Resolution (7A)

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4/21/18

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Argument

$$\frac{\begin{array}{c}(p \vee q) \\ (\neg p \vee r)\end{array}}{q \vee r}$$

$$(p \vee q) \wedge (\neg p \vee r) \rightarrow q \vee r$$

Truth Table

p	q	r	$p \vee q$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	F

p	q	r	$\neg p$	$\neg p \vee r$
T	T	T	F	T
T	T	F	F	F
T	F	T	F	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

p	q	r	$(p \vee q) \wedge (\neg p \vee r)$	$q \vee r$
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

$$(p \vee q) \wedge (\neg p \vee r) \rightarrow q \vee r$$

Interpretation of this truth table

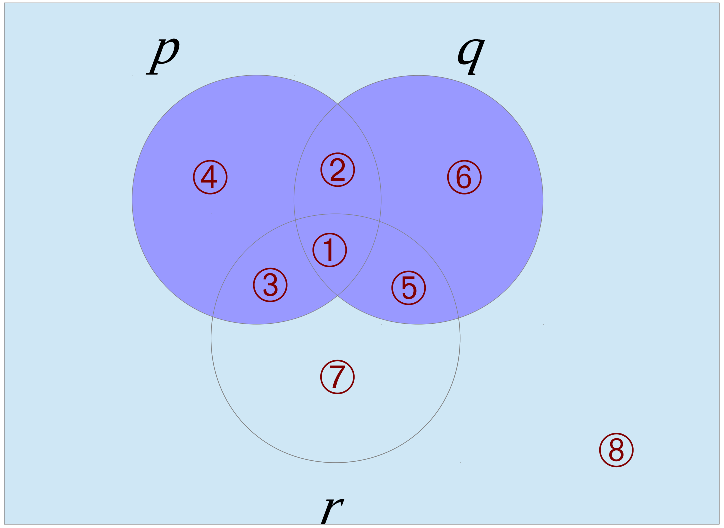
	p	q	r	$p \vee q$	$\neg p \vee r$	A: $(p \vee q) \wedge (\neg p \vee r)$		B: $q \vee r$	$A \rightarrow B$
case ①	T	T	T	T	T	T	→	T	T
case ②	T	T	F	T	F	F	→	T	T
case ③	T	F	T	T	T	T	→	T	T
case ④	T	F	F	T	F	F	→	F	T
case ⑤	F	T	T	T	T	T	→	T	T
case ⑥	F	T	F	T	T	T	→	T	T
case ⑦	F	F	T	F	T	F	→	T	T
case ⑧	F	F	F	F	T	F	→	F	T

Whenever $p \vee q$ and $\neg p \vee r$ are **true**, $q \vee r$ is **true**

$$(p \vee q) \wedge (\neg p \vee r) \rightarrow q \vee r$$

Venn diagram for $p \vee q$

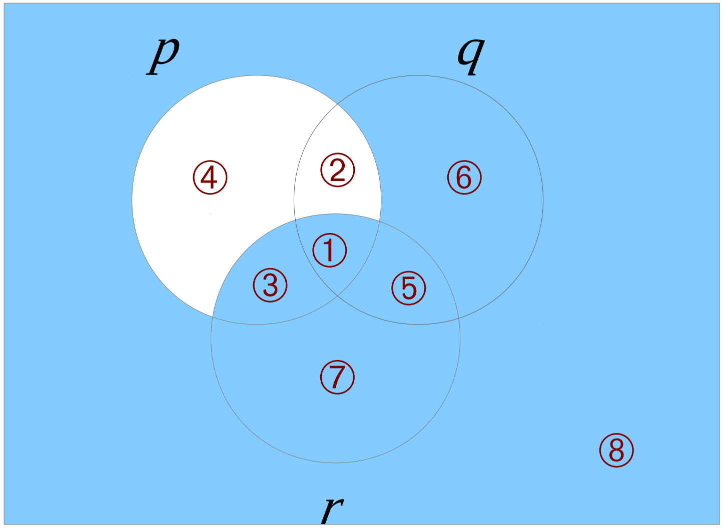
$p \vee q$



	p	q	r	$p \vee q$
case ①	T	T	T	T
case ②	T	T	F	T
case ③	T	F	T	T
case ④	T	F	F	T
case ⑤	F	T	T	T
case ⑥	F	T	F	T
case ⑦	F	F	T	F
case ⑧	F	F	F	F

