Resolution (7A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice and Octave.

Example B – (1)

$p \rightarrow q \vee r$
$p \vee \neg q$
$r \vee q$

$\neg p \vee q \vee r$
$p \vee \neg q$
$r \vee q$

$q \vee \neg q \vee r$
$r \vee q$

$r \vee q$

$q \vee \neg q \vee r = T \vee r = T$

$p \rightarrow q \vee r$
$p \vee \neg q$
$r \vee q$

$\neg p \vee q \vee r$
$p \vee \neg q$
$r \vee q$

$p \vee r$

Discrete Mathematics, Johnsonbough

Example B – (2)

$p \rightarrow q \vee r$
$p \vee \neg q$
$r \vee q$

$p \rightarrow q \vee r$
$\neg p \rightarrow \neg q$
$q \vee r$

$p \rightarrow q \vee r$
$q \rightarrow p$
$q \vee r$

$p \vee r$

Discrete Mathematics, Johnsonbough

Truth Table

p	q	r	$\neg p$	$\neg p \vee q \vee r$
T	T	T	F	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

p	q	r	$\neg q$	$p \vee \neg q$
T	T	T	F	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	T
F	T	T	F	F
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

p	q	r	$q \vee r$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	F

p	q	r	$\neg p \vee q \vee r$	$p \vee \neg q$	$q \vee r$	$H1 \wedge H2 \wedge H3$	$p \vee r$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	F	T	F	F	T
F	T	T	T	F	T	F	T
F	T	F	T	F	T	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	F	F	F

$H1 = \neg p \vee q \vee r$
$H2 = p \vee \neg q$
$H3 = q \vee r$

$H1 \wedge H2 \wedge H3 \rightarrow H3$
$H1 \wedge H2 \wedge H3 \rightarrow H2$
$H1 \wedge H2 \wedge H3 \rightarrow H1$
$H1 \wedge H2 \wedge H3 \rightarrow (p \vee r)$

Truth Table and K-Map

p	q	r	$H1 \wedge H2 \wedge H3$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

p	q	r	$H1 \wedge H2 \wedge H3$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0

p	q	r	$H1 \wedge H2 \wedge H3$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

p \ $q\ r$

	0 0	0 1	1 1	1 0
0	0	1	0	0
1	0	1	1	1

K-Map and Logic Minimization

p $q\ r$

	0 0	0 1	1 1	1 0
0	0	1	0	0
1	0	1	1	1

	0 0	0 1	1 1	1 0
0		$\bar{p}\bar{q}r$		
1		$p\bar{q}r$	pqr	$pq\bar{r}$

	0 0	0 1	1 1	1 0
0		$\bar{q}r$		
1			pq	

$$\bar{p}\bar{q}r + p\bar{q}r = (\bar{p} + p)\bar{q}r = \bar{q}r$$

$$pqr + pq\bar{r} = pq(r + \bar{r}) = pq$$

K-Map : Verification

	0 0	0 1	1 1	1 0
0		$\bar{q}r$		
1			pq	

$$H1 \wedge H2 \wedge H3 \equiv \bar{q}r + pq$$

p	q	r	$\bar{q}$	$\bar{q}r$	pq	$\bar{q}r + pq$
0	0	0	1	0	0	0
0	0	1	1	1	0	1
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	0	1	1	1	0	1
1	1	0	0	0	1	1
1	1	1	0	0	1	1

p	q	r	$H1 \wedge H2 \wedge H3$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Adding two don’t care conditions

p q r

p \ qr	0 0	0 1	1 1	1 0
0	0	1	X	X
1	0	1	1	1

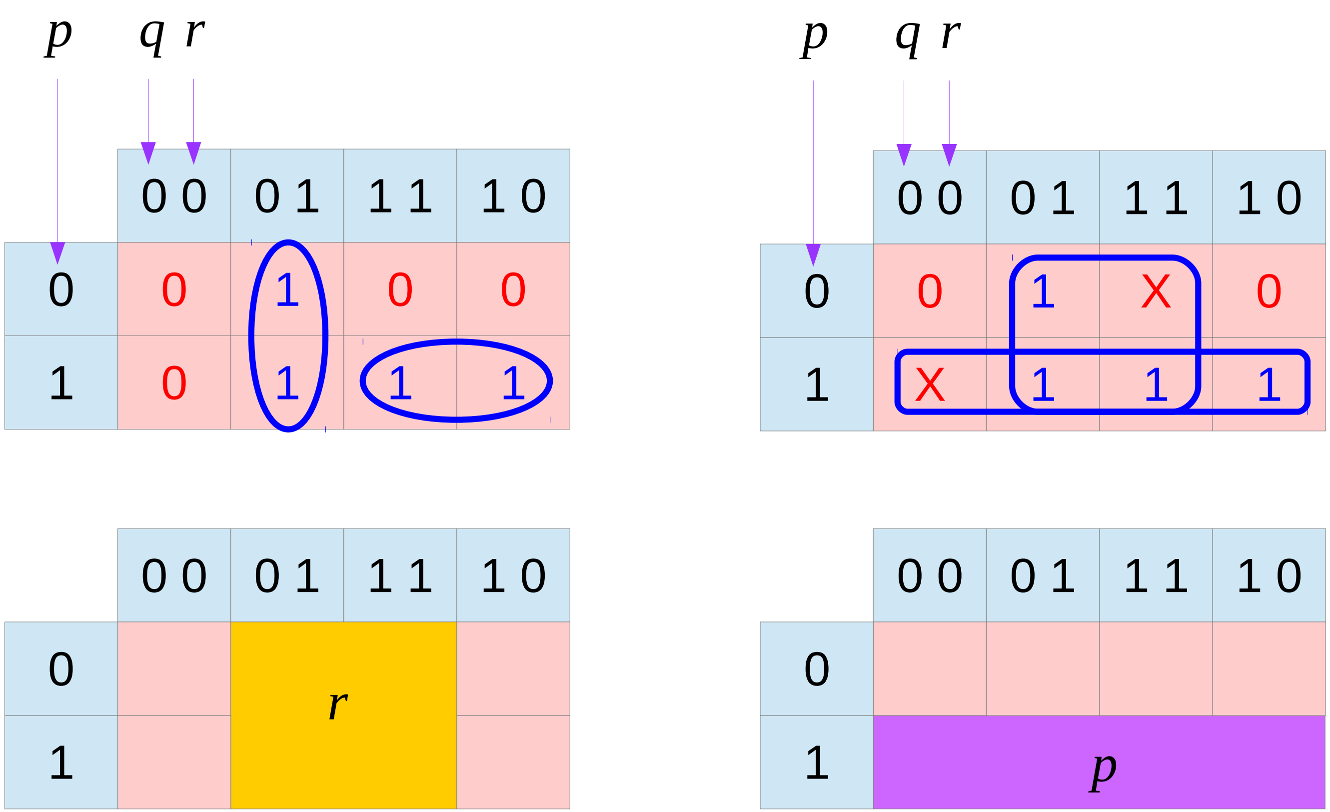
p \ qr	0 0	0 1	1 1	1 0
0	0	1	X	X
1	0	1	1	1

	0 0	0 1	1 1	1 0
0		r	r	
1		r	r	

	0 0	0 1	1 1	1 0
0			q	q
1			q	q

$q \vee r$

Adding two don't care conditions



$p \lor r$

K-Map : Verification

	00	01	11	10
0		r		
1				

	00	01	11	10
0				
1		p		

$H1 \wedge H2 \wedge H3 \rightarrow p \vee r$

p	q	r	$p \vee r$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

p	q	r	$H1 \wedge H2 \wedge H3$	$p \vee r$
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

References

[1] http://en.wikipedia.org/
[2]

Boolean Algebra (8A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice and Octave.

Argument

Boolean Algebra

In mathematics and mathematical logic, **Boolean algebra** is the branch of algebra in which the values of the variables are the truth values *true* and *false*, usually denoted 1 and 0 respectively. Instead of elementary algebra where the values of the variables are numbers, and the prime operations are addition and multiplication, the main operations of Boolean algebra are the conjunction *and* denoted as ∧, the disjunction *or* denoted as ∨, and the negation *not* denoted as ¬. It is thus a formalism for describing logical relations in the same way that ordinary algebra describes numeric relations.

https://en.wikipedia.org/wiki/Boolean_algebra

Operators

x	y	$x \wedge y$	$x \vee y$
0	**0**	0	0
1	**0**	0	1
0	**1**	0	1
1	**1**	1	1

x	$\neg x$
0	1
1	0

x	y	$x \rightarrow y$	$x \oplus y$	$x \equiv y$
0	**0**	1	0	1
1	**0**	0	1	0
0	**1**	1	1	0
1	**1**	1	0	1

https://en.wikipedia.org/wiki/Boolean_algebra

Laws (1)

Associativity of $\vee$:	$x \vee (y \vee z) = (x \vee y) \vee z$	x+(y+z) = (x+y)+z
Associativity of $\wedge$:	$x \wedge (y \wedge z) = (x \wedge y) \wedge z$	x(yz) = (xy)z
Commutativity of $\vee$:	$x \vee y = y \vee x$	x+y = y+x
Commutativity of $\wedge$:	$x \wedge y = y \wedge x$	xy = yz
Distributivity of $\wedge$ over $\vee$:	$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$	x(y+z) = xy + xz
Identity for $\vee$:	$x \vee 0 = x$	x+0=x
Identity for $\wedge$:	$x \wedge 1 = x$	x*1=x
Annihilator for $\wedge$:	$x \wedge 0 = 0$	x*0=0

https://en.wikipedia.org/wiki/Boolean_algebra

Laws (2)

Annihilator for ∨:	$x \vee 1 = 1$	x+1=1
Idempotence of ∨:	$x \vee x = x$	x+x=x
Idempotence of ∧:	$x \wedge x = x$	x*x=x
Absorption 1:	$x \wedge (x \vee y) = x$	x(x+y)=x
Absorption 2:	$x \vee (x \wedge y) = x$	x+xy=x
Distributivity of ∨ over ∧:	$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$	x+yz=(x+y)(x+z)

Complementation 1	$x \wedge \neg x = 0$	$x\,\overline{x} = 0$
Complementation 2	$x \vee \neg x = 1$	$x + \overline{x} = 1$

De Morgan 1	$\neg x \wedge \neg y = \neg(x \vee y)$	$\overline{x}\,\overline{y} = \overline{(x+y)}$
De Morgan 2	$\neg x \vee \neg y = \neg(x \wedge y)$	$\overline{x}+\overline{y} = \overline{(x\,y)}$

https://en.wikipedia.org/wiki/Boolean_algebra

Digital Logic Gates

Digital logic is the application of the Boolean algebra of 0 and 1 to electronic hardware consisting of logic gates connected to form a circuit diagram. Each gate implements a Boolean operation, and is depicted schematically by a shape indicating the operation. The shapes associated with the gates for conjunction (AND-gates), disjunction (OR-gates), and complement (inverters) are as follows.[17]

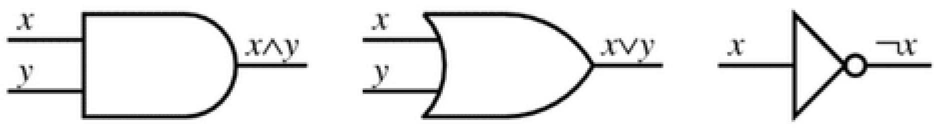
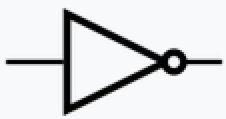
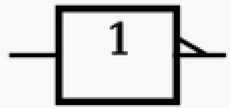


Figure 3. Logic gates

The lines on the left of each gate represent input wires or *ports*. The value of the input is represented by a voltage on the lead. For so-called "active-high" logic, 0 is represented by a voltage close to zero or "ground", while 1 is represented by a voltage close to the supply voltage; active-low reverses this. The line on the right of each gate represents the output port, which normally follows the same voltage conventions as the input ports.

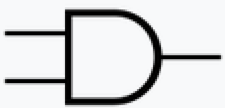
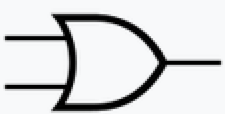
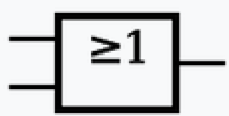
https://en.wikipedia.org/wiki/Boolean_algebra

Negation				
NOT			$\overline{A}$ or $\sim A$	see truth table below

INPUT	**OUTPUT**
A	NOT A
0	1
1	0

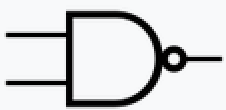
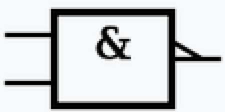
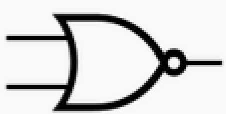
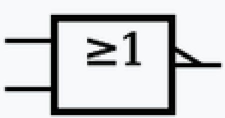
https://en.wikipedia.org/wiki/Logic_gate

AND, OR Gates

<table>
<tr><th colspan="5">Conjunction and Disjunction</th></tr>
<tr>
<td>AND</td>
<td></td>
<td>
</td>
<td>$A \cdot B$</td>
<td>
<table>
<tr><th colspan="2">INPUT</th><th>OUTPUT</th></tr>
<tr><td>A</td><td>B</td><td>A AND B</td></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
</table>
</td>
</tr>
<tr>
<td>OR</td>
<td></td>
<td>
</td>
<td>$A + B$</td>
<td>
<table>
<tr><th colspan="2">INPUT</th><th>OUTPUT</th></tr>
<tr><td>A</td><td>B</td><td>A OR B</td></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
</table>
</td>
</tr>
</table>

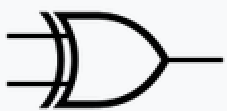
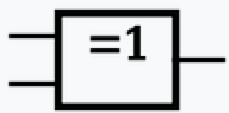
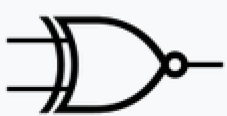
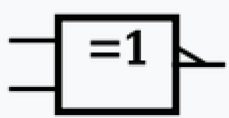
https://en.wikipedia.org/wiki/Logic_gate

NAND, NOR Gates

<table>
<tr><th colspan="5">Alternative denial and Joint denial</th></tr>
<tr><td>NAND</td><td></td><td></td><td>$\overline{A \cdot B}$ or $A \uparrow B$</td><td>
<table>
<tr><th colspan="2">INPUT</th><th>OUTPUT</th></tr>
<tr><td>A</td><td>B</td><td>A NAND B</td></tr>
<tr><td>0</td><td>0</td><td>1</td></tr>
<tr><td>0</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td></tr>
<tr><td>1</td><td>1</td><td>0</td></tr>
</table>
</td></tr>
<tr><td>NOR</td><td></td><td></td><td>$\overline{A + B}$ or $A \downarrow B$</td><td>
<table>
<tr><th colspan="2">INPUT</th><th>OUTPUT</th></tr>
<tr><td>A</td><td>B</td><td>A NOR B</td></tr>
<tr><td>0</td><td>0</td><td>1</td></tr>
<tr><td>0</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td><td>0</td></tr>
</table>
</td></tr>
</table>

