Boolean Functions (8B)

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3/30/18

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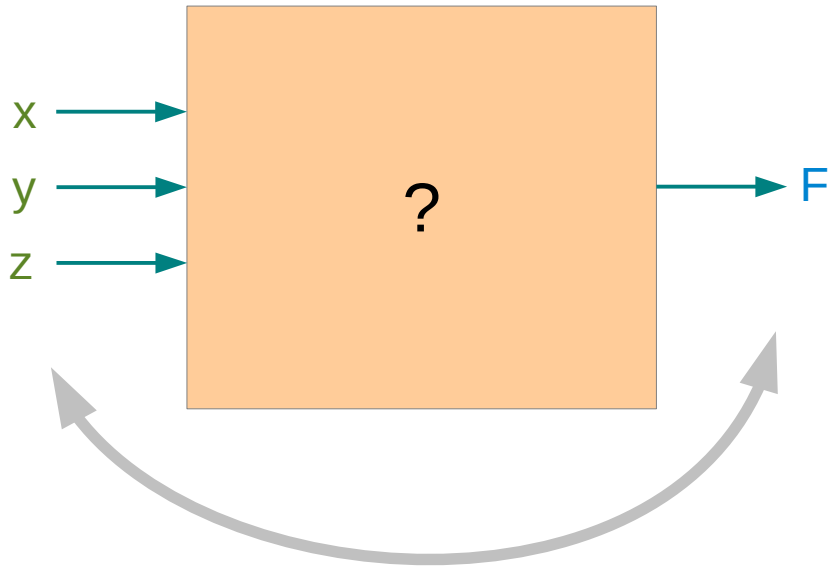
Truth Table