Venn diagram for $\neg p \vee r$

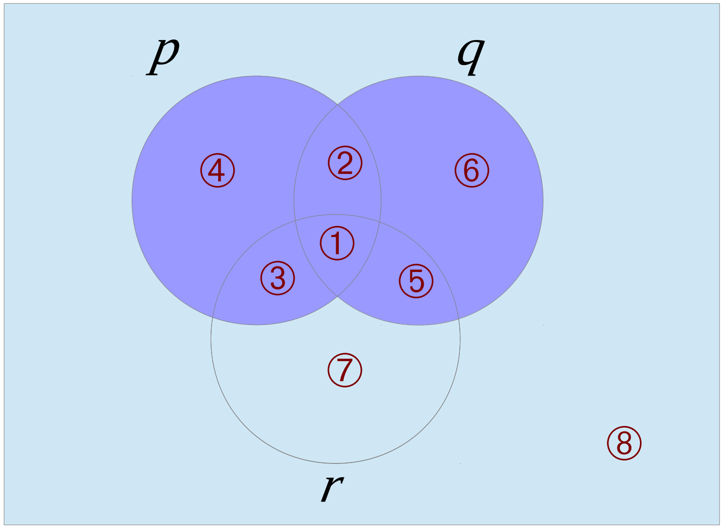
$\neg p \vee r$

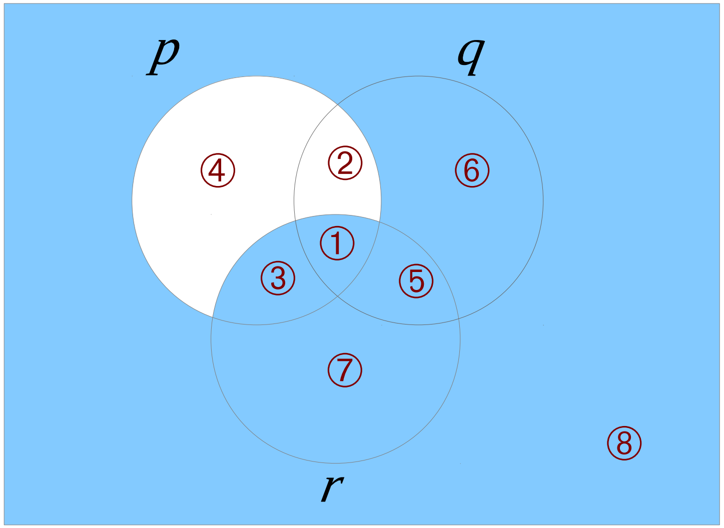


	p	q	r	$\neg p$	$\neg p \vee q$
case ①	T	T	T	F	T
case ②	T	T	F	F	F
case ③	T	F	T	F	T
case ④	T	F	F	F	F
case ⑤	F	T	T	T	T
case ⑥	F	T	F	T	T
case ⑦	F	F	T	T	T
case ⑧	F	F	F	T	T