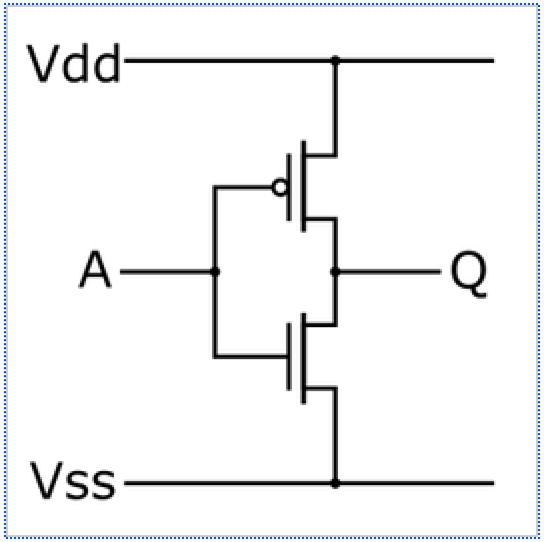
https://en.wikipedia.org/wiki/Logic_gate

XOR, XNOR Gates

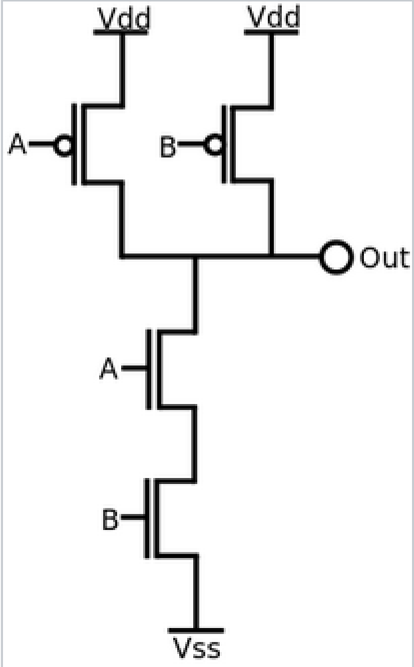
<table>
<tr><th colspan="5">Exclusive or and Biconditional</th></tr>
<tr>
<td>XOR</td>
<td></td>
<td>
</td>
<td>$A \oplus B$</td>
<td>
<table>
<tr><th colspan="2">INPUT</th><th>OUTPUT</th></tr>
<tr><td>A</td><td>B</td><td>A XOR B</td></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td></tr>
<tr><td>1</td><td>1</td><td>0</td></tr>
</table>
</td>
</tr>
<tr>
<td>XNOR</td>
<td></td>
<td>
</td>
<td>$\overline{A \oplus B}$ or $A \odot B$</td>
<td>
<table>
<tr><th colspan="2">INPUT</th><th>OUTPUT</th></tr>
<tr><td>A</td><td>B</td><td>A XNOR B</td></tr>
<tr><td>0</td><td>0</td><td>1</td></tr>
<tr><td>0</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
</table>
</td>
</tr>
</table>

https://en.wikipedia.org/wiki/Logic_gate

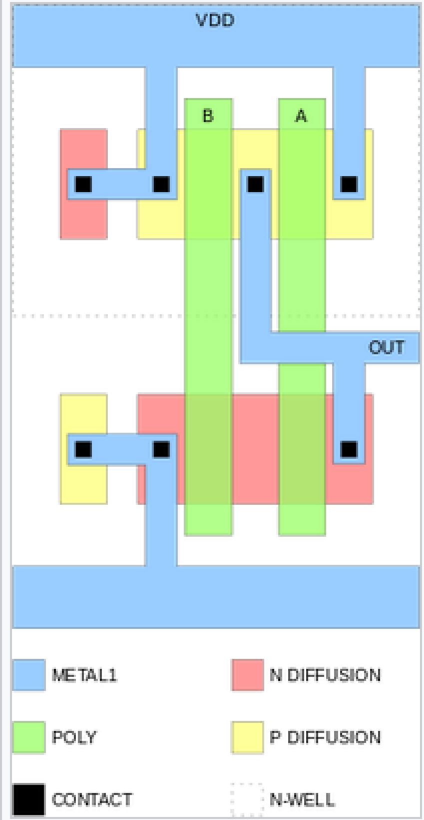
CMOS Logic Gates



CMOS inverter (NOT logic gate)

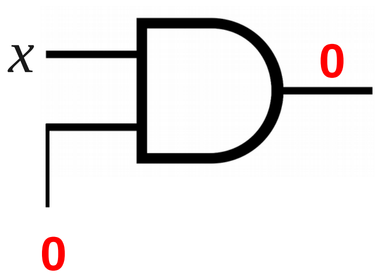


NAND gate in CMOS logic

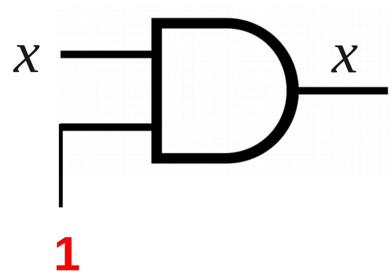


https://en.wikipedia.org/wiki/CMOS

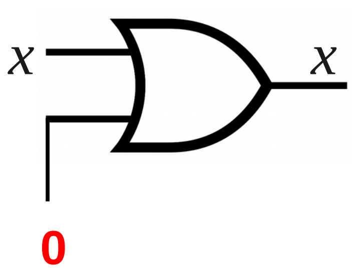
Identity and Null Element Theorem



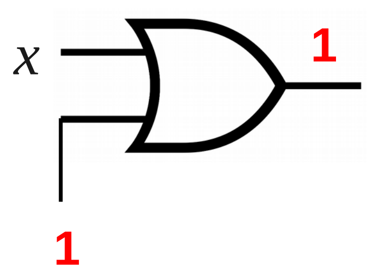
$$x \cdot 0 = 0$$



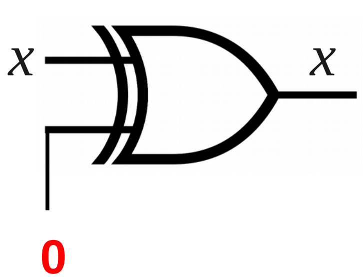
$$x \cdot 1 = x$$



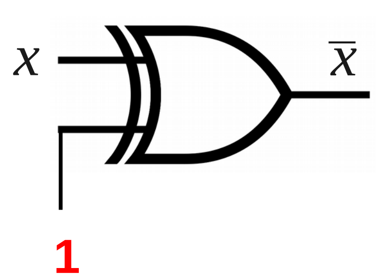
$$x + 0 = x$$



$$x + 1 = 1$$



$$x \oplus 0 = x$$



$$x \oplus 1 = \bar{x}$$

https://en.wikiversity.org/wiki/Discrete_Mathematics_in_plain_view#Algorithms

Distributive

$$x \cdot (y + z) = x \cdot y + x \cdot z \qquad \neq x \cdot y + z$$

This parenthesis cannot be deleted

$$x + (y \cdot z) = (x + y) \cdot (x + z) \qquad = x + y \cdot z$$

This parenthesis can be deleted

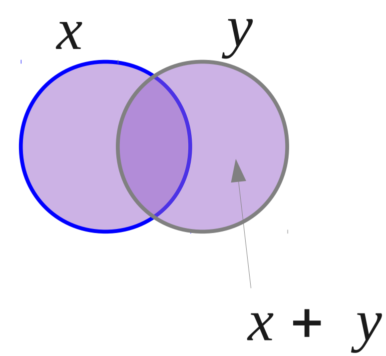
Operator precedence : $\cdot > +$

https://en.wikiversity.org/wiki/Discrete_Mathematics_in_plain_view#Algorithms

Inclusion

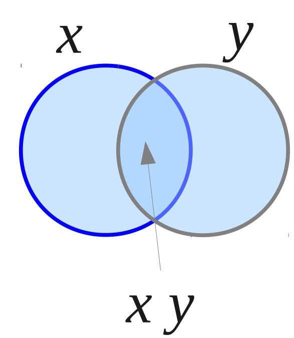
$$x \cdot (x + y) = x$$

$$\begin{aligned} x \cdot (x + y) &= x \cdot x + x \cdot y \\ &= x + x \cdot y \\ &= x \cdot (1 + y) \\ &= x \end{aligned}$$



$$x + xy = x$$

$$\begin{aligned} x + xy &= x \cdot 1 + x \cdot y \\ &= x \cdot (1 + y) \\ &= x \end{aligned}$$

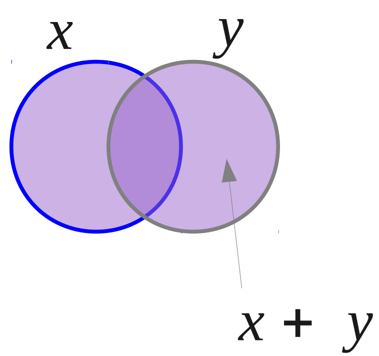


https://en.wikiversity.org/wiki/Discrete_Mathematics_in_plain_view#Algorithms

Inclusion

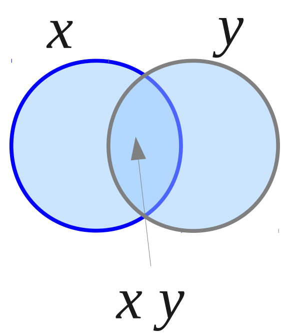
$$x \cdot (x + y) = x$$

$$\begin{aligned} x \cdot (x + y) &= x \cdot x + x \cdot y \\ &= x + x \cdot y \\ &= x \cdot (1 + y) \\ &= x \end{aligned}$$



$$x + xy = x$$

$$\begin{aligned} x + xy &= x \cdot 1 + x \cdot y \\ &= x \cdot (1 + y) \\ &= x \end{aligned}$$



https://en.wikiversity.org/wiki/Discrete_Mathematics_in_plain_view#Algorithms

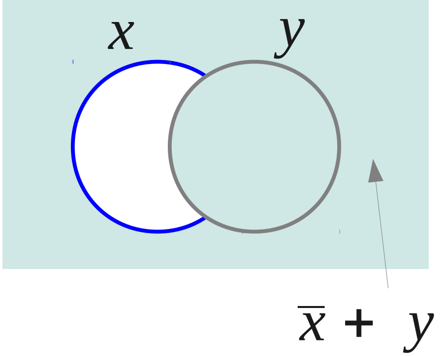
Eliminate

$$x \cdot (\overline{x} + y) = x\,y$$

$$x \cdot (\overline{x} + y) = x \cdot \overline{x} + x \cdot y$$

$$= 0 + x \cdot y$$

$$= x \cdot y$$

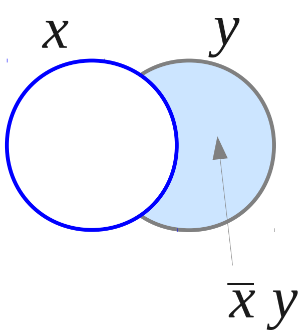


$$x + \overline{x}\,y = x + y$$

$$x + \overline{x}\,y = (x + \overline{x}) \cdot (x + y)$$

$$= 1 \cdot (x + y)$$

$$= x + y$$



https://en.wikiversity.org/wiki/Discrete_Mathematics_in_plain_view#Algorithms

References

[1] http://en.wikipedia.org/
[2]

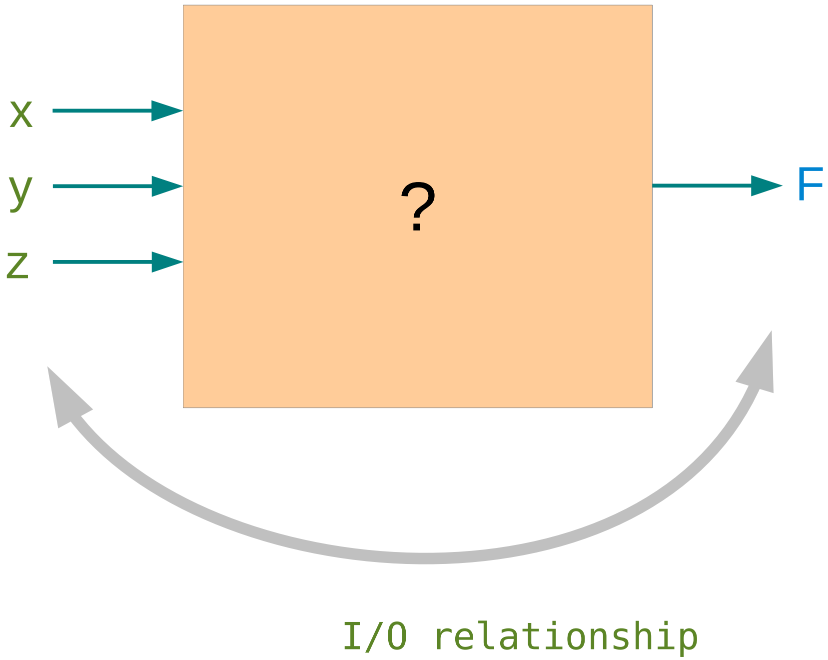
Boolean Functions (8B)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice and Octave.