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



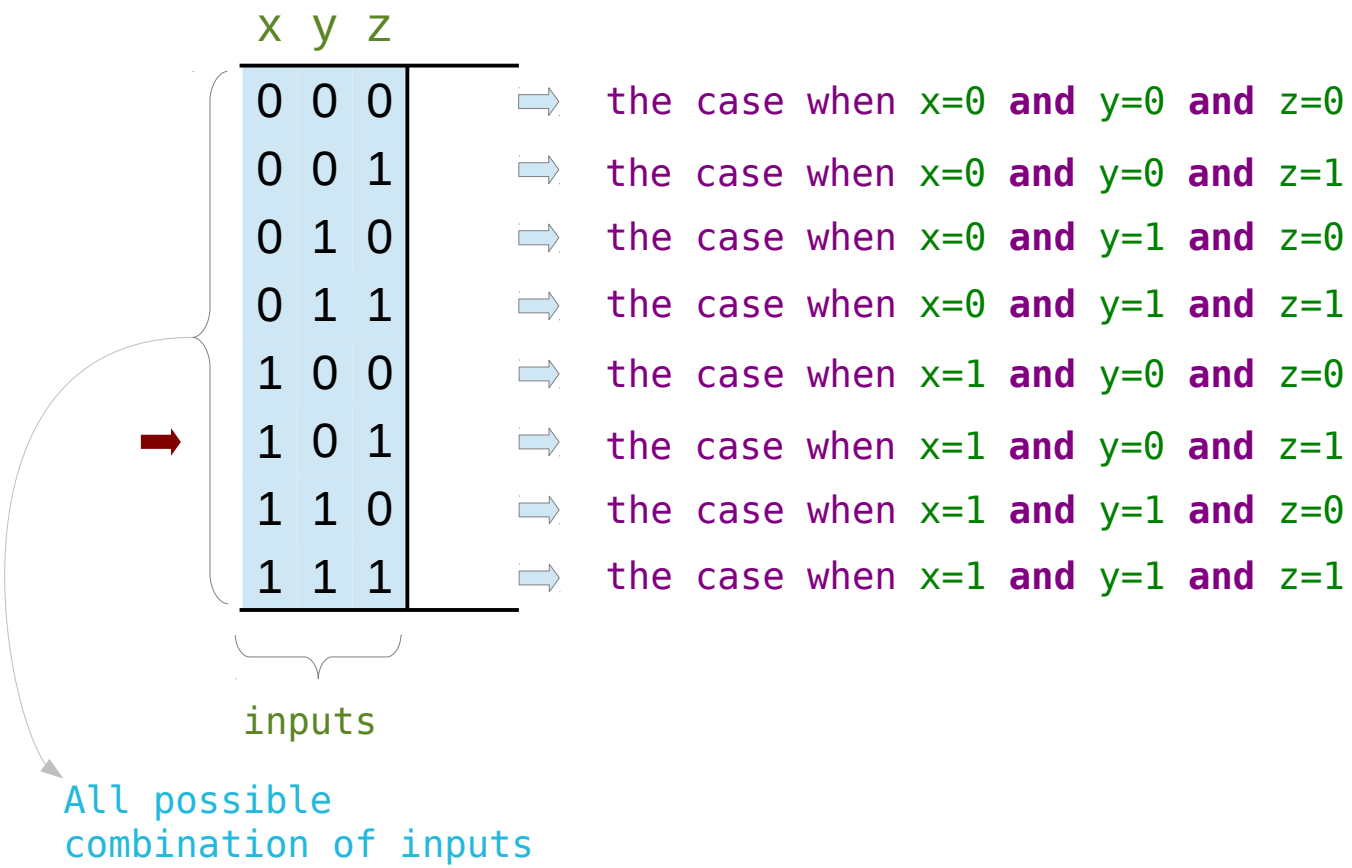
I/O relationship

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

inputs output

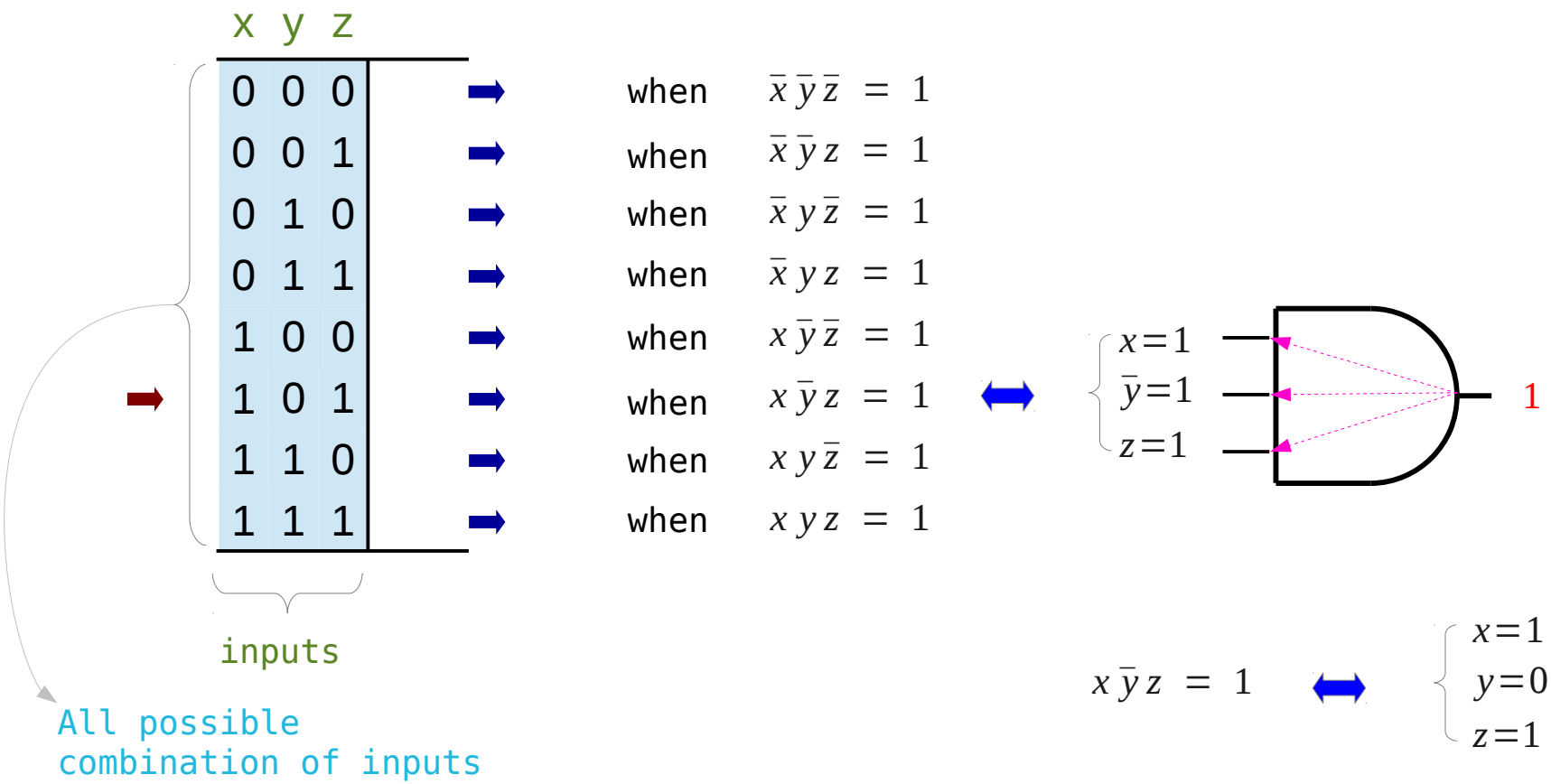
All possible input cases

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



All possible input cases using **minterms**

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

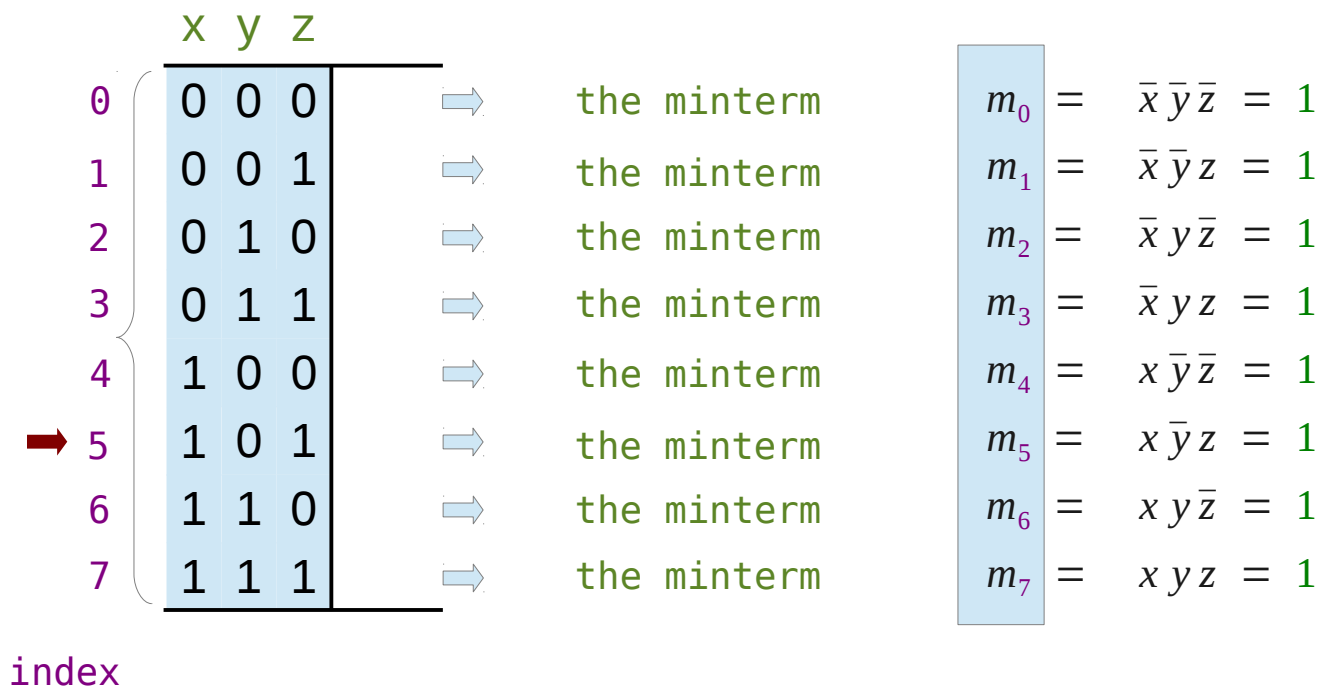


$x\,\bar{y}\,z = 1 \Leftrightarrow \begin{cases} x=1 \\ y=0 \\ z=1 \end{cases}$

For the output of an **and** gate to be 1,
all inputs must be 1

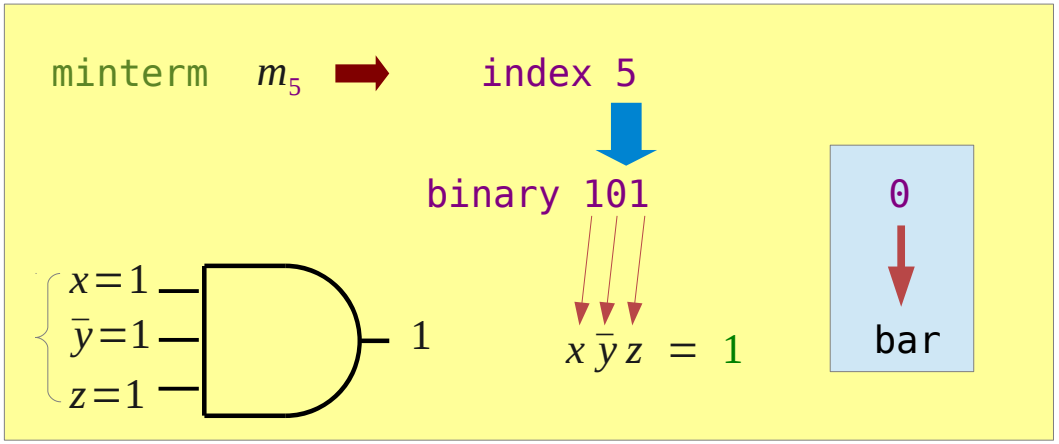
Naming **minterms**

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



Computing **minterms**

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



x y z

1 0 1

$x\ \bar{y}\ z$

$$m_5 = x\ \bar{y}\ z$$

Truth Table and **minterms** (1)

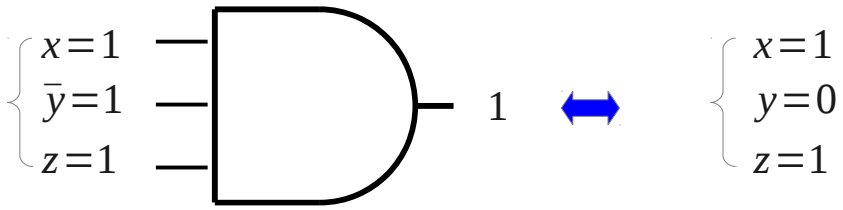
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

x	y	z			
0	0	0	⇨	the case when x=0 **and** y=0 **and** z=0	⇔ $\bar{x}\,\bar{y}\,\bar{z} = 1$
0	0	1	⇨	the case when x=0 **and** y=0 **and** z=1	⇔ $\bar{x}\,\bar{y}\,z = 1$
0	1	0	⇨	the case when x=0 **and** y=1 **and** z=0	⇔ $\bar{x}\,y\,\bar{z} = 1$