When $(p \lor q) \land (\neg p \lor r)$ is true

$p \lor q$



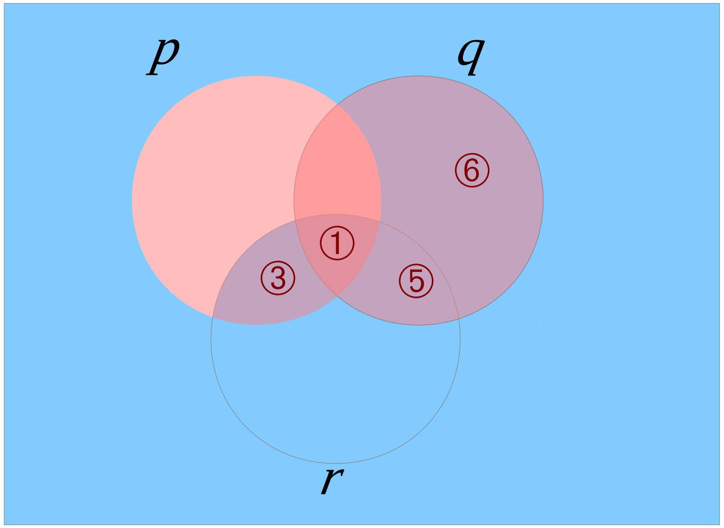


$\neg p \lor r$

When $p \lor q$ is true
and $\neg p \lor r$ is true

$(p \lor q) \land (\neg p \lor r)$

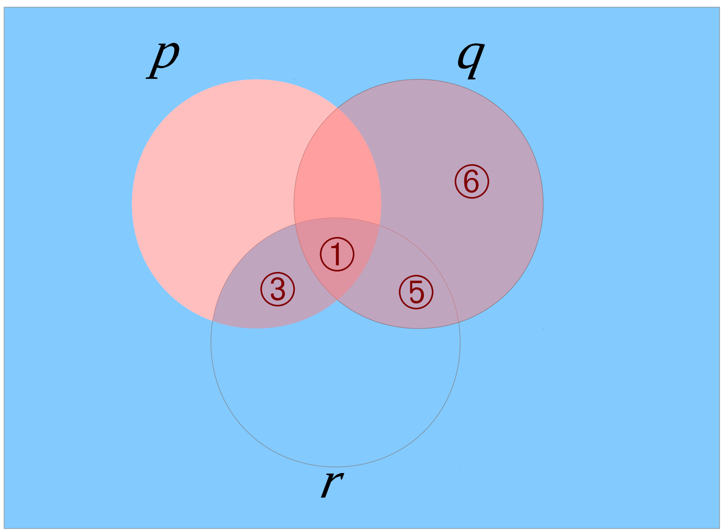
cases ①+③+⑤+⑥



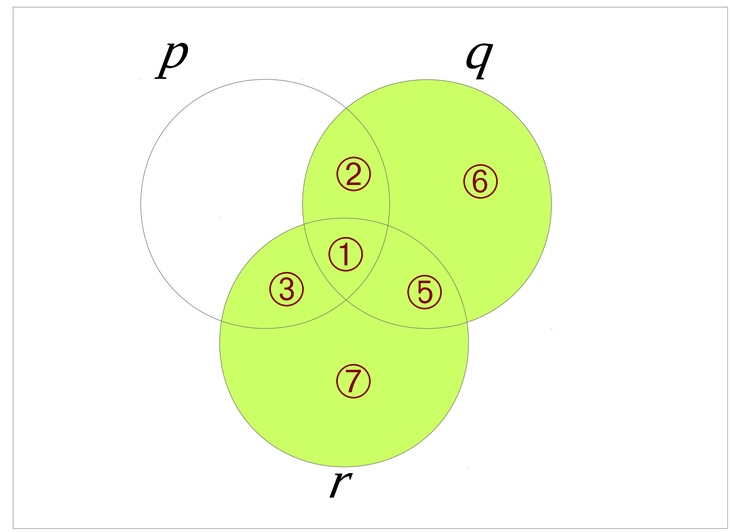
When $(p \lor q) \land (\neg p \lor r)$ is true, $q \lor r$ is also true

$p \lor q$

$\neg p \lor r$



$q \lor r$



cases ①+③+⑤+⑥ $\subset$ cases ①+③+⑤+⑥+②+⑦

$(p \lor q) \land (\neg p \lor r) \longrightarrow q \lor r$

Argument

Case 1: ***p*** is false

Case 2: ***p*** is true

$$\frac{\begin{array}{c} p \lor q \\ \neg p \lor r \end{array}}{q \lor r}$$

$$\frac{\begin{array}{c} F \lor q \\ T \lor r \end{array}}{q}$$

$$\frac{\begin{array}{c} T \lor q \\ F \lor r \end{array}}{r}$$

when ***p*** is false,
q must be true.

when ***p*** is true,
r must be true.