Truth Table

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

inputs output

All possible input cases

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

x	y	z	
0	0	0	the case when x=0 **and** y=0 **and** z=0
0	0	1	the case when x=0 **and** y=0 **and** z=1
0	1	0	the case when x=0 **and** y=1 **and** z=0
0	1	1	the case when x=0 **and** y=1 **and** z=1
1	0	0	the case when x=1 **and** y=0 **and** z=0
1	0	1	the case when x=1 **and** y=0 **and** z=1
1	1	0	the case when x=1 **and** y=1 **and** z=0
1	1	1	the case when x=1 **and** y=1 **and** z=1

inputs

All possible
combination of inputs

All possible input cases using **minterms**

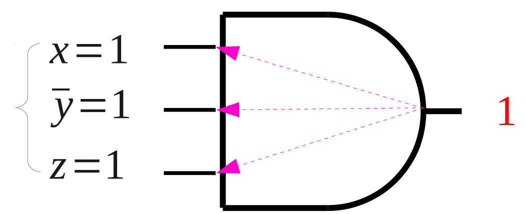
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

x	y	z		
0	0	0	when	$\bar{x}\,\bar{y}\,\bar{z} = 1$
0	0	1	when	$\bar{x}\,\bar{y}\,z = 1$
0	1	0	when	$\bar{x}\,y\,\bar{z} = 1$
0	1	1	when	$\bar{x}\,y\,z = 1$
1	0	0	when	$x\,\bar{y}\,\bar{z} = 1$
1	0	1	when	$x\,\bar{y}\,z = 1$
1	1	0	when	$x\,y\,\bar{z} = 1$
1	1	1	when	$x\,y\,z = 1$

inputs

All possible
combination of inputs

$x\,\bar{y}\,z = 1 \iff$



$x\,\bar{y}\,z = 1 \iff x=1,\ y=0,\ z=1$

For the output of an **and** gate to be 1,
all inputs must be 1

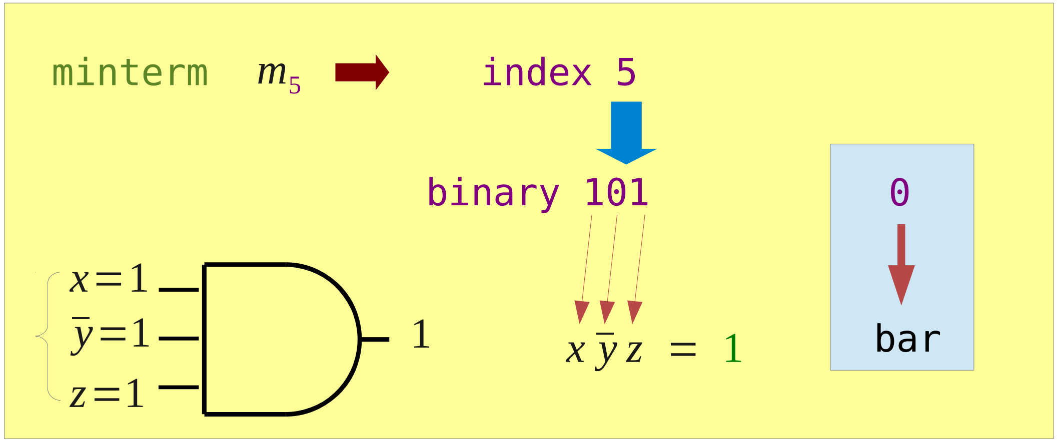
Naming **minterms**

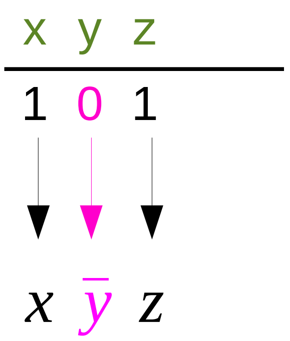
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z		
0	0	0	0	the minterm	$m_0 = \bar{x}\,\bar{y}\,\bar{z} = 1$
1	0	0	1	the minterm	$m_1 = \bar{x}\,\bar{y}\,z = 1$
2	0	1	0	the minterm	$m_2 = \bar{x}\,y\,\bar{z} = 1$
3	0	1	1	the minterm	$m_3 = \bar{x}\,y\,z = 1$
4	1	0	0	the minterm	$m_4 = x\,\bar{y}\,\bar{z} = 1$
➡ 5	1	0	1	the minterm	$m_5 = x\,\bar{y}\,z = 1$
6	1	1	0	the minterm	$m_6 = x\,y\,\bar{z} = 1$
7	1	1	1	the minterm	$m_7 = x\,y\,z = 1$

Computing **minterms**

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design





$$m_5 = x\ \bar{y}\ z$$

Truth Table and **minterms** (1)

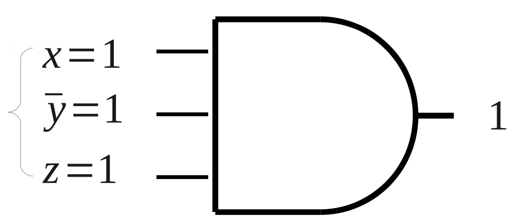
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

x	y	z		
0	0	0	the case when x=0 **and** y=0 **and** z=0 ⟺	$\bar{x}\,\bar{y}\,\bar{z} = 1$
0	0	1	the case when x=0 **and** y=0 **and** z=1 ⟺	$\bar{x}\,\bar{y}\,z = 1$
0	1	0	the case when x=0 **and** y=1 **and** z=0 ⟺	$\bar{x}\,y\,\bar{z} = 1$
0	1	1	the case when x=0 **and** y=1 **and** z=1 ⟺	$\bar{x}\,y\,z = 1$
1	0	0	the case when x=1 **and** y=0 **and** z=0 ⟺	$x\,\bar{y}\,\bar{z} = 1$
1	0	1	the case when x=1 **and** y=0 **and** z=1 ⟺	$x\,\bar{y}\,z = 1$
1	1	0	the case when x=1 **and** y=1 **and** z=0 ⟺	$x\,y\,\bar{z} = 1$
1	1	1	the case when x=1 **and** y=1 **and** z=1 ⟺	$x\,y\,z = 1$

inputs

All possible combination of inputs

$x\,\bar{y}\,z = 1$ ⟺



⟺ $\{x=1,\ y=0,\ z=1\}$

For the output of an **and** gate to be 1, all inputs must be 1

Truth Table and **minterms** (2)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z		minterm
0	0	0	0	the case when the minterm	$m_0 = \bar{x}\,\bar{y}\,\bar{z} = 1$
1	0	0	1	the case when the minterm	$m_1 = \bar{x}\,\bar{y}\,z = 1$
2	0	1	0	the case when the minterm	$m_2 = \bar{x}\,y\,\bar{z} = 1$
3	0	1	1	the case when the minterm	$m_3 = \bar{x}\,y\,z = 1$
4	1	0	0	the case when the minterm	$m_4 = x\,\bar{y}\,\bar{z} = 1$
➡ 5	1	0	1	the case when the minterm	$m_5 = x\,\bar{y}\,z = 1$
6	1	1	0	the case when the minterm	$m_6 = x\,y\,\bar{z} = 1$
7	1	1	1	the case when the minterm	$m_7 = x\,y\,z = 1$

index

inputs

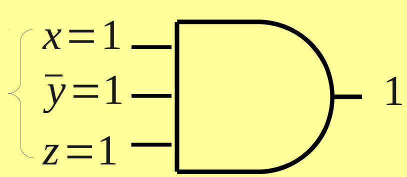
All possible
combination of inputs

minterm m_5 ➡ index 5

binary 101

$x\,\bar{y}\,z = 1$

0 → bar



Truth Table and **MAXterms** (1)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

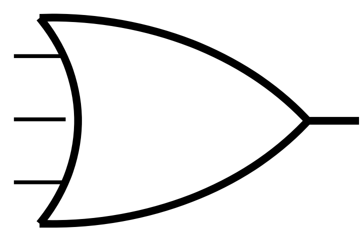
x	y	z		
0	0	0	the case when x=0 **and** y=0 **and** z=0 ⟺	$x + y + z = 0$
0	0	1	the case when x=0 **and** y=0 **and** z=1 ⟺	$x + y + \overline{z} = 0$
0	1	0	the case when x=0 **and** y=1 **and** z=0 ⟺	$x + \overline{y} + z = 0$
0	1	1	the case when x=0 **and** y=1 **and** z=1 ⟺	$x + \overline{y} + \overline{z} = 0$
1	0	0	the case when x=1 **and** y=0 **and** z=0 ⟺	$\overline{x} + y + z = 0$
1	0	1	the case when x=1 **and** y=0 **and** z=1 ⟺	$\overline{x} + y + \overline{z} = 0$
1	1	0	the case when x=1 **and** y=1 **and** z=0 ⟺	$\overline{x} + \overline{y} + z = 0$
1	1	1	the case when x=1 **and** y=1 **and** z=1 ⟺	$\overline{x} + \overline{y} + \overline{z} = 0$

inputs

All possible combination of inputs

$\overline{x} + y + \overline{z} = 0$ ⟺

$\overline{x}=0$, $y=0$, $\overline{z}=0$



0 ⟺ $x=1$, $y=0$, $z=1$

For the output of an **or** gate to be 0,
all inputs must be 0

Truth Table and **MAXterms** (2)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z		MAXterm
0	0	0	0	the case when the MAXterm	$M_0 = x + y + z = 0$
1	0	0	1	the case when the MAXterm	$M_1 = x + y + \bar{z} = 0$
2	0	1	0	the case when the MAXterm	$M_2 = x + \bar{y} + z = 0$
3	0	1	1	the case when the MAXterm	$M_3 = x + \bar{y} + \bar{z} = 0$
4	1	0	0	the case when the MAXterm	$M_4 = \bar{x} + y + z = 0$
5	1	0	1	the case when the MAXterm	$M_5 = \bar{x} + y + \bar{z} = 0$
6	1	1	0	the case when the MAXterm	$M_6 = \bar{x} + \bar{y} + z = 0$
7	1	1	1	the case when the MAXterm	$M_7 = \bar{x} + \bar{y} + \bar{z} = 0$

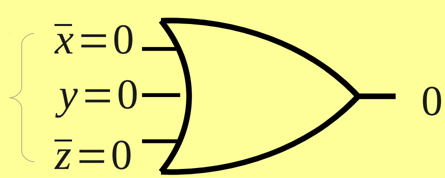
index

inputs

All possible
combination of inputs

minterm M_5 ➡ index 5

binary 101

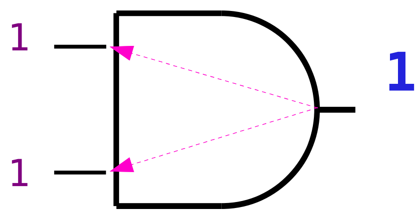


$\bar{x} + y + \bar{z}$

1 → bar

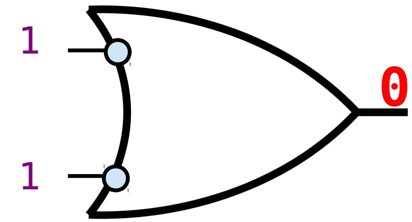
Maxterm and **minterm** Conditions

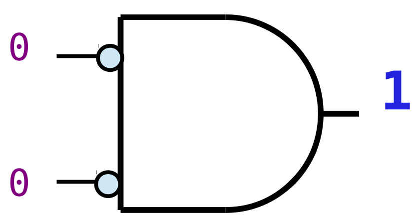
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



x	y	xy
0	0	0
0	1	0
1	0	0
1	1	1

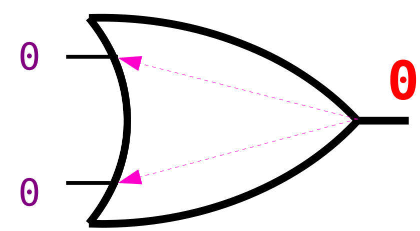
$\bar{x}$	$\bar{y}$	$\bar{x}+\bar{y}$
1	1	1
1	0	1
0	1	1
0	0	0





$\bar{x}$	$\bar{y}$	$\bar{x}\bar{y}$
1	1	1
1	0	0
0	1	0
0	0	0

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1



Boolean functions defined by a truth table

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index — inputs (x y z) — output (F)

All possible
combination of inputs

When the output becomes 1

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

inputs (x, y, z) output (F)

All possible combination of inputs

The output F becomes 1,
for one of the three following cases

(the case when x=0 **and** y=0 **and** z=1)

or (the case when x=0 **and** y=1 **and** z=1)

or (the case when x=1 **and** y=0 **and** z=0)

Function output values and **minterms**

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index

inputs output

All possible
combination of inputs

The output F becomes 1,
for one of the three following cases

⬌ $m_1 = \bar{x}\,\bar{y}\,z = 1$

⬌ $m_3 = \bar{x}\,y\,z = 1$

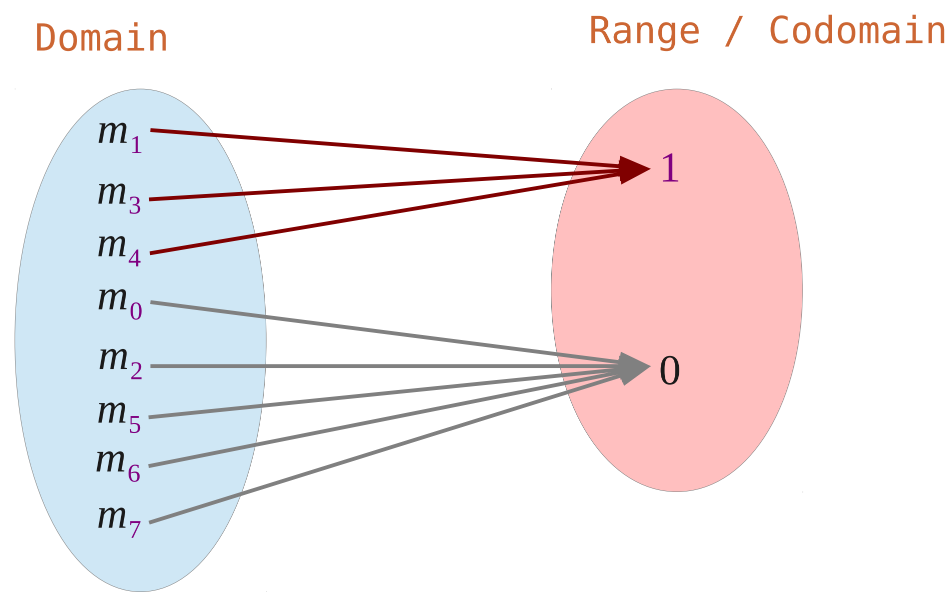
⬌ $m_4 = x\,\bar{y}\,\bar{z} = 1$

Mapping Set Diagram

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index inputs output



Boolean function definition using **minterms**

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

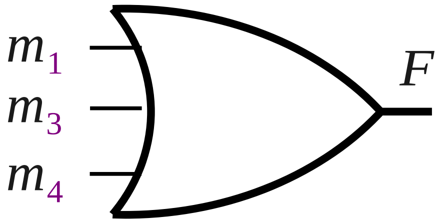
index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index — inputs — output

All possible
combination of inputs

The output F becomes 1,
either $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

$m_1 + m_3 + m_4 = 1$ ⇄ $F = 1$



For the output of an **or** gate to be 1,
at least one must be 1

⟺ $F = m_1 + m_3 + m_4$

Boolean Function with **minterms** (1)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index — inputs (x y z) — output (F)

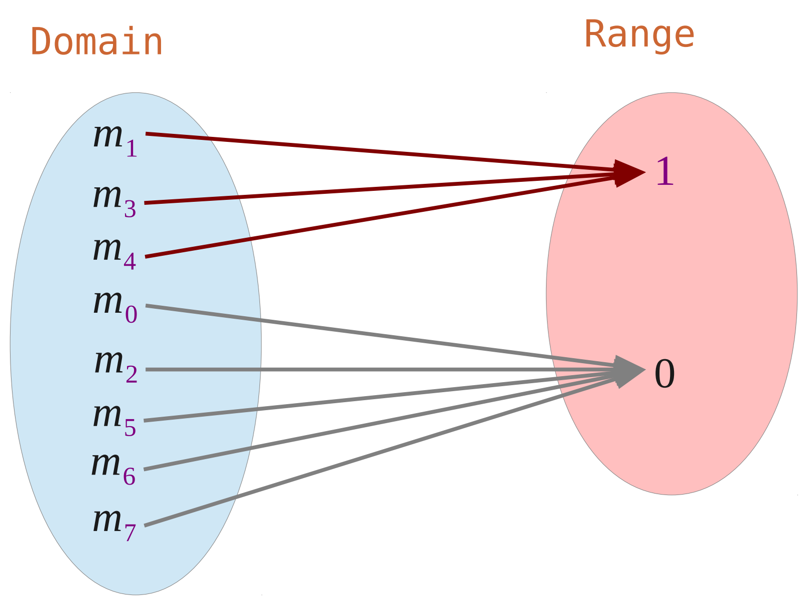
All possible combination of inputs

The output F becomes 1,
for one of the three following cases

(the case when x=0 **and** y=0 **and** z=1) ⟺ $m_1 = \bar{x}\,\bar{y}\,z = 1$

or (the case when x=0 **and** y=1 **and** z=1) ⟺ $m_3 = \bar{x}\,y\,z = 1$

or (the case when x=1 **and** y=0 **and** z=0) ⟺ $m_4 = x\,\bar{y}\,\bar{z} = 1$



Boolean Function with **minterms** (2)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

