A' \cup B'$**

The complement of A intersected with B is equal to the complement of A union to the complement of B.

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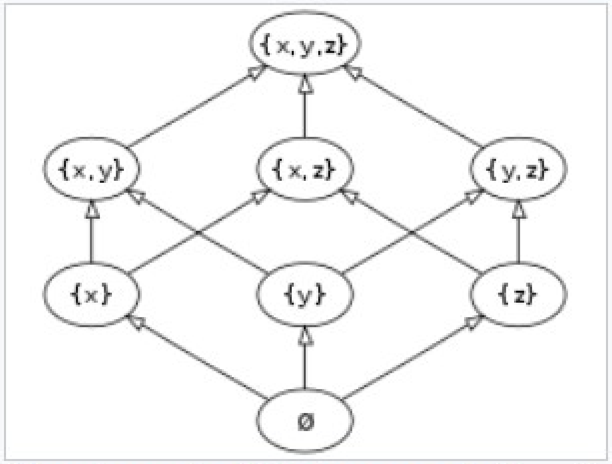
Power Set

In mathematics, the **power set** (or **powerset**) of any set S is the set of all subsets of S, including the empty set and S itself, variously denoted as $\mathcal{P}(S)$, $\mathscr{P}(S)$, $\wp(S)$ (using the "Weierstrass p"), $P(S)$, $\mathbb{P}(S)$, or, identifying the powerset of S with the set of all functions from S to a given set of two elements, 2^S. In axiomatic set theory (as developed, for example, in the ZFC axioms), the existence of the power set of any set is postulated by the axiom of power set.[1]

Any subset of $\mathcal{P}(S)$ is called a *family of sets* over S.



The elements of the power set of the set $\{x, y, z\}$ ordered with respect to inclusion.

https://en.wikipedia.org/wiki/Power_set

Power Set Example

If S is the set $\{x, y, z\}$, then the subsets of S are

- $\{\}$ (also denoted $\varnothing$ or $\emptyset$, the empty set or the null set)
- $\{x\}$
- $\{y\}$
- $\{z\}$
- $\{x, y\}$
- $\{x, z\}$
- $\{y, z\}$
- $\{x, y, z\}$

and hence the power set of S is $\{\{\}, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$.[2]

https://en.wikipedia.org/wiki/Power_set

Function

https://en.wikipedia.org/wiki/Cantor%27s_diagonal_argument

Function

https://en.wikipedia.org/wiki/Cantor%27s_diagonal_argument

References

[1] http://en.wikipedia.org/
[2]