Therefore regardless of truth value of ***p***,
If both premises hold,
then the conclusion $q \lor r$ is true

http://en.wikipedia.org/wiki/Derivative

Resolution Examples

$$\begin{array}{c} p \lor q \\ \neg p \lor r \\ \hline q \lor r \end{array} \quad \Rightarrow \quad \begin{array}{c} \cancel{p} \lor q \\ \neg \cancel{p} \lor r \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{c} \cancel{p} \lor q \\ \neg \cancel{p} \lor r \\ \hline q \lor r \end{array}$$

$$\begin{array}{c} p \lor q \\ \neg p \\ \hline q \end{array} \quad \Rightarrow \quad \begin{array}{c} \cancel{p} \lor q \\ \neg \cancel{p} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{c} \cancel{p} \lor q \\ \neg \cancel{p} \\ \hline q \end{array}$$

$$\begin{array}{c} p \\ \neg p \lor r \\ \hline r \end{array} \quad \Rightarrow \quad \begin{array}{c} \cancel{p} \\ \neg \cancel{p} \lor r \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{c} \cancel{p} \\ \neg \cancel{p} \lor r \\ \hline r \end{array}$$

Conjunctive Norm Form (CNF) is assumed

$$(\cdots \vee \cdots) \wedge (\cdots \vee \cdots \vee \cdots) \wedge (\cdots \vee \cdots)$$

the variables A, B, C, D, and E are in conjunctive normal form:

¬ A ∧ (B ∨ C)
(A ∨ B) ∧ (¬ B ∨ C ∨ ¬ D) ∧ (D ∨ ¬ E)
A ∨ B
A ∧ B

The following formulas are not in conjunctive normal form:

¬ (B ∨ C)
(A ∧ B) ∨ C
A ∧ (B ∨ (D ∧ E))

https://en.wikipedia.org/wiki/Conjunctive_normal_form

General Rules of Inference (1)

Discrete Mathematics and Its Applications, Rosen

$$\frac{\begin{array}{l} p \\ p \rightarrow q \end{array}}{q}$$

$$(p \land (p \rightarrow q)) \rightarrow q$$

Modus ponens

Modus: manner, way, mode
Ponens: to affirm

$$\frac{\begin{array}{l} \neg q \\ p \rightarrow q \end{array}}{\neg p}$$

$$(\neg q \land (p \rightarrow q)) \rightarrow \neg p$$

Modus tollens

Modus: manner, way, mode
tollens: to remove

$$\frac{\begin{array}{l} p \rightarrow q \\ q \rightarrow r \end{array}}{p \rightarrow r}$$

$$((p \rightarrow q) \land (q \rightarrow r)) \rightarrow p \rightarrow r$$

Hypothetical syllogism

Syllogism: inference, conclusion

$$\frac{\begin{array}{l} p \lor q \\ \neg p \end{array}}{q}$$

$$((p \lor q) \land (\neg p)) \rightarrow q$$

Disjunctive syllogism

General Rules of Inference (2)

Discrete Mathematics and Its Applications, Rosen

$$\frac{p}{p \lor q}$$

$(p) \rightarrow p \lor q$

Addition

$$\frac{p \land q}{p}$$

$(p \land q) \rightarrow p$

Simplification

$$\frac{\begin{array}{l} p \\ q \end{array}}{p \land q}$$

$(p \land q) \rightarrow p \land q$

Conjunction

$$\frac{\begin{array}{l} p \lor q \\ \neg p \lor q \end{array}}{q \lor r}$$

$((p \lor q) \land (\neg p \lor r)) \rightarrow q \lor r$

Resolution

Breaking into parts, a loosening

Example A

$$p \vee q$$
$$p \vee \neg r$$
$$\neg p \vee q$$
$$\neg q \vee r$$

$$q$$
$$p \vee \neg r$$
$$\neg q \vee r$$

$$p \vee \neg r$$
$$r$$

$$p$$

$$(p \vee q) \wedge (p \vee \neg r) \wedge (\neg p \vee q) \wedge (\neg q \vee r)$$
$$\vdash (p \vee q) \wedge (p \vee \neg r) \wedge (\neg p \vee q) \wedge (\neg q \vee r) \wedge (q)$$
$$\vdash (p \vee q) \wedge (p \vee \neg r) \wedge (\neg p \vee q) \wedge (\neg q \vee r) \wedge (q) \wedge (r)$$
$$\vdash (p \vee q) \wedge (p \vee \neg r) \wedge (\neg p \vee q) \wedge (\neg q \vee r) \wedge (q) \wedge (r) \wedge (p)$$