0	1	1	⇨	the case when x=0 **and** y=1 **and** z=1	⇔ $\bar{x}\,y\,z = 1$
1	0	0	⇨	the case when x=1 **and** y=0 **and** z=0	⇔ $x\,\bar{y}\,\bar{z} = 1$
1	0	1	⇨	the case when x=1 **and** y=0 **and** z=1	⇔ $x\,\bar{y}\,z = 1$
1	1	0	⇨	the case when x=1 **and** y=1 **and** z=0	⇔ $x\,y\,\bar{z} = 1$
1	1	1	⇨	the case when x=1 **and** y=1 **and** z=1	⇔ $x\,y\,z = 1$

inputs

All possible
combination of inputs

$x\,\bar{y}\,z = 1$ ⇔



For the output of an **and** gate to be 1,
all inputs must be 1

Truth Table and **minterms** (2)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

	x	y	z	
0	0	0	0	⇨ the case when the minterm $m_0 = \bar{x}\,\bar{y}\,\bar{z} = 1$
1	0	0	1	⇨ the case when the minterm $m_1 = \bar{x}\,\bar{y}\,z = 1$
2	0	1	0	⇨ the case when the minterm $m_2 = \bar{x}\,y\,\bar{z} = 1$
3	0	1	1	⇨ the case when the minterm $m_3 = \bar{x}\,y\,z = 1$
4	1	0	0	⇨ the case when the minterm $m_4 = x\,\bar{y}\,\bar{z} = 1$
➡ 5	1	0	1	⇨ the case when the minterm $m_5 = x\,\bar{y}\,z = 1$
6	1	1	0	⇨ the case when the minterm $m_6 = x\,y\,\bar{z} = 1$
7	1	1	1	⇨ the case when the minterm $m_7 = x\,y\,z = 1$

index

inputs

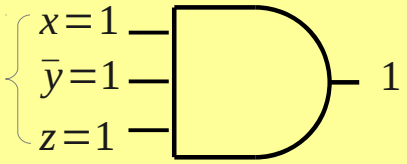
All possible
combination of inputs

minterm m_5 ➡ index 5

binary 101

$x\,\bar{y}\,z = 1$

0 ⇩ bar



Truth Table and **MAXterms** (1)