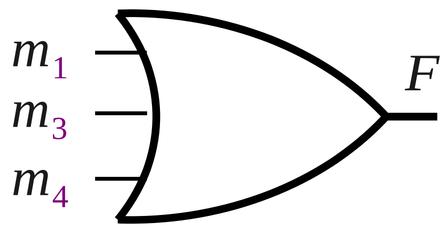
index — inputs — output

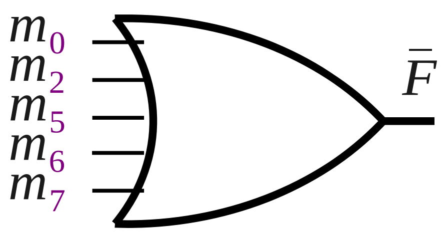
All possible combination of inputs

The output F becomes 1,
either $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

$m_1 + m_3 + m_4 = 1 \rightleftarrows F = 1$

$\Leftrightarrow \quad F = m_1 + m_3 + m_4$





For the output of an **or** gate to be 1,
at least one must be 1

Boolean Function with **minterms** (3)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

inputs output

All possible combination of inputs

The output F becomes 1,

either $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

$m_1 + m_3 + m_4 = 1 \rightleftarrows F = 1$

$\Leftrightarrow \quad F = m_1 + m_3 + m_4$

The output F becomes 0,

either $m_0 = 1$ or $m_2 = 1$ or $m_5 = 1$ or $m_6 = 1$ or $m_7 = 1$

$m_0 + m_2 + m_5 + m_6 + m_7 = 1 \rightleftarrows F = 0$

$\Leftrightarrow \quad \overline{F} = m_0 + m_2 + m_5 + m_6 + m_7$

For the output of an **or** gate to be 1,
at least one must be 1

Boolean Function with **Maxterms** (1)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

The output F becomes 0,
for one of the five following cases

index	x	y	z	F
➡ 0	0	0	0	0
1	0	0	1	1
➡ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
➡ 5	1	0	1	0
➡ 6	1	1	0	0
➡ 7	1	1	1	0

index — inputs — output

(the case when x=0 **and** y=0 **and** z=0) ⟺ $x+y+z = 0$

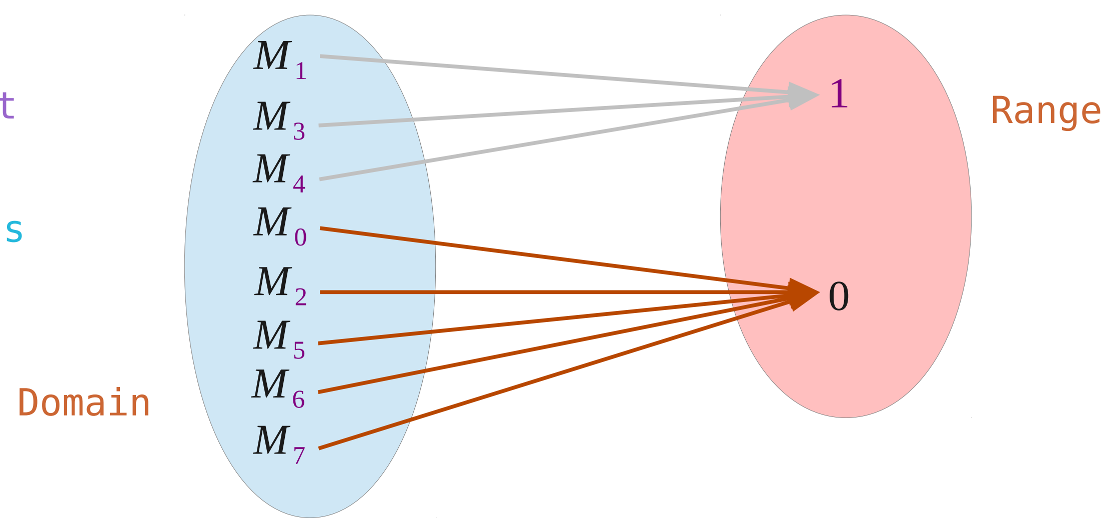
or (the case when x=0 **and** y=1 **and** z=0) ⟺ $x+\bar{y}+z = 0$

or (the case when x=1 **and** y=0 **and** z=1) ⟺ $\bar{x}+y+\bar{z} = 0$

or (the case when x=1 **and** y=1 **and** z=0) ⟺ $\bar{x}+\bar{y}+z = 0$

or (the case when x=1 **and** y=1 **and** z=1) ⟺ $\bar{x}+\bar{y}+\bar{z} = 0$

All possible
combination of inputs



Boolean Function with **Maxterms** (2)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z	F
➡ 0	0	0	0	0
1	0	0	1	1
➡ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
➡ 5	1	0	1	0
➡ 6	1	1	0	0
➡ 7	1	1	1	0

index — inputs — output

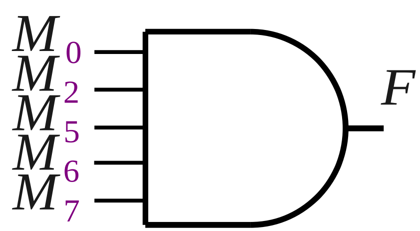
All possible combination of inputs

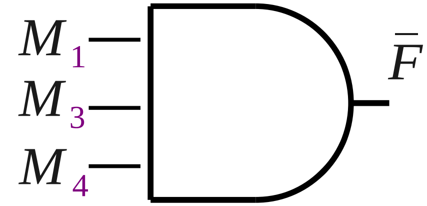
The output F becomes 0,

either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0 \rightleftarrows F = 0$$

$$\Leftrightarrow \quad F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$





For the output of an **and** gate to be 0,
at least one input must be 0

Boolean Function with **Maxterms** (2)

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

index	x	y	z	F
➡ 0	0	0	0	0
1	0	0	1	1
➡ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
➡ 5	1	0	1	0
➡ 6	1	1	0	0
➡ 7	1	1	1	0

index — inputs — output

All possible combination of inputs

The output F becomes 0,

either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0 \rightleftarrows F = 0$$

$$\Leftrightarrow \quad F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$

The output F becomes 1,

either $M_1=0$ or $M_3=0$ or $M_4=0$

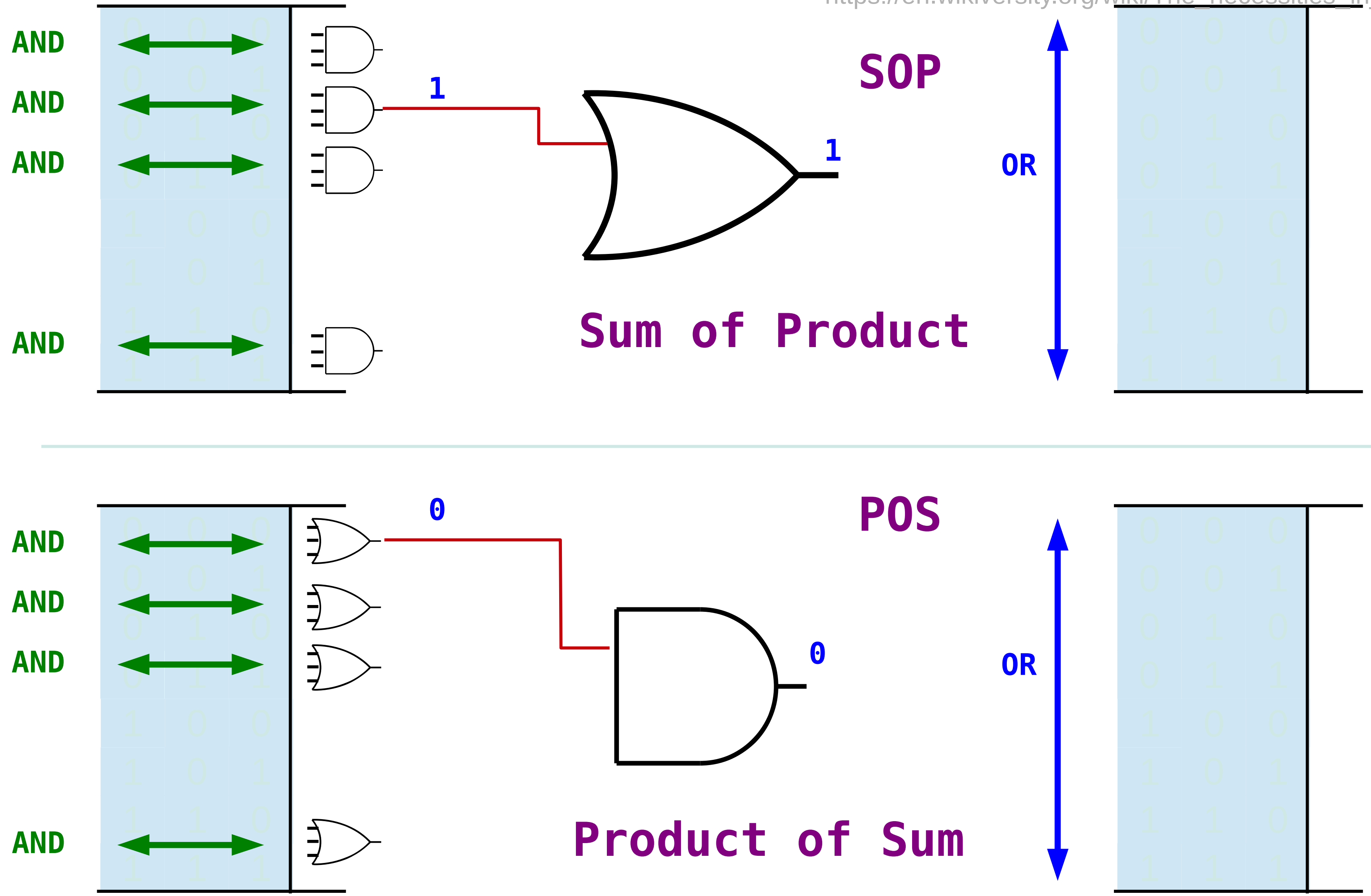
$$M_1 \cdot M_3 \cdot M_4 = 0 \rightleftarrows F = 1$$

$$\Leftrightarrow \quad \overline{F} = M_1 \cdot M_3 \cdot M_4$$

For the output of an **and** gate to be 0,
at least one input must be 0

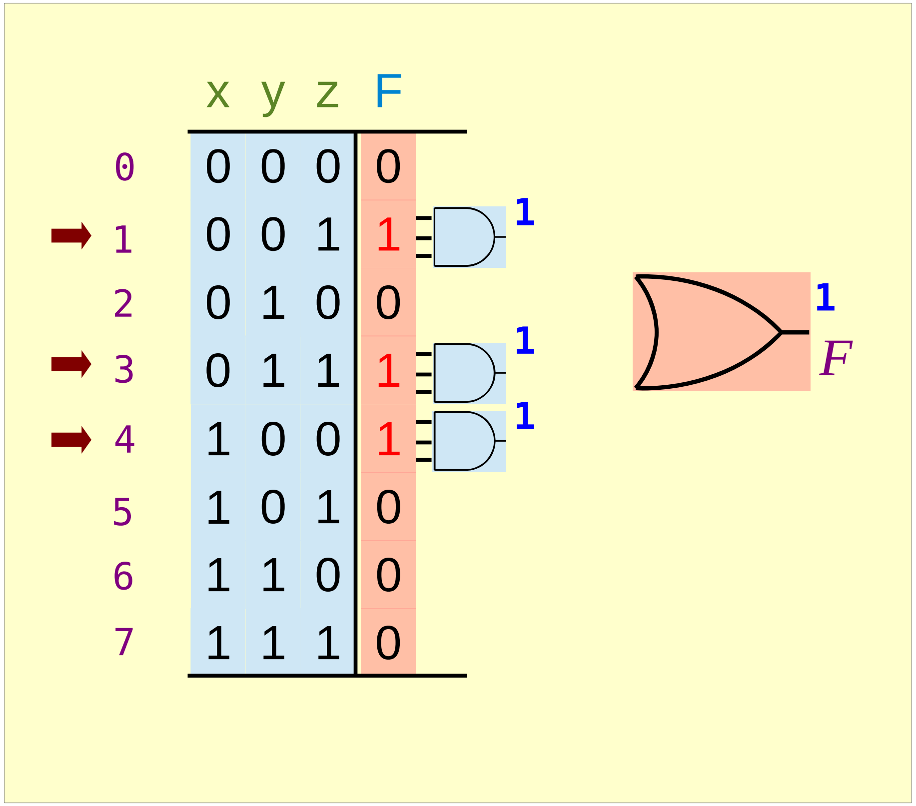
SOP and POS

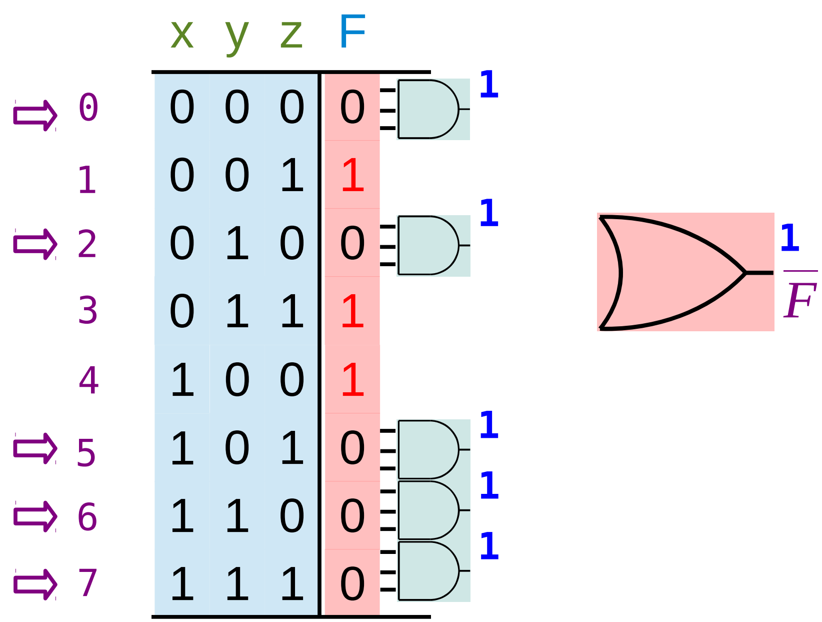
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