James Aspnes, Notes on Discrete Mathematics, CS 202: Fall 2013

Example B – (1)

$p \rightarrow q \lor r$

$p \lor \neg q$

$r \lor q$

$\neg p \lor q \lor r$

$p \lor \neg q$

$r \lor q$

$q \lor \neg q \lor r$

$r \lor q$

$r \lor q$

$q \lor \neg q \lor r = T \lor r = T$

$p \rightarrow q \lor r$

$p \lor \neg q$

$r \lor q$

$\neg p \lor q \lor r$

$p \lor \neg q$

$r \lor q$

$p \lor r$

Discrete Mathematics, Johnsonbough

Example B – (2)

$p \rightarrow q \vee r$
$p \vee \neg q$
$r \vee q$

$p \rightarrow q \vee r$
$\neg p \rightarrow \neg q$
$q \vee r$

$p \rightarrow q \vee r$
$q \rightarrow p$
$q \vee r$

$p \vee r$

Discrete Mathematics, Johnsonbough

Truth Table

p	q	r	$\neg p$	$\neg p \vee q \vee r$
T	T	T	F	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

p	q	r	$\neg q$	$p \vee \neg q$
T	T	T	F	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	T
F	T	T	F	F
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

p	q	r	$q \vee r$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	F

p	q	r	$\neg p \vee q \vee r$	$p \vee \neg q$	$q \vee r$	$H1 \wedge H2 \wedge H3$	$p \vee r$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	F	T	F	F	T
F	T	T	T	F	T	F	T
F	T	F	T	F	T	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	F	F	F

$H1 = \neg p \vee q \vee r$
$H2 = p \vee \neg q$
$H3 = q \vee r$

$H1 \wedge H2 \wedge H3 \rightarrow H3$
$H1 \wedge H2 \wedge H3 \rightarrow H2$
$H1 \wedge H2 \wedge H3 \rightarrow H1$
$H1 \wedge H2 \wedge H3 \rightarrow (p \vee r)$

Truth Table and K-Map

p	q	r	$H1 \wedge H2 \wedge H3$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

p	q	r	$H1 \wedge H2 \wedge H3$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0

p	q	r	$H1 \wedge H2 \wedge H3$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