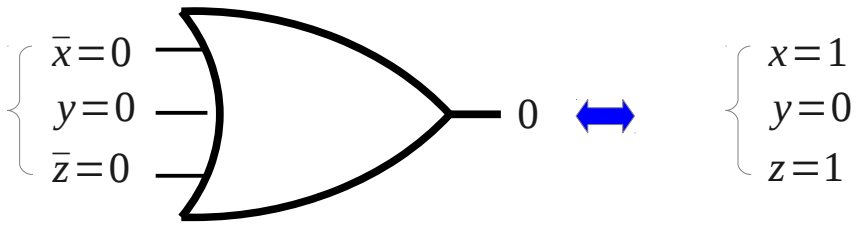
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

x	y	z			
0	0	0	⇨	the case when x=0 **and** y=0 **and** z=0	⇔ $x+y+z=0$
0	0	1	⇨	the case when x=0 **and** y=0 **and** z=1	⇔ $x+y+\bar{z}=0$
0	1	0	⇨	the case when x=0 **and** y=1 **and** z=0	⇔ $x+\bar{y}+z=0$
0	1	1	⇨	the case when x=0 **and** y=1 **and** z=1	⇔ $x+\bar{y}+\bar{z}=0$
1	0	0	⇨	the case when x=1 **and** y=0 **and** z=0	⇔ $\bar{x}+y+z=0$
1	0	1	⇨	the case when x=1 **and** y=0 **and** z=1	⇔ $\bar{x}+y+\bar{z}=0$
1	1	0	⇨	the case when x=1 **and** y=1 **and** z=0	⇔ $\bar{x}+\bar{y}+z=0$
1	1	1	⇨	the case when x=1 **and** y=1 **and** z=1	⇔ $\bar{x}+\bar{y}+\bar{z}=0$

inputs

All possible
combination of inputs

$$\bar{x}+y+\bar{z}=0 \Leftrightarrow$$



For the output of an **or** gate to be 0,
all inputs must be 0

Truth Table and **MAXterms** (2)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

	x	y	z			
0	0	0	0	⇨	the case when the MAXterm	$M_0 = x + y + z = 0$
1	0	0	1	⇨	the case when the MAXterm	$M_1 = x + y + \bar{z} = 0$
2	0	1	0	⇨	the case when the MAXterm	$M_2 = x + \bar{y} + z = 0$
3	0	1	1	⇨	the case when the MAXterm	$M_3 = x + \bar{y} + \bar{z} = 0$
4	1	0	0	⇨	the case when the MAXterm	$M_4 = \bar{x} + y + z = 0$
➡ 5	1	0	1	⇨	the case when the MAXterm	$M_5 = \bar{x} + y + \bar{z} = 0$
6	1	1	0	⇨	the case when the MAXterm	$M_6 = \bar{x} + \bar{y} + z = 0$
7	1	1	1	⇨	the case when the MAXterm	$M_7 = \bar{x} + \bar{y} + \bar{z} = 0$

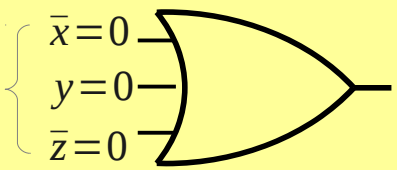
index

inputs

All possible
combination of inputs

minterm M_5 ➡ index 5

binary 101



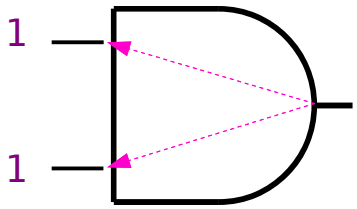
$\bar{x} + y + \bar{z}$

1

bar

Maxterm and **minterm** Conditions

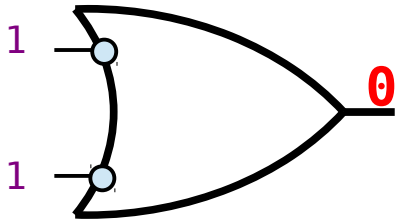
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

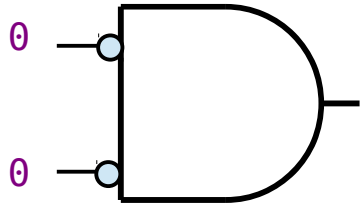


x	y	xy
0	0	0
0	1	0
1	0	0
1	1	1

1

$\bar{x}$	$\bar{y}$	$\bar{x}+\bar{y}$
1	1	1
1	0	1
0	1	1
0	0	0

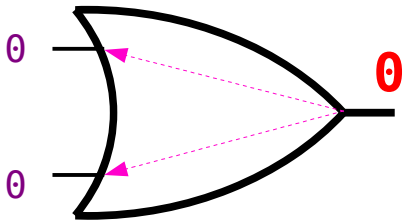




$\bar{x}$	$\bar{y}$	$\bar{x}\bar{y}$
1	1	1
1	0	0
0	1	0
0	0	0

1

x	y	x+y
0	0	0
0	1	1
1	0	1
1	1	1



Boolean functions defined by a truth table

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index

inputs output

All possible
combination of inputs

When the output becomes 1

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index

inputs output

All possible
combination of inputs

The output F becomes 1,
for one of the three following cases

(the case when x=0 **and** y=0 **and** z=1)

or (the case when x=0 **and** y=1 **and** z=1)

or (the case when x=1 **and** y=0 **and** z=0)