Boolean Function with **minterms**

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

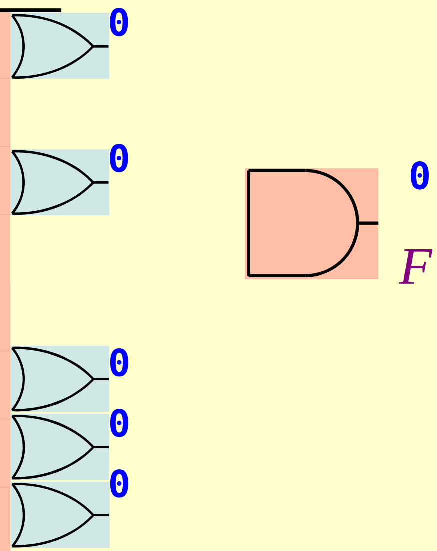




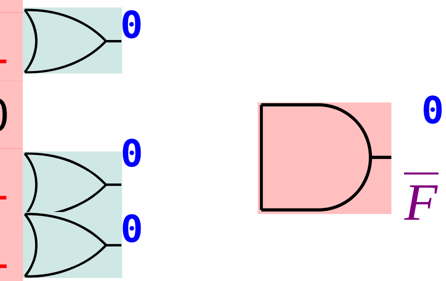
Boolean Function with **Maxterms**

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

		x	y	z	F
⇨	0	0	0	0	0
	1	0	0	1	1
⇨	2	0	1	0	0
	3	0	1	1	1
	4	1	0	0	1
⇨	5	1	0	1	0
⇨	6	1	1	0	0
⇨	7	1	1	1	0



		x	y	z	F
	0	0	0	0	0
➡	1	0	0	1	1
	2	0	1	0	0
➡	3	0	1	1	1
➡	4	1	0	0	1
	5	1	0	1	0
	6	1	1	0	0
	7	1	1	1	0



References

[1] http://en.wikipedia.org/
[2]

K-Map (8C)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

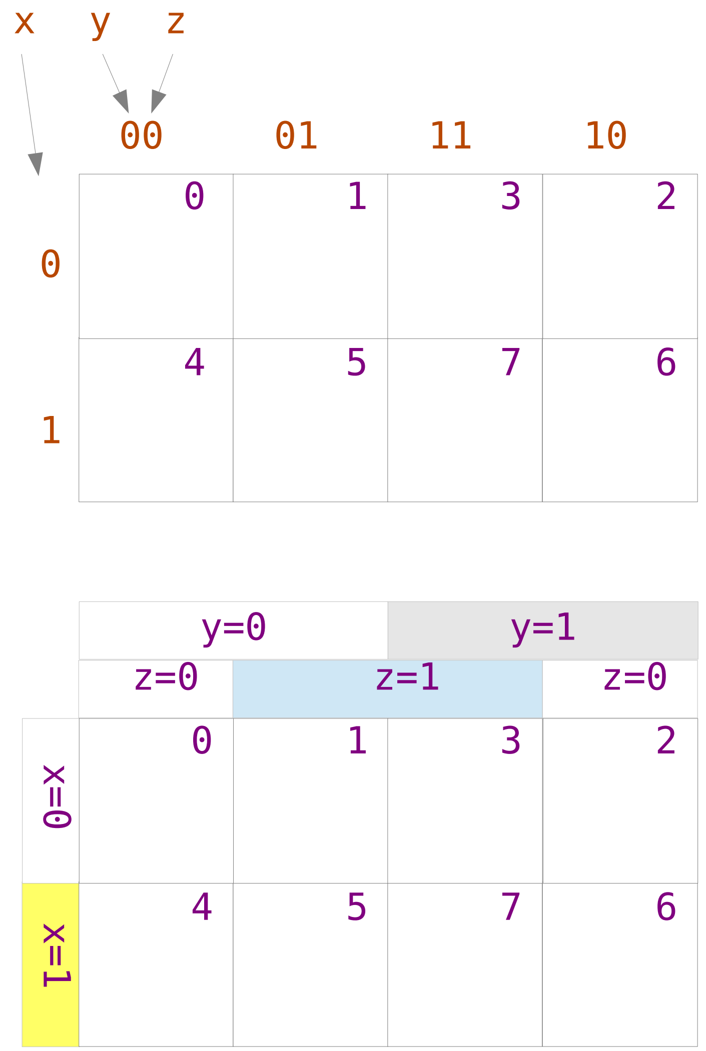
This document was produced by using LibreOffice and Octave.

Boolean Function with Maxterms

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

K-Map 3 variables (1)

index				minterms
0	0	0	0	$\overline{x}\,\overline{y}\,\overline{z}$
1	0	0	1	$\overline{x}\,\overline{y}\,z$
2	0	1	0	$\overline{x}\,y\,\overline{z}$
3	0	1	1	$\overline{x}\,y\,z$
4	1	0	0	$x\,\overline{y}\,\overline{z}$
5	1	0	1	$x\,\overline{y}\,z$
6	1	1	0	$x\,y\,\overline{z}$
7	1	1	1	$x\,y\,z$

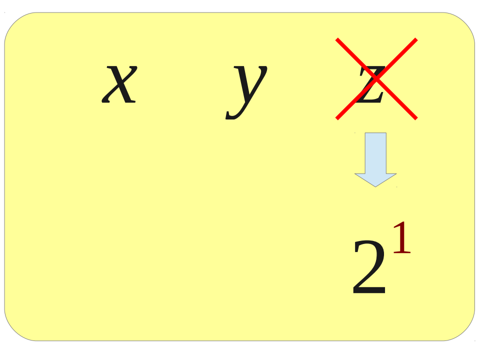


K-Map 3 variables (2)

index				minterms	
0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$	$\bar{x}\,\bar{y}$
1	0	0	1	$\bar{x}\,\bar{y}\,z$	
2	0	1	0	$\bar{x}\,y\,\bar{z}$	$\bar{x}\,y$
3	0	1	1	$\bar{x}\,y\,z$	
4	1	0	0	$x\,\bar{y}\,\bar{z}$	$x\,\bar{y}$
5	1	0	1	$x\,\bar{y}\,z$	
6	1	1	0	$x\,y\,\bar{z}$	$x\,y$
7	1	1	1	$x\,y\,z$	

$$\bar{x}\,\bar{y}\,\bar{z} + \bar{x}\,\bar{y}\,z = \bar{x}\,\bar{y}(\bar{z} + z) = \bar{x}\,\bar{y}$$

a group of 2 minterms



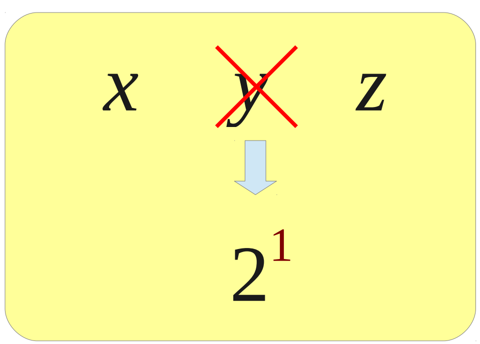
		y=0		y=1	
		z=0	z=1	z=1	z=0
		00	01	11	10
x=0	0	0 $\bar{x}\,\bar{y}$	1	3 $\bar{x}\,y$	2
x=1	1	4 $x\,\bar{y}$	5	7 $x\,y$	6

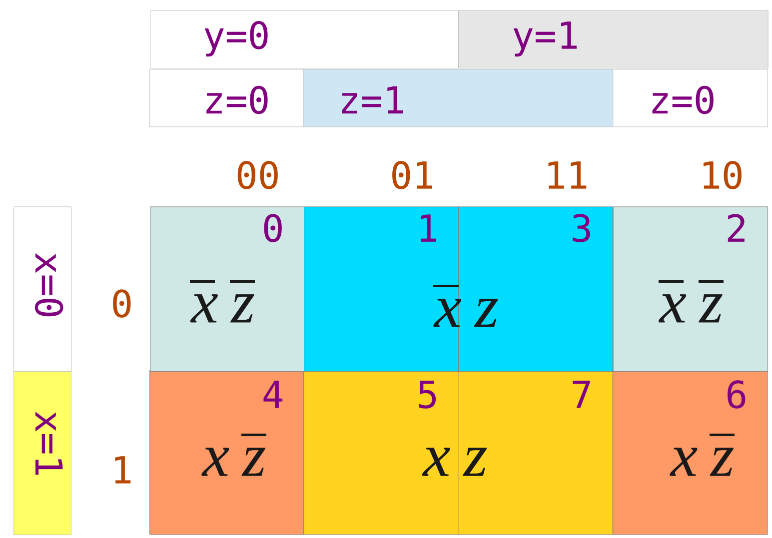
K-Map 3 variables (3)

index				minterms	
0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$	$\bar{x}\,\bar{z}$
1	0	0	1	$\bar{x}\,\bar{y}\,z$	$\bar{x}\,z$
2	0	1	0	$\bar{x}\,y\,\bar{z}$	
3	0	1	1	$\bar{x}\,y\,z$	
4	1	0	0	$x\,\bar{y}\,\bar{z}$	$x\,\bar{z}$
5	1	0	1	$x\,\bar{y}\,z$	$x\,z$
6	1	1	0	$x\,y\,\bar{z}$	
7	1	1	1	$x\,y\,z$	

$$\bar{x}\,\bar{y}\,\bar{z} + \bar{x}\,y\,\bar{z} = \bar{x}\,\bar{z}(\bar{y}+y) = \bar{x}\,\bar{z}$$

a group of 2 minterms



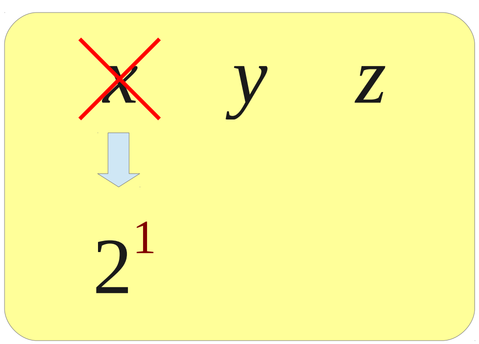


K-Map 3 variables (4)

index				minterms	
0	0	0	0	$\bar{x}\bar{y}\bar{z}$	$\bar{y}\bar{z}$
1	0	0	1	$\bar{x}\bar{y}z$	$\bar{y}z$
2	0	1	0	$\bar{x}y\bar{z}$	$y\bar{z}$
3	0	1	1	$\bar{x}yz$	yz
4	1	0	0	$x\bar{y}\bar{z}$	
5	1	0	1	$x\bar{y}z$	
6	1	1	0	$xy\bar{z}$	
7	1	1	1	xyz	

$$\bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z} = \bar{y}\bar{z}(\bar{x}+x) = \bar{y}\bar{z}$$

a group of 2 minterms

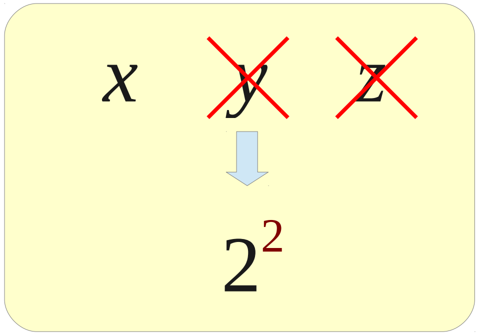


		y=0		y=1	
		z=0	z=1		z=0
		00	01	11	10
x=0	0	0 $\bar{y}\bar{z}$	1 $\bar{y}z$	3 yz	2 $y\bar{z}$
x=1	1	4	5	7	6

K-Map 3 variables (5)

index				minterms	
0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$	$\bar{x}$
1	0	0	1	$\bar{x}\,\bar{y}\,z$	
2	0	1	0	$\bar{x}\,y\,\bar{z}$	
3	0	1	1	$\bar{x}\,y\,z$	
4	1	0	0	$x\,\bar{y}\,\bar{z}$	x
5	1	0	1	$x\,\bar{y}\,z$	
6	1	1	0	$x\,y\,\bar{z}$	
7	1	1	1	$x\,y\,z$	

a group of 4 minterms



		y=0		y=1	
		z=0	z=1		z=0
		00	01	11	10
x=0	0	0	1	3 $\bar{x}$	2
x=1	1	4	5	7 x	6

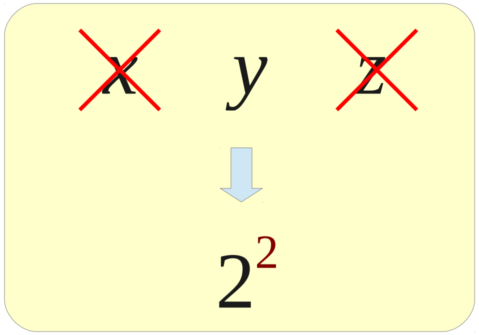
K-Map 3 variables (5)

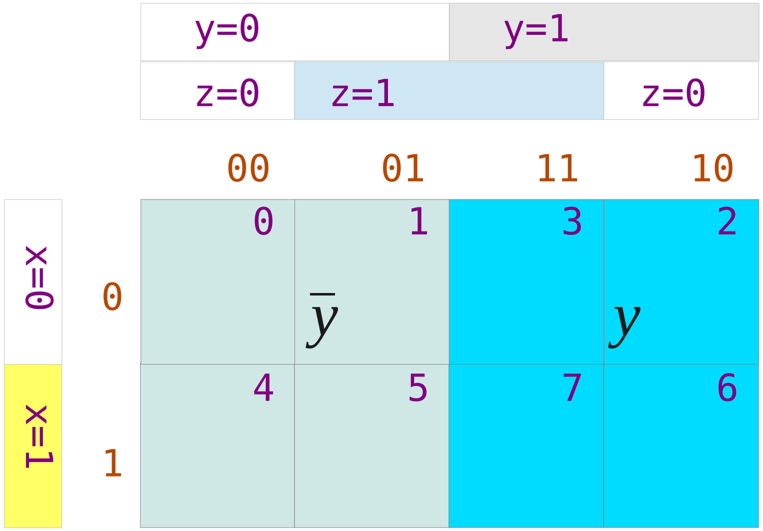
index				minterms
0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$
1	0	0	1	$\bar{x}\,\bar{y}\,z$
2	0	1	0	$\bar{x}\,y\,\bar{z}$
3	0	1	1	$\bar{x}\,y\,z$
4	1	0	0	$x\,\bar{y}\,\bar{z}$
5	1	0	1	$x\,\bar{y}\,z$
6	1	1	0	$x\,y\,\bar{z}$
7	1	1	1	$x\,y\,z$