p q r

p \ $q\,r$	0 0	0 1	1 1	1 0
0	0	1	0	0
1	0	1	1	1

K-Map and Logic Minimization

p $\quad$ $q\ r$

	00	01	11	10
0	0	1	0	0
1	0	1	1	1

	00	01	11	10
0		$\bar{p}\bar{q}r$		
1		$p\bar{q}r$	pqr	$pq\bar{r}$

	00	01	11	10
0		$\bar{q}r$		
1			pq	

$$\bar{p}\bar{q}r + p\bar{q}r = (\bar{p} + p)\bar{q}r = \bar{q}r$$

$$pqr + pq\bar{r} = pq(r + \bar{r}) = pq$$

K-Map : Verification

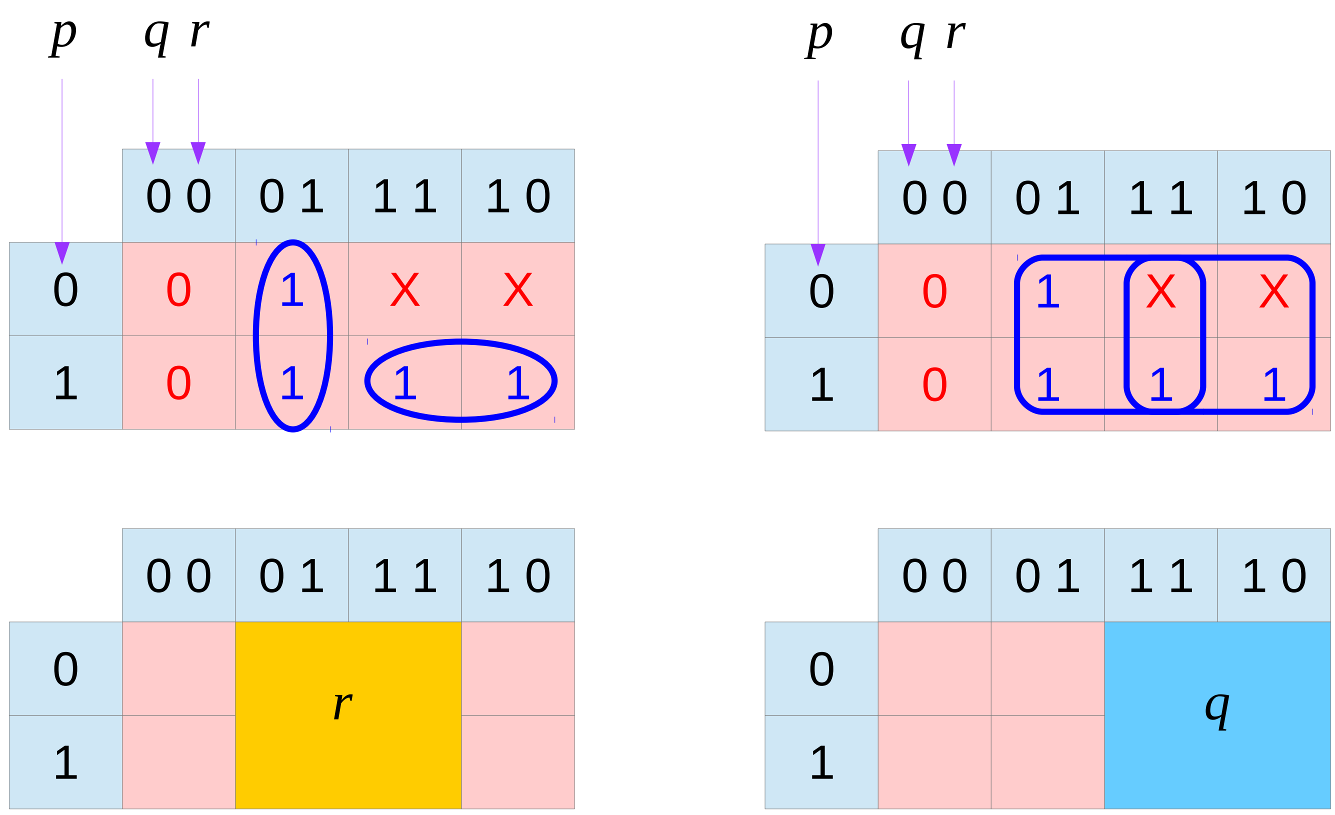
	00	01	11	10
0		$\bar{q}r$		
1		$\bar{q}r$		pq

$$H1 \wedge H2 \wedge H3 \equiv \bar{q}r + pq$$

p	q	r	$\bar{q}$	$\bar{q}r$	pq	$\bar{q}r + pq$
0	0	0	1	0	0	0
0	0	1	1	1	0	1
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	0	1	1	1	0	1
1	1	0	0	0	1	1
1	1	1	0	0	1	1