Function output values and **minterms**

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index

inputs output

All possible
combination of inputs

The output F becomes 1,
for one of the three following cases

$\Leftrightarrow \quad m_1 = \bar{x}\,\bar{y}\,z = 1$

$\Leftrightarrow \quad m_3 = \bar{x}\,y\,z = 1$

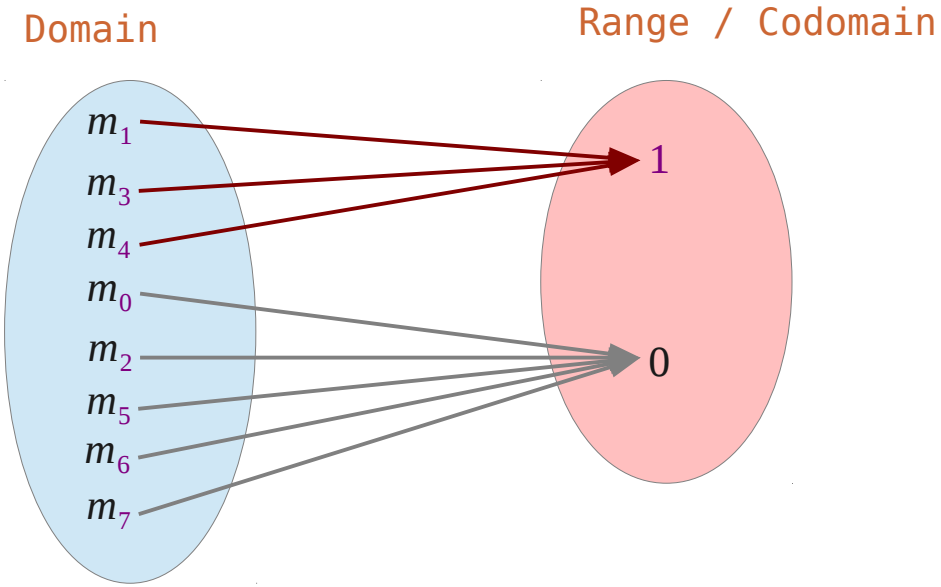
$\Leftrightarrow \quad m_4 = x\,\bar{y}\,\bar{z} = 1$

Mapping Set Diagram

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

inputs output



Boolean function definition using **minterms**

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index

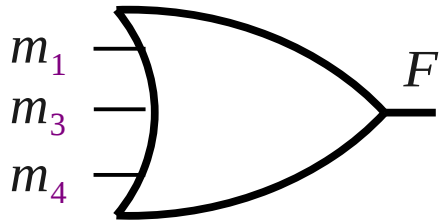
inputs output

All possible
combination of inputs

The output F becomes 1,

either $m_1=1$ or $m_3=1$ or $m_4=1$

$$m_1 + m_3 + m_4 = 1 \quad \rightleftharpoons \quad F = 1$$



For the output of an **or** gate to be 1,
at least one must be 1

$$\Leftrightarrow \quad F = m_1 + m_3 + m_4$$

Boolean Function with **minterms** (1)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index

inputs output

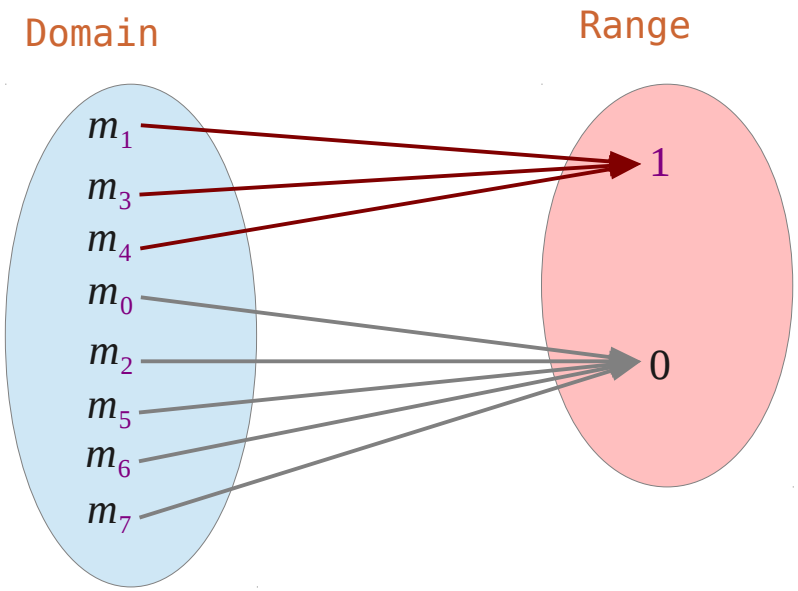
All possible
combination of inputs

The output F becomes 1,
for one of the three following cases

(the case when x=0 and y=0 and z=1) $\Leftrightarrow m_1 = \bar{x}\,\bar{y}\,z = 1$

or (the case when x=0 and y=1 and z=1) $\Leftrightarrow m_3 = \bar{x}\,y\,z = 1$

or (the case when x=1 and y=0 and z=0) $\Leftrightarrow m_4 = x\,\bar{y}\,\bar{z} = 1$



Boolean Function with **minterms** (2)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

inputs output

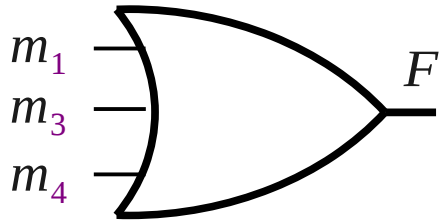
All possible
combination of inputs

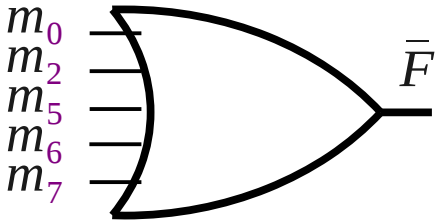
The output F becomes 1,

either $m_1=1$ or $m_3=1$ or $m_4=1$

$$m_1 + m_3 + m_4 = 1 \quad \rightleftharpoons \quad F = 1$$

$$\Leftrightarrow \quad F = m_1 + m_3 + m_4$$





For the output of an **or** gate to be 1,
at least one must be 1

Boolean Function with **minterms** (3)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

inputs output

All possible
combination of inputs

The output F becomes 1,

either $m_1=1$ or $m_3=1$ or $m_4=1$

$$m_1 + m_3 + m_4 = 1 \quad \rightleftharpoons \quad F = 1$$

$$\Leftrightarrow \quad F = m_1 + m_3 + m_4$$

The output F becomes 0,

either $m_0=1$ or $m_2=1$ or $m_5=1$ or $m_6=1$ or $m_7=1$

$$m_0+m_2+m_5+m_6+m_7=1 \quad \rightleftharpoons \quad F = 0$$

$$\Leftrightarrow \quad \overline{F} = m_0+m_2+m_5+m_6+m_7$$

For the output of an **or** gate to be 1,
at least one must be 1

Boolean Function with **Maxterms** (1)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