$\bar{y}$

y

a group of 4 minterms





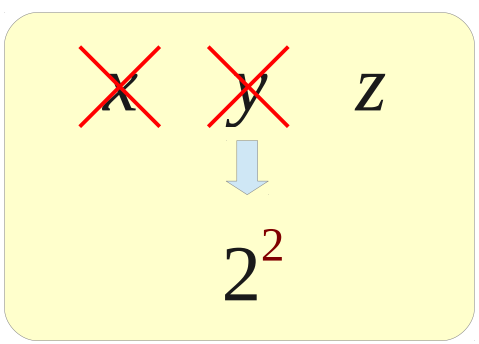
K-Map 3 variables (5)

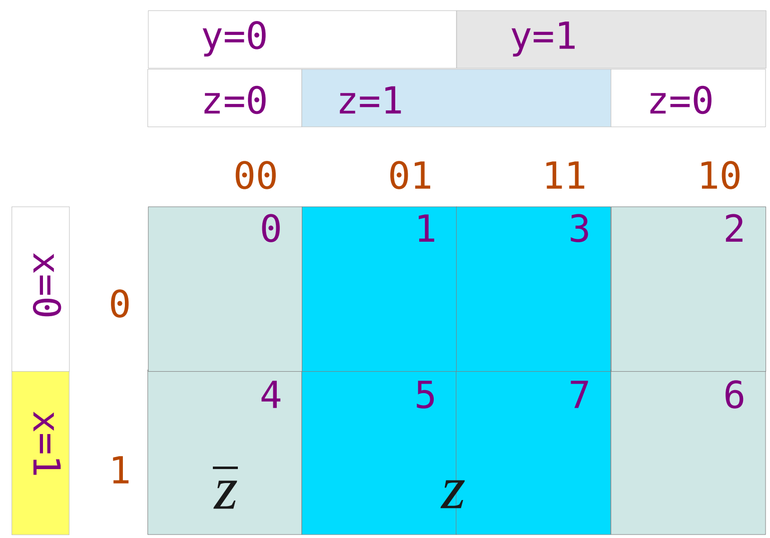
index	x	y	z	minterms
0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$
1	0	0	1	$\bar{x}\,\bar{y}\,z$
2	0	1	0	$\bar{x}\,y\,\bar{z}$
3	0	1	1	$\bar{x}\,y\,z$
4	1	0	0	$x\,\bar{y}\,\bar{z}$
5	1	0	1	$x\,\bar{y}\,z$
6	1	1	0	$x\,y\,\bar{z}$
7	1	1	1	$x\,y\,z$

$\bar{z}$

z

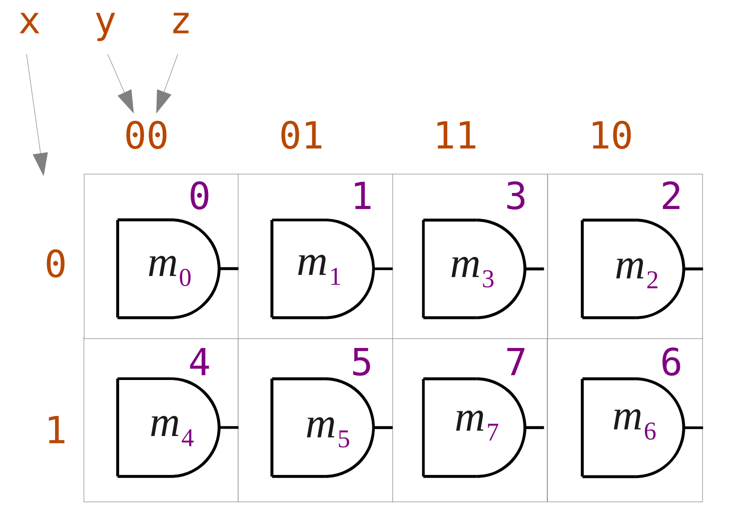
a group of 4 minterms

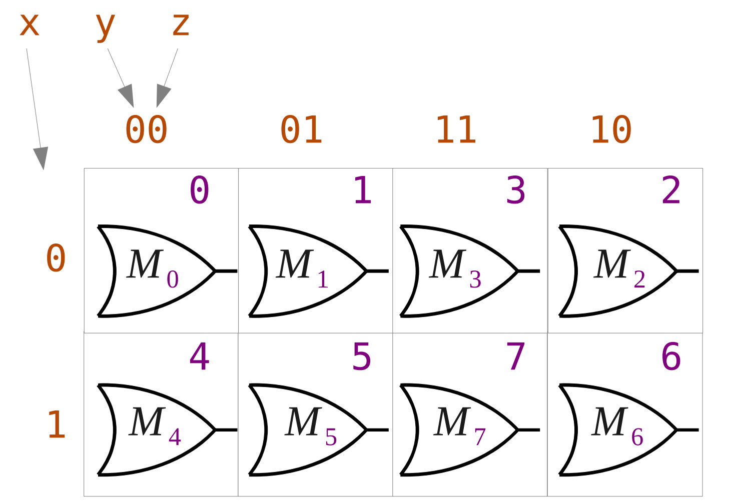




K-Map, minterms, and Maxterms

index				minterms
0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}$
1	0	0	1	$\bar{x}\,\bar{y}\,z$
2	0	1	0	$\bar{x}\,y\,\bar{z}$
3	0	1	1	$\bar{x}\,y\,z$
4	1	0	0	$x\,\bar{y}\,\bar{z}$
5	1	0	1	$x\,\bar{y}\,z$
6	1	1	0	$x\,y\,\bar{z}$
7	1	1	1	$x\,y\,z$



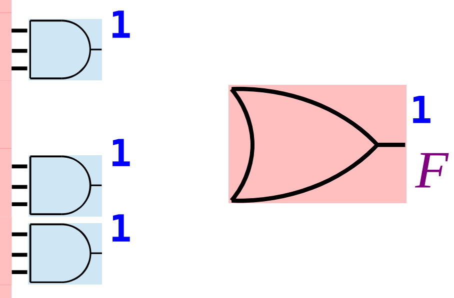


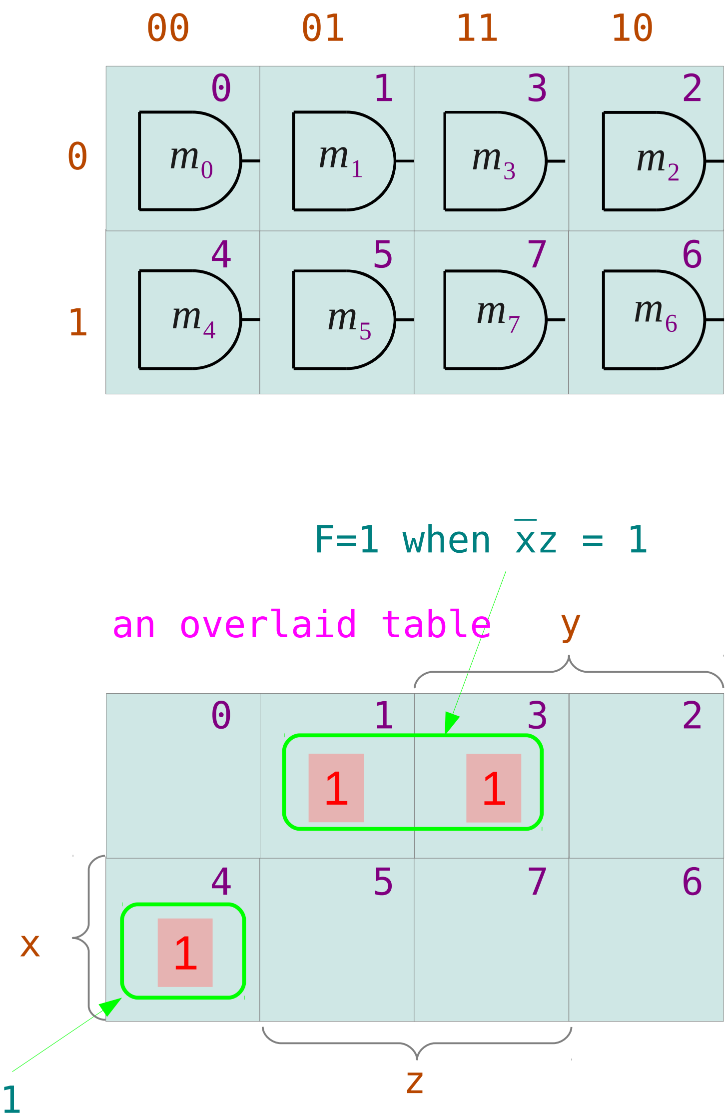
Each rectangle is associated with a minterm or a maxterm which represents a particular input variable conditions.

In this table, output function value is overlaid

Boolean Function with minterms

	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0



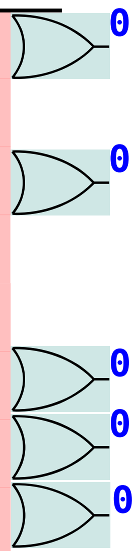


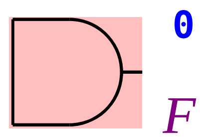
a simplified function

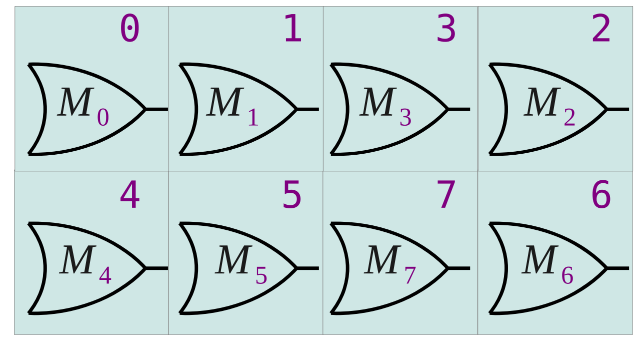
$$F = \bar{x}z + x\bar{y}\bar{z}$$

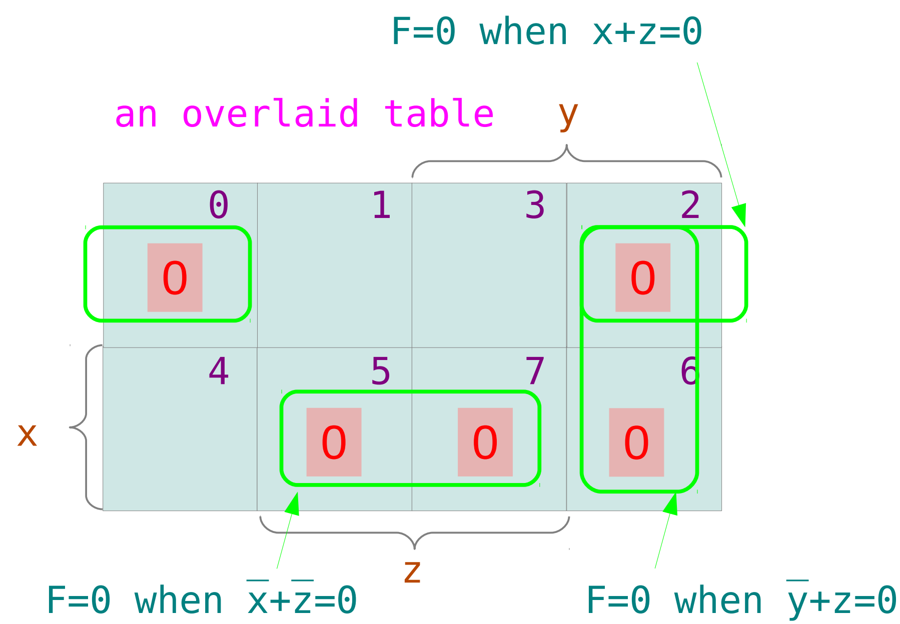
Boolean Function with Maxterms

	x	y	z	F
⇨ 0	0	0	0	0
1	0	0	1	1
⇨ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
⇨ 5	1	0	1	0
⇨ 6	1	1	0	0
⇨ 7	1	1	1	0









a simplified function

$$F = (\overline{x} + \overline{z})(\overline{y} + z)(x + z)$$

K-Map 4 variables (1)

index					minterms
0	0	0	0	0	$\bar{x}\bar{y}\bar{z}\bar{w}$
1	0	0	0	1	$\bar{x}\bar{y}\bar{z}w$
2	0	0	1	0	$\bar{x}\bar{y}z\bar{w}$
3	0	0	1	1	$\bar{x}\bar{y}zw$
4	0	1	0	0	$\bar{x}y\bar{z}\bar{w}$
5	0	1	0	1	$\bar{x}y\bar{z}w$
6	0	1	1	0	$\bar{x}yz\bar{w}$
7	0	1	1	1	$\bar{x}yzw$
8	1	0	0	0	$x\bar{y}\bar{z}\bar{w}$
9	1	0	0	1	$x\bar{y}\bar{z}w$
10	1	0	1	0	$x\bar{y}z\bar{w}$
11	1	0	1	1	$x\bar{y}zw$
12	1	1	0	0	$xy\bar{z}\bar{w}$
13	1	1	0	1	$xy\bar{z}w$
14	1	1	1	0	$xyz\bar{w}$
15	1	1	1	1	$xyzw$

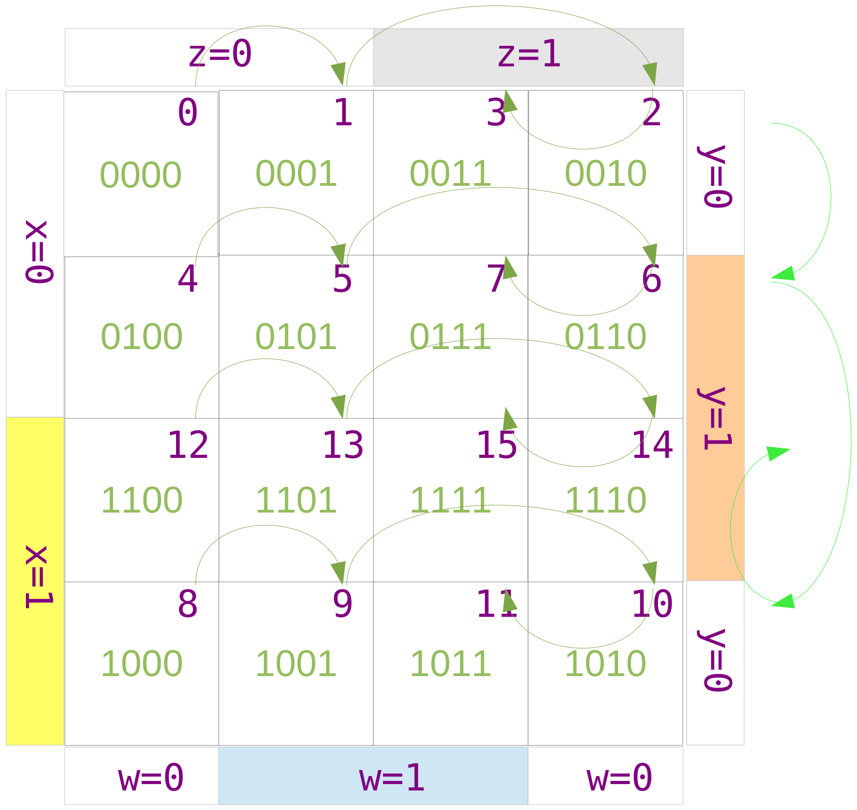
z=0		z=1	
w=0	w=1		w=0

x=0, x=1; y=0, y=1, y=0

	00	01	11	10
00	0 0000	1 0001	3 0011	2 0010
01	4 0100	5 0101	7 0111	6 0110
11	12 1100	13 1101	15 1111	14 1110
10	8 1000	9 1001	11 1011	10 1010