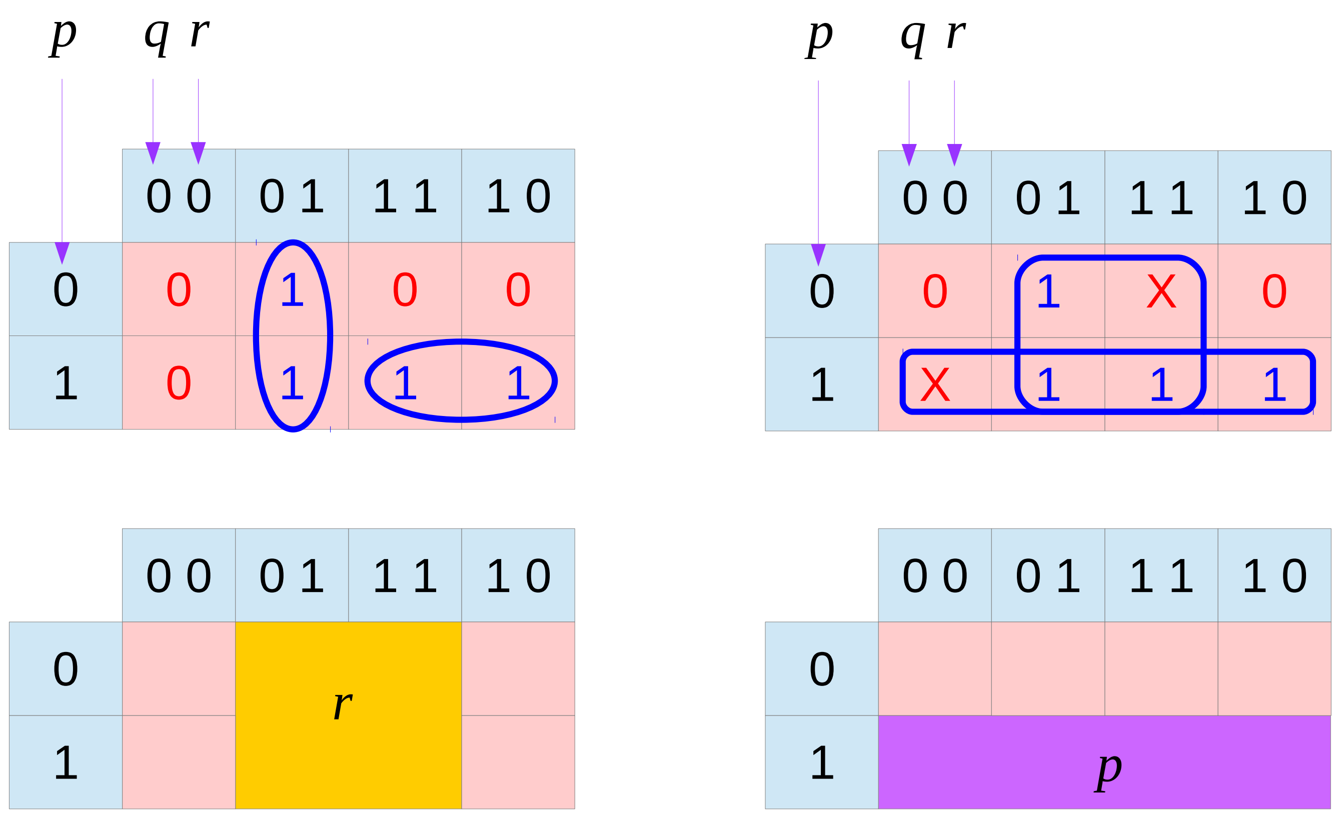
p	q	r	$H1 \wedge H2 \wedge H3$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Adding two don’t care conditions



$q \lor r$

Adding two don’t care conditions



$p \lor r$

K-Map : Verification

	00	01	11	10
0		r	r	
1		r	r	

	00	01	11	10
0				
1	p	p	p	p

$$H1 \wedge H2 \wedge H3 \rightarrow p \vee r$$

p	q	r	$p \vee r$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

p	q	r	$H1 \wedge H2 \wedge H3$	$p \vee r$
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Example C

$$p \rightarrow q$$
$$\neg q$$

$$\neg p \lor q$$
$$\neg q$$

$$\neg p$$

$$p \rightarrow q$$
$$\neg q$$

$$\neg p \lor q$$
$$\neg q$$

$$(\neg p \lor q) \land \neg q$$

$$= (\neg p \land \neg q) \lor (q \land \neg q)$$

$$= (\neg p \land \neg q)$$

Simplification Rule

$$\neg p$$

Discrete Mathematics, Johnsonbough

References

[1] http://en.wikipedia.org/
[2]