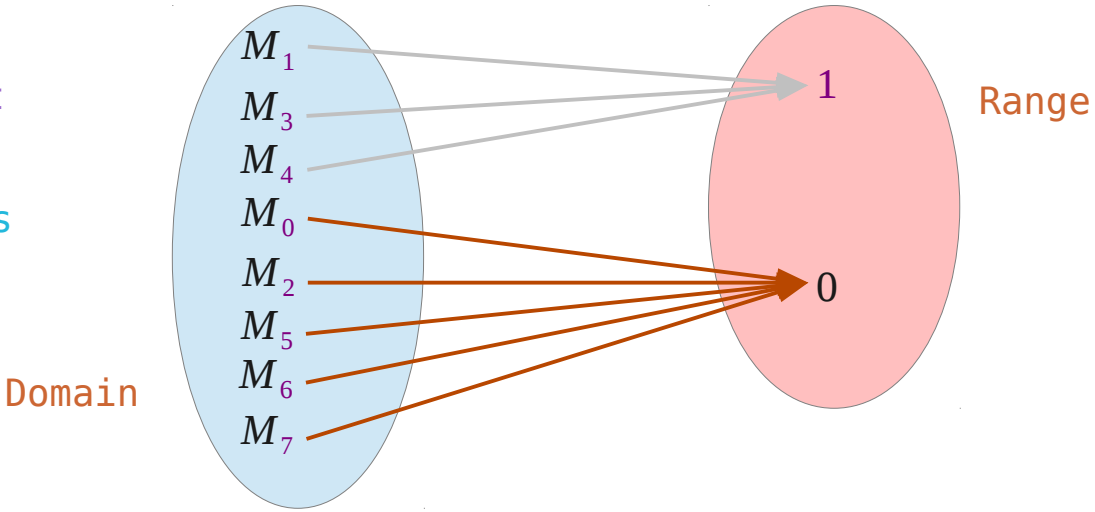
The output F becomes 0,
for one of the five following cases

	x	y	z	F	
➡ 0	0	0	0	0	(the case when x=0 and y=0 and z=0) ⟺ $x + y + z = 0$
1	0	0	1	1	
➡ 2	0	1	0	0	or (the case when x=0 and y=1 and z=0) ⟺ $x + \bar{y} + z = 0$
3	0	1	1	1	
4	1	0	0	1	
➡ 5	1	0	1	0	or (the case when x=1 and y=0 and z=1) ⟺ $\bar{x} + y + \bar{z} = 0$
➡ 6	1	1	0	0	or (the case when x=1 and y=1 and z=0) ⟺ $\bar{x} + \bar{y} + z = 0$
➡ 7	1	1	1	0	or (the case when x=1 and y=1 and z=1) ⟺ $\bar{x} + \bar{y} + \bar{z} = 0$

index

inputs output

All possible
combination of inputs



Boolean Function with **Maxterms** (2)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

	x	y	z	F
➡ 0	0	0	0	0
1	0	0	1	1
➡ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
➡ 5	1	0	1	0
➡ 6	1	1	0	0
➡ 7	1	1	1	0

index

inputs output

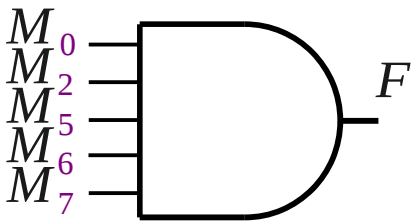
All possible
combination of inputs

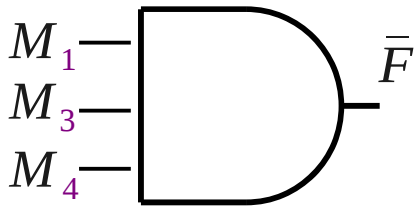
The output F becomes 0,

either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0 \quad \rightleftharpoons \quad F = 0$$

$$\Leftrightarrow \quad F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$





For the output of an **and** gate to be 0,
at least one input must be 0

Boolean Function with **Maxterms** (2)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

	x	y	z	F
➡ 0	0	0	0	0
1	0	0	1	1
➡ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
➡ 5	1	0	1	0
➡ 6	1	1	0	0
➡ 7	1	1	1	0

index

inputs output

All possible
combination of inputs

The output F becomes 0,

either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0 \quad \rightleftharpoons \quad F = 0$$