K-Map 4 variables (2)

index					minterms
0	0	0	0	0	$\bar{x}\,\bar{y}\,\bar{z}\,\bar{w}$
1	0	0	0	1	$\bar{x}\,\bar{y}\,\bar{z}\,w$
2	0	0	1	0	$\bar{x}\,\bar{y}\,z\,\bar{w}$
3	0	0	1	1	$\bar{x}\,\bar{y}\,z\,w$
4	0	1	0	0	$\bar{x}\,y\,\bar{z}\,\bar{w}$
5	0	1	0	1	$\bar{x}\,y\,\bar{z}\,w$
6	0	1	1	0	$\bar{x}\,y\,z\,\bar{w}$
7	0	1	1	1	$\bar{x}\,y\,z\,w$
8	1	0	0	0	$x\,\bar{y}\,\bar{z}\,\bar{w}$
9	1	0	0	1	$x\,\bar{y}\,\bar{z}\,w$
10	1	0	1	0	$x\,\bar{y}\,z\,\bar{w}$
11	1	0	1	1	$x\,\bar{y}\,z\,w$
12	1	1	0	0	$x\,y\,\bar{z}\,\bar{w}$
13	1	1	0	1	$x\,y\,\bar{z}\,w$
14	1	1	1	0	$x\,y\,z\,\bar{w}$
15	1	1	1	1	$x\,y\,z\,w$



References

[1] http://en.wikipedia.org/
[2]

Set Operations (1A)

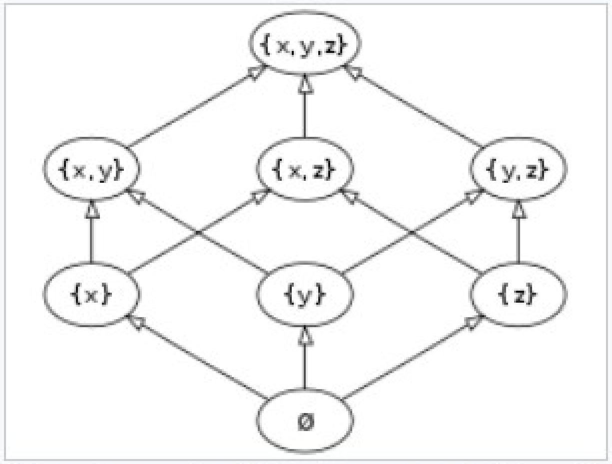
Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice and Octave.

Power Set

In mathematics, the **power set** (or **powerset**) of any set S is the set of all subsets of S, including the empty set and S itself, variously denoted as $\mathcal{P}(S)$, $\mathscr{P}(S)$, $\wp(S)$ (using the "Weierstrass p"), $P(S)$, $\mathbb{P}(S)$, or, identifying the powerset of S with the set of all functions from S to a given set of two elements, 2^S. In axiomatic set theory (as developed, for example, in the ZFC axioms), the existence of the power set of any set is postulated by the axiom of power set.[1]

Any subset of $\mathcal{P}(S)$ is called a *family of sets* over S.



The elements of the power set of the set {*x*, *y*, *z*} ordered with respect to inclusion.

https://en.wikipedia.org/wiki/Power_set

Power Set Example

If S is the set $\{x, y, z\}$, then the subsets of S are

- $\{\,\}$ (also denoted $\varnothing$ or $\emptyset$, the empty set or the null set)
- $\{x\}$
- $\{y\}$
- $\{z\}$
- $\{x, y\}$
- $\{x, z\}$
- $\{y, z\}$
- $\{x, y, z\}$

and hence the power set of S is $\{\{\,\}, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$.[2]

https://en.wikipedia.org/wiki/Power_set

References

[1] http://en.wikipedia.org/
[2]

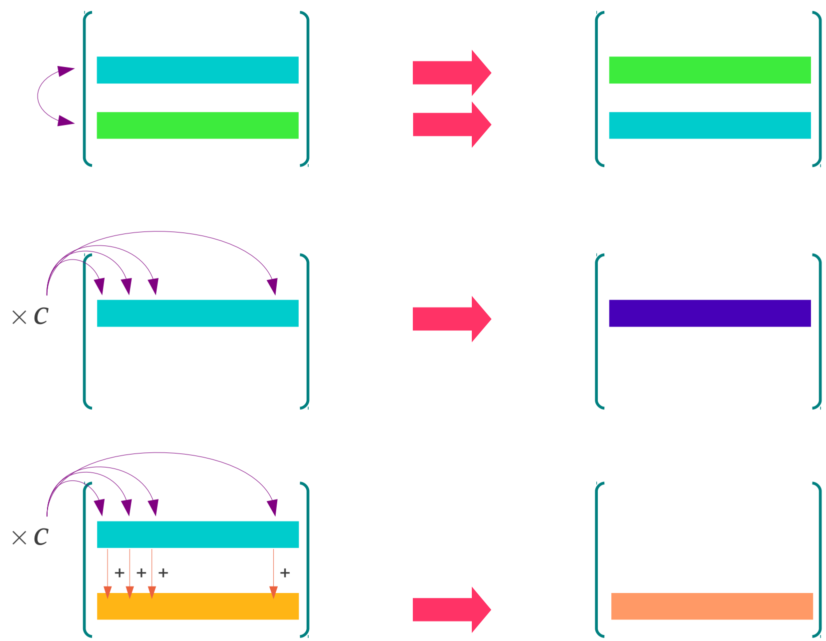
Matrix (2A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice and Octave.

Gauss-Jordan Elimination

$$\begin{pmatrix} +2 & +1 & -1 \\ -3 & -1 & +2 \\ -2 & +1 & +2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} +8 \\ -11 \\ -3 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} * \\ * \\ * \end{pmatrix}$$



https://en.wikiversity.org/wiki/Linear_Algebra_in_plain_view

Gauss-Jordan Elimination – Step 1

$$+2x_1 + x_2 - x_3 = 8 \qquad (L_1)$$
$$-3x_1 - x_2 + 2x_3 = -11 \qquad (L_2)$$
$$-2x_1 + x_2 + 2x_3 = -3 \qquad (L_3)$$

$$\left[\begin{array}{ccc|c} +2 & +1 & -1 & +8 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right]$$

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = 4 \qquad (\tfrac{1}{2} \times L_1)$$

+2/2 +1/2 –1/2 +8/2

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = 4 \qquad (\tfrac{1}{2} \times L_1)$$
$$-3x_1 - x_2 + 2x_3 = -11 \qquad (L_2)$$
$$-2x_1 + x_2 + 2x_3 = -3 \qquad (L_3)$$

$$\left[\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right]$$

https://en.wikiversity.org/wiki/Linear_Algebra_in_plain_view

Gauss-Jordan Elimination – Step 2

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \quad (L_1)$$
$$-3x_1 - x_2 + 2x_3 = -11 \quad (L_2)$$
$$-2x_1 + x_2 + 2x_3 = -3 \quad (L_3)$$

$$\left[\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ \boxed{-3} & -1 & +2 & -11 \\ \boxed{-2} & +1 & +2 & -3 \end{array}\right]$$

$$+3x_1 + \tfrac{3}{2}x_2 - \tfrac{3}{2}x_3 = +12 \quad (\boxed{3 \times L_1})$$
$$-3x_1 - x_2 + 2x_3 = -11 \quad (L_2)$$

+3	+3/2	–3/2	+12
–3	–1	+2	–11

$$+2x_1 + \tfrac{2}{2}x_2 - \tfrac{2}{2}x_3 = +8 \quad (\boxed{2 \times L_1})$$
$$-2x_1 + x_2 + 2x_3 = -3 \quad (L_3)$$

+2	+2/2	–2/2	+8
–2	+1	+2	–3

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \quad (L_1)$$
$$0x_1 + \tfrac{1}{2}x_2 + \tfrac{1}{2}x_3 = +1 \quad (\boxed{3 \times L_1} + L_2)$$
$$0x_1 + 2x_2 + 1x_3 = +5 \quad (\boxed{2 \times L_1} + L_3)$$

$$\left[\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ \boxed{0} & +1/2 & +1/2 & +1 \\ \boxed{0} & +2 & +1 & +5 \end{array}\right]$$

https://en.wikiversity.org/wiki/Linear_Algebra_in_plain_view

Gauss-Jordan Elimination – Step 3

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$
$$0x_1 + \tfrac{1}{2}x_2 + \tfrac{1}{2}x_3 = +1 \qquad (L_2)$$
$$0x_1 + 2x_2 + 1x_3 = +5 \qquad (L_3)$$

$$\left[\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1/2 & +1/2 & +1 \\ 0 & +2 & +1 & +5 \end{array}\right]$$

$$0x_1 + 1x_2 + 1x_3 = +2 \qquad (2 \times L_2)$$

0	+1	+1	+2

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$
$$0x_1 + 1x_2 + 1x_3 = +2 \qquad (2 \times L_2)$$
$$0x_1 + 2x_2 + 1x_3 = +5 \qquad (L_3)$$

$$\left[\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & +2 & +1 & +5 \end{array}\right]$$

https://en.wikiversity.org/wiki/Linear_Algebra_in_plain_view

Gauss-Jordan Elimination – Step 4

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \quad (L_1)$$
$$0x_1 + 1x_2 + 1x_3 = +2 \quad (L_2)$$
$$0x_1 + 2x_2 + 1x_3 = +5 \quad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & \boxed{+2} & +1 & +5 \end{array}\right)$$

$$0x_1 - 2x_2 - 2x_3 = -4 \quad \boxed{(-2 \times L_2)}$$
$$0x_1 + 2x_2 + 1x_3 = +5 \quad (L_3)$$

$$\begin{array}{cccc} 0 & -2 & -2 & -4 \\ 0 & +2 & +1 & +5 \end{array}$$

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \quad (L_1)$$
$$0x_1 + 1x_2 + 1x_3 = +2 \quad (L_2)$$
$$0x_1 + 0x_2 - 1x_3 = +1 \quad \boxed{(-2 \times L_2)} + (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & \boxed{0} & -1 & +1 \end{array}\right)$$

https://en.wikiversity.org/wiki/Linear_Algebra_in_plain_view

Gauss-Jordan Elimination – Step 5

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$
$$0x_1 + 1x_2 + 1x_3 = +2 \qquad (L_2)$$
$$0x_1 + 0x_2 - 1x_3 = +1 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & -1 & +1 \end{array}\right)$$

$$0x_1 - 0x_2 + 1x_3 = -1 \qquad (-1 \times L_3)$$

0	0	+1	–1

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$
$$0x_1 + 1x_2 + 1x_3 = +2 \qquad (L_2)$$
$$0x_1 + 0x_2 + 1x_3 = -1 \qquad (-1 \times L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

https://en.wikiversity.org/wiki/Linear_Algebra_in_plain_view

Forward Phase

$$\left[\begin{array}{ccc|c} +2 & +1 & -1 & +8 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1/2 & +1/2 & +1 \\ 0 & +2 & +1 & +5 \end{array}\right] \Rightarrow$$

$$\left[\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & +2 & +1 & +5 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & -1 & +1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & +1 & -1 \end{array}\right] \Rightarrow$$

Forward Phase - Gaussian Elimination

https://en.wikiversity.org/wiki/Linear_Algebra_in_plain_view

Gauss-Jordan Elimination – Step 6

$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$
$$0x_1 + 1x_2 + 1x_3 = +2 \qquad (L_2)$$
$$0x_1 + 0x_2 + 1x_3 = -1 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

$$0x_1 + 0x_2 + \tfrac{1}{2}x_3 = -\tfrac{1}{2} \qquad (+\tfrac{1}{2} \times L_3)$$
$$+1x_1 + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 = +4 \qquad (L_1)$$

0	0	+1/2	–1/2
+1	+1/2	–1/2	+4

$$0x_1 + 0x_2 - 1x_3 = +1 \qquad (-1 \times L_3)$$
$$0x_1 + 1x_2 + 1x_3 = +2 \qquad (L_2)$$

0	0	–1	+1
0	+1	+1	+2

$$+1x_1 + \tfrac{1}{2}x_2 + 0x_3 = +\tfrac{7}{2} \qquad (+\tfrac{1}{2} \times L_3 + L_1)$$
$$0x_1 + 1x_2 + 0x_3 = +3 \qquad (-1 \times L_3 + L_2)$$
$$0x_1 + 0x_2 + 1x_3 = -1 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & 0 & +7/2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

https://en.wikiversity.org/wiki/Linear_Algebra_in_plain_view

Gauss-Jordan Elimination – Step 7

$$+1x_1 + \tfrac{1}{2}x_2 + 0x_3 = +\tfrac{7}{2} \qquad (L_1)$$
$$0x_1 + 1x_2 + 0x_3 = +3 \qquad (L_2)$$
$$0x_1 + 0x_2 + 1x_3 = -1 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & 0 & +7/2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

$$0x_1 - \tfrac{1}{2}x_2 + 0x_3 = -\tfrac{3}{2} \qquad (-\tfrac{1}{2} \times L_2)$$
$$+1x_1 + 0x_2 - 0x_3 = +2 \qquad (L_1)$$

0	–1/2	0	–3/2
+1	+1/2	0	+7/2

$$+1x_1 + 0x_2 - 0x_3 = +2 \qquad (-\tfrac{1}{2} \times L_2 + L_1)$$
$$0x_1 + 1x_2 + 0x_3 = +3 \qquad (L_2)$$
$$0x_1 + 0x_2 + 1x_3 = -1 \qquad (L_3)$$

$$\left(\begin{array}{ccc|c} +1 & 0 & 0 & +2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right)$$

https://en.wikiversity.org/wiki/Linear_Algebra_in_plain_view

Backward Phase

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & 0 & +7/2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & 0 & 0 & +2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow$$

https://en.wikiversity.org/wiki/Linear_Algebra_in_plain_view

Gauss-Jordan Elimination

Forward Phase – Gaussian Elimination

$$\left(\begin{array}{ccc|c} +2 & +1 & -1 & +8 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ -3 & -1 & +2 & -11 \\ -2 & +1 & +2 & -3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1/2 & +1/2 & +1 \\ 0 & +2 & +1 & +5 \end{array}\right) \Rightarrow$$

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & +2 & +1 & +5 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & -1 & +1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow$$

Backward Phase – Guass-Jordan Elimination

$$\left(\begin{array}{ccc|c} +1 & +1/2 & -1/2 & +4 \\ 0 & +1 & +1 & +2 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & +1/2 & 0 & +7/2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} +1 & 0 & 0 & +2 \\ 0 & +1 & 0 & +3 \\ 0 & 0 & +1 & -1 \end{array}\right) \Rightarrow$$

https://en.wikiversity.org/wiki/Linear_Algebra_in_plain_view

References

[1] http://en.wikipedia.org/
[2]

The Growth of Functions (2A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice and Octave.