$$\Leftrightarrow \quad F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$

The output F becomes 1,

either $M_1=0$ or $M_3=0$ or $M_4=0$

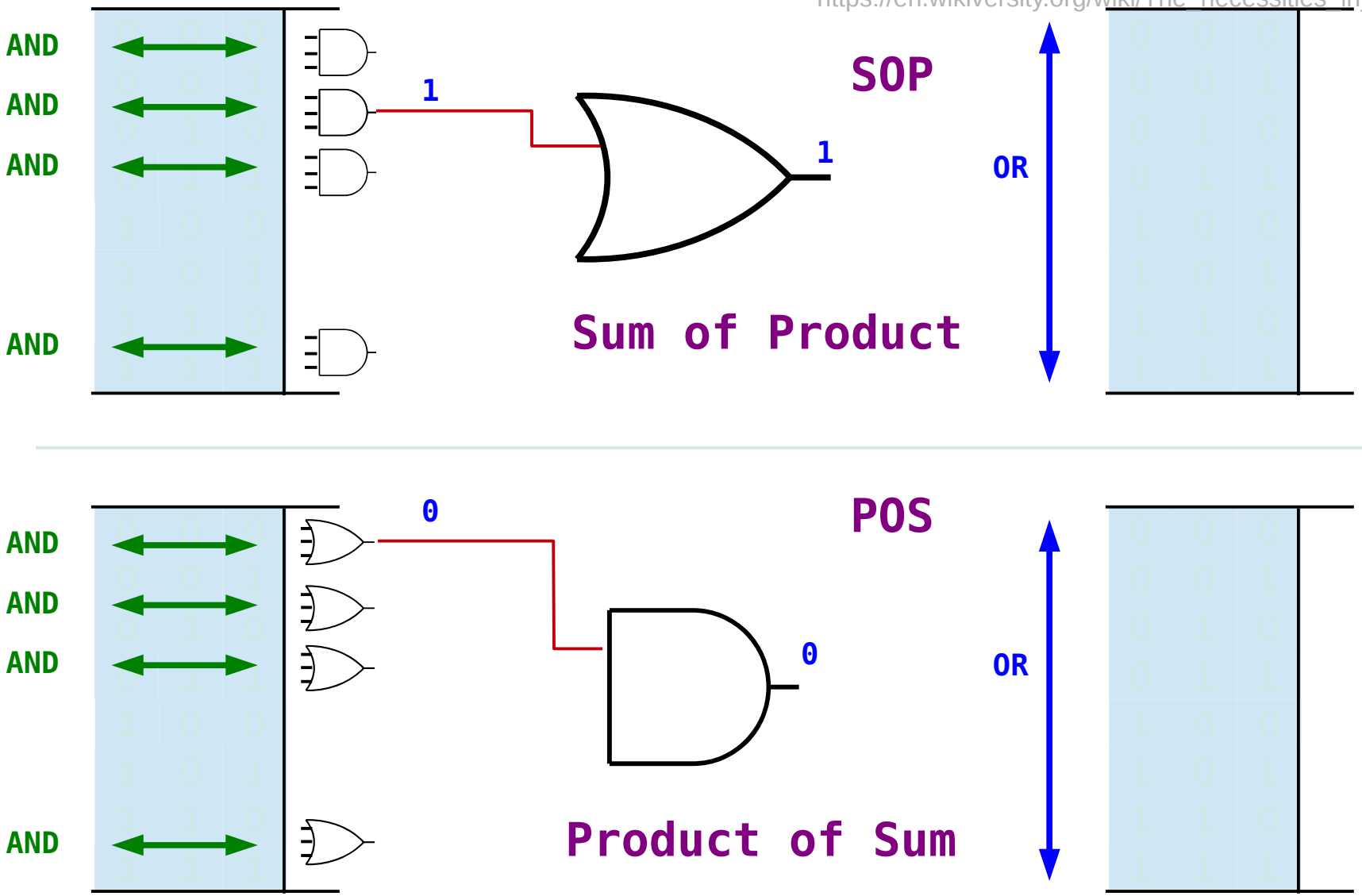
$$M_1 \cdot M_3 \cdot M_4 = 0 \quad \rightleftharpoons \quad F = 1$$

$$\Leftrightarrow \quad \overline{F} = M_1 \cdot M_3 \cdot M_4$$

For the output of an **and** gate to be 0,
at least one input must be 0

SOP and POS

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



Boolean Function with **minterms**

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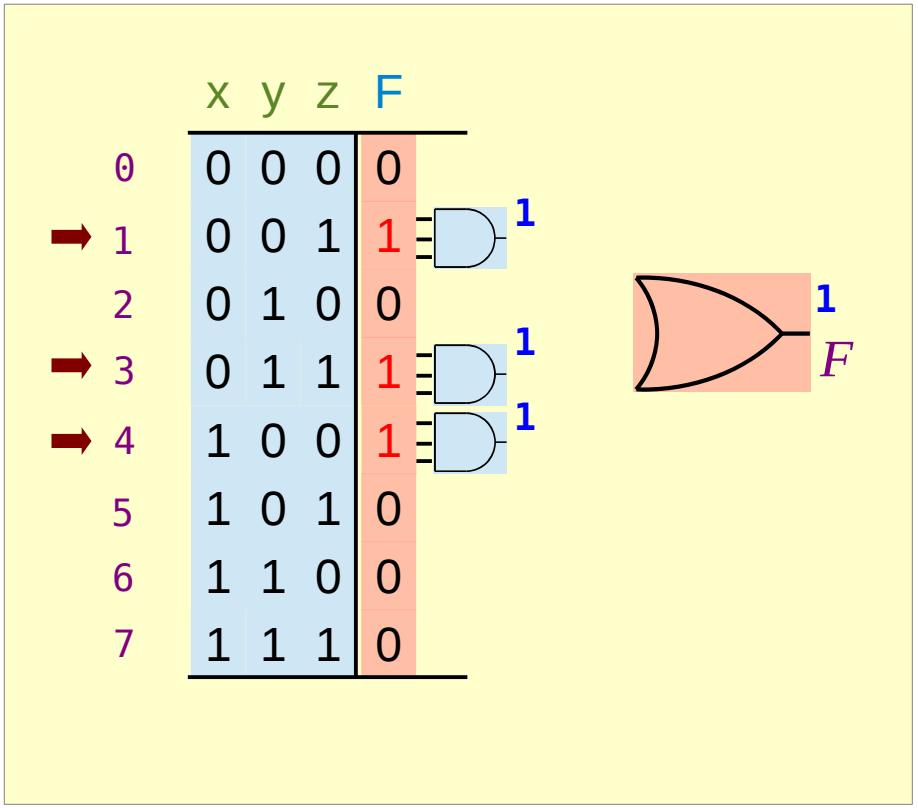

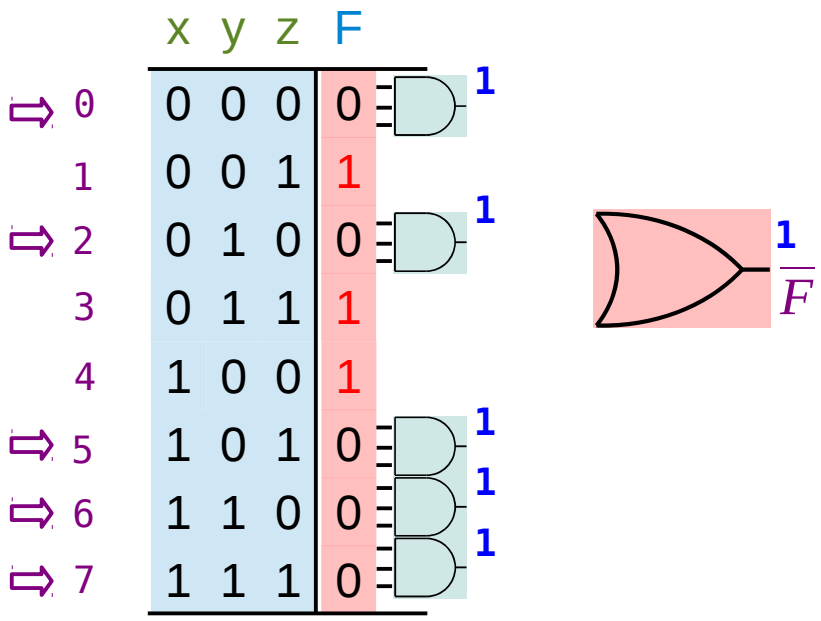

Boolean Function with **Maxterms**

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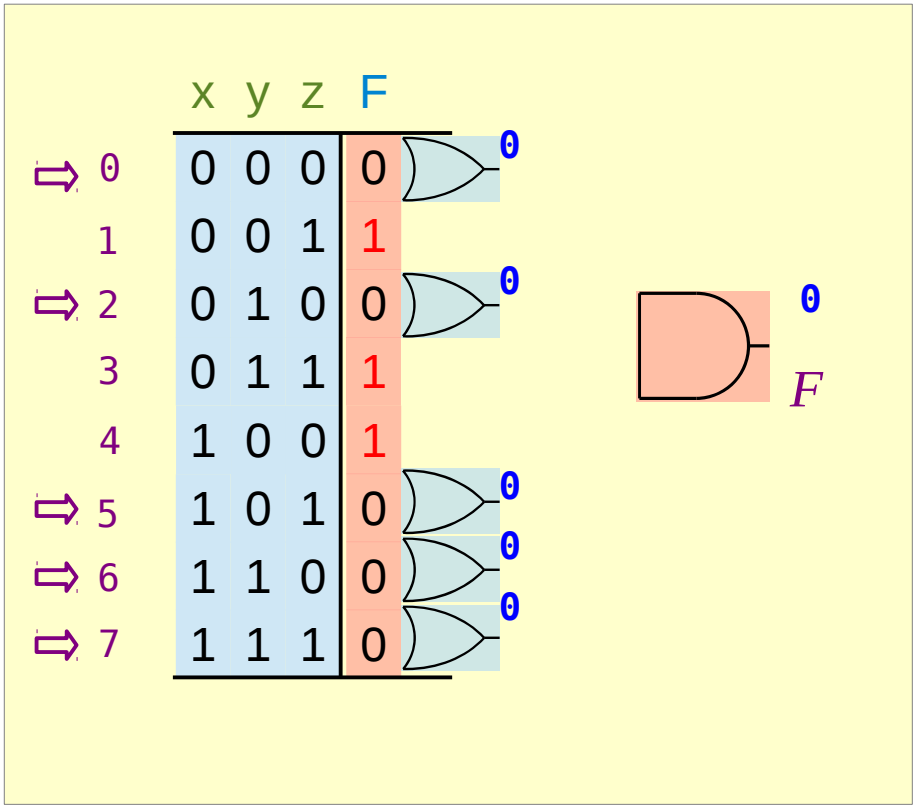

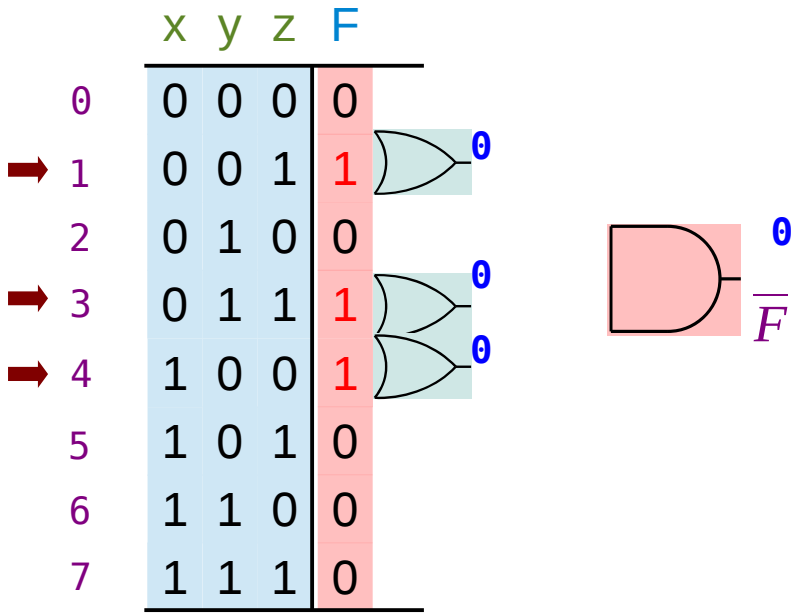

References

[1] http://en.wikipedia.org/
[2]