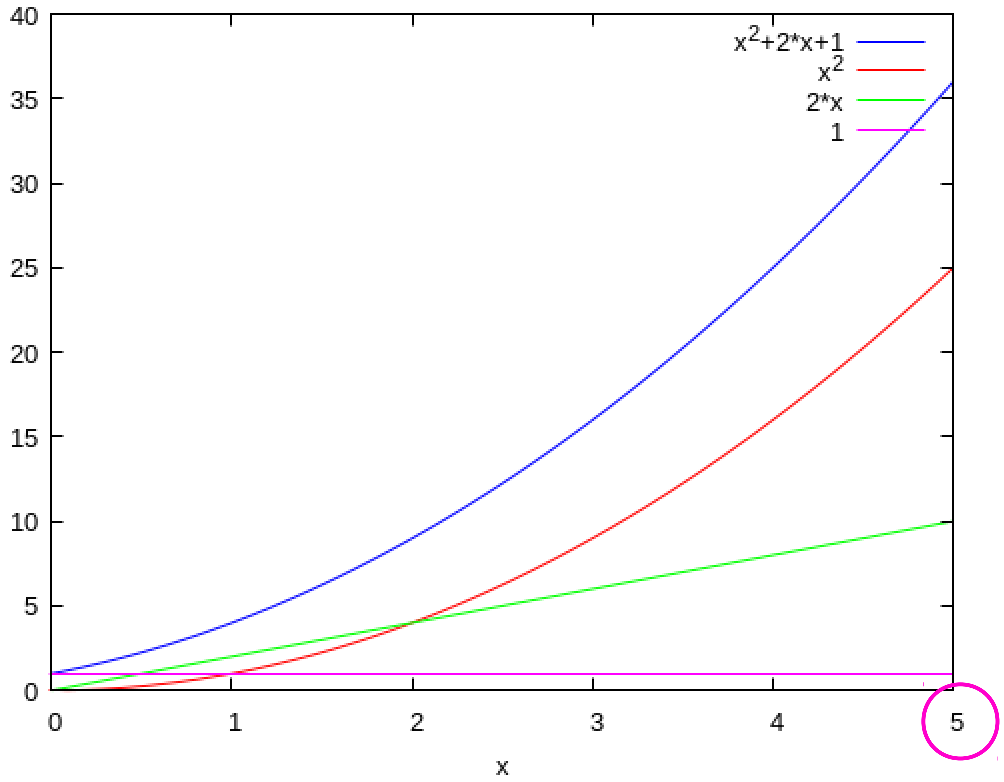
Functions and Ranges

$$\left\{ \begin{array}{l} x^2 + 2x + 1 \\ x^2 \\ 2x \\ 1 \end{array} \right.$$

$$A_1 = [0, 5]$$
$$A_2 = [0, 100]$$
$$A_3 = [0, 500]$$

Small Range

All are distinguishable



x^2+2x+1

x^2

$2x$

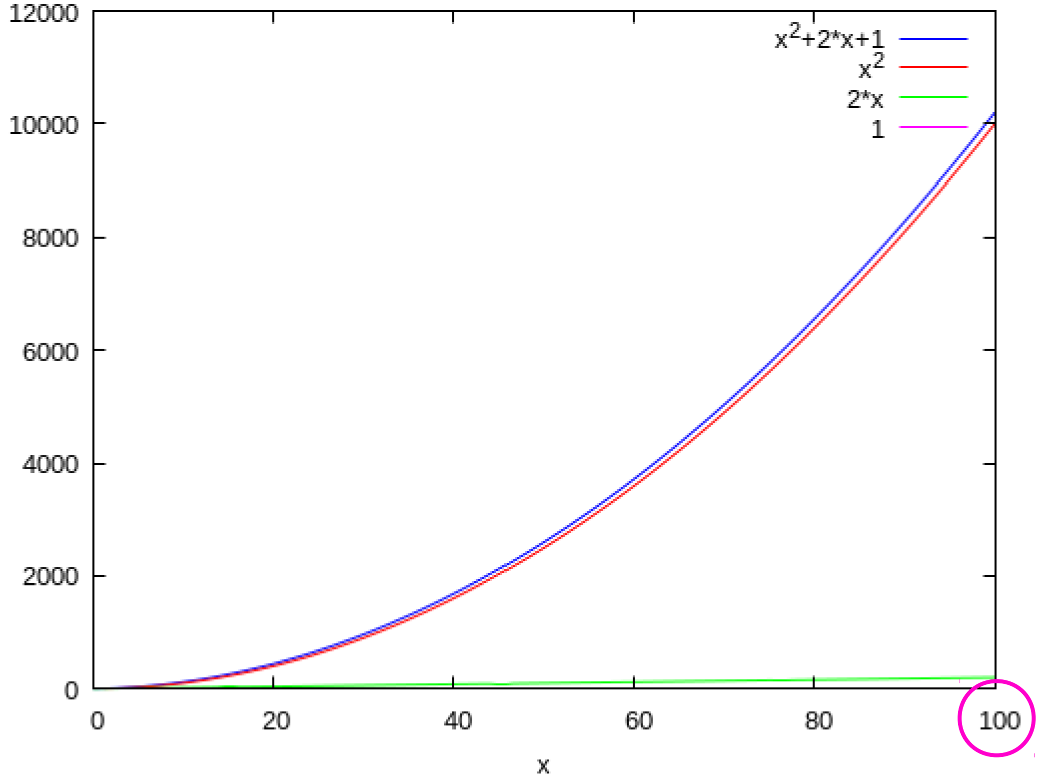
1

Zoom Out

$for \quad x > -0.5 \qquad x^2 < x^2+2x+1$

Medium Range

similar



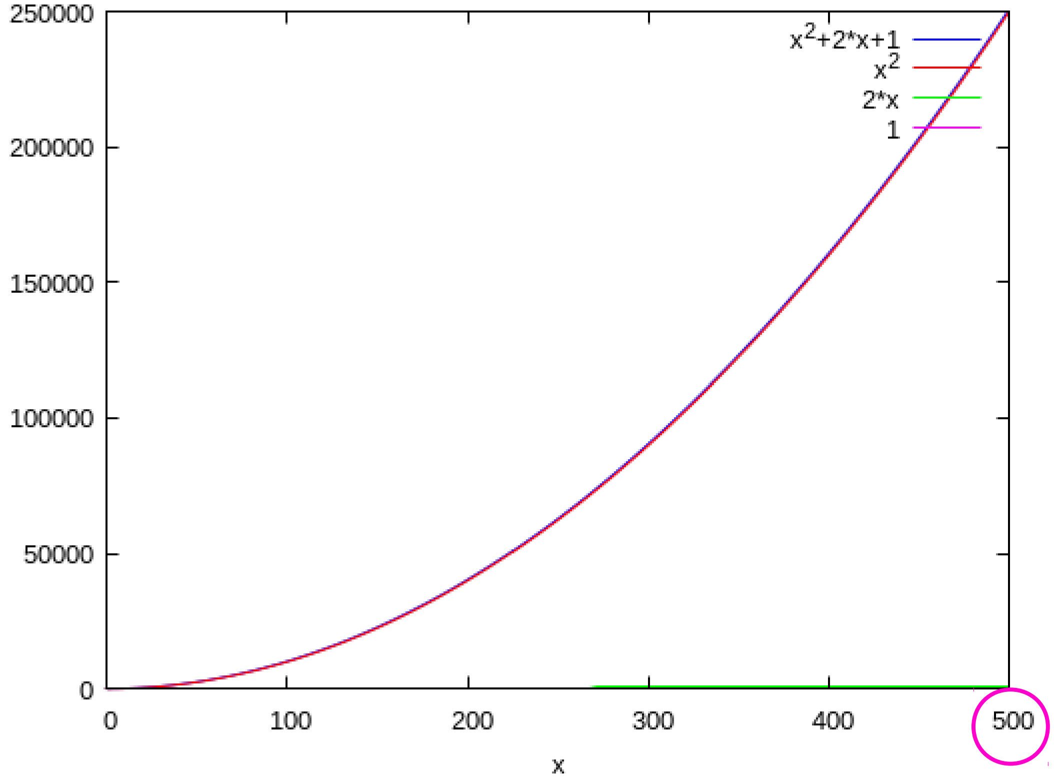
$x^2 + 2x + 1$ $\quad$ $10000 + 201$

x^2 $\quad$ 10000

$2x$

Zoom Out More

$for \quad x > -0.5 \qquad\qquad x^2 < x^2 + 2x + 1$



Indistinguishable

$x^2 + 2x + 1$ $\quad 250000 + 1001$

x^2 $\quad 250000$

$for \quad x > -0.5$ $\qquad x^2 < x^2 + 2x + 1$

Functions and Ranges

$$\begin{cases} 2 \cdot x^2 \\ x^2 + 2x + 1 \\ 2x \\ 1 \end{cases}$$

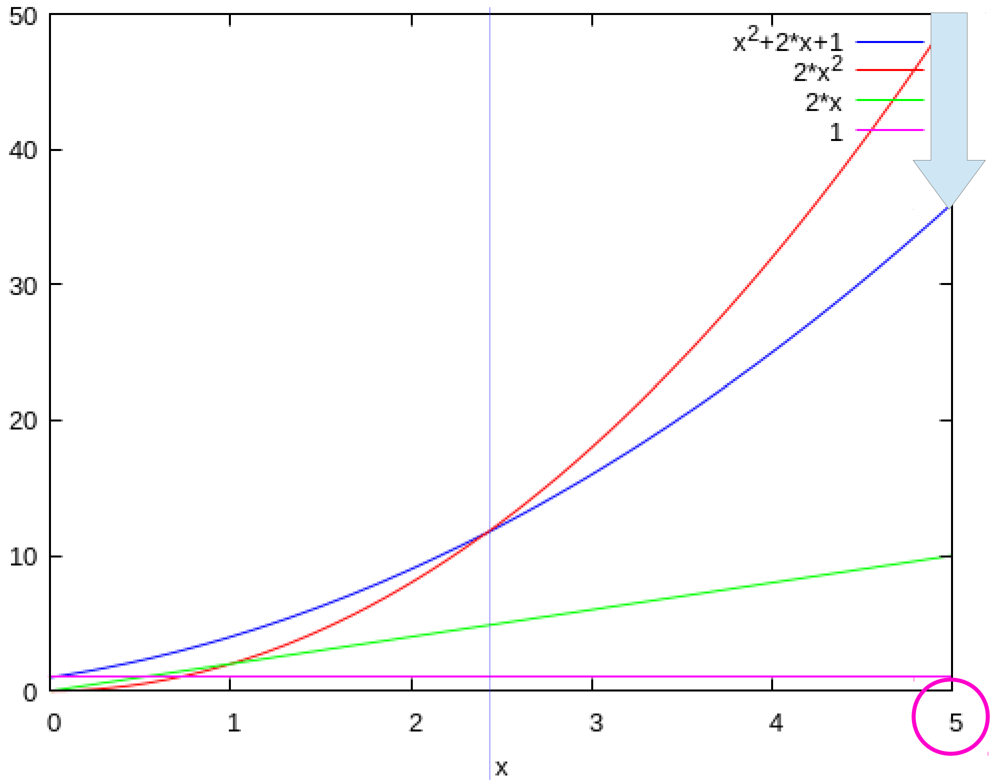
$$B_1 = [0, 5]$$

$$B_2 = [0, 100]$$

$$B_3 = [0, 500]$$

Small Range, $2x^2$

distinguishable



$2x^2$ 50

x^2+2x+1 $25+11$

$2x$

1

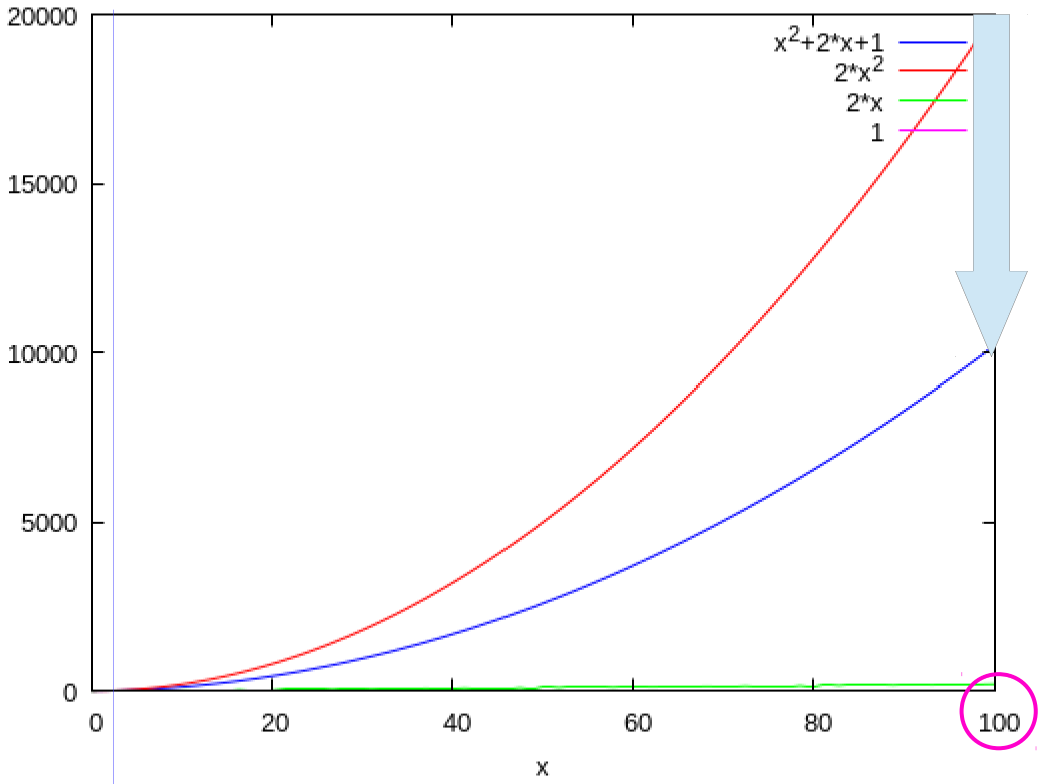
Zoom Out

for $x > 2.414$ $x^2+2x+1 < 2x^2$

for $x > k$ $f(n) < C g(n)$

Medium Range, $2x^2$

distinguishable



$2x^2$ 20000

x^2+2x+1 10000+201

$2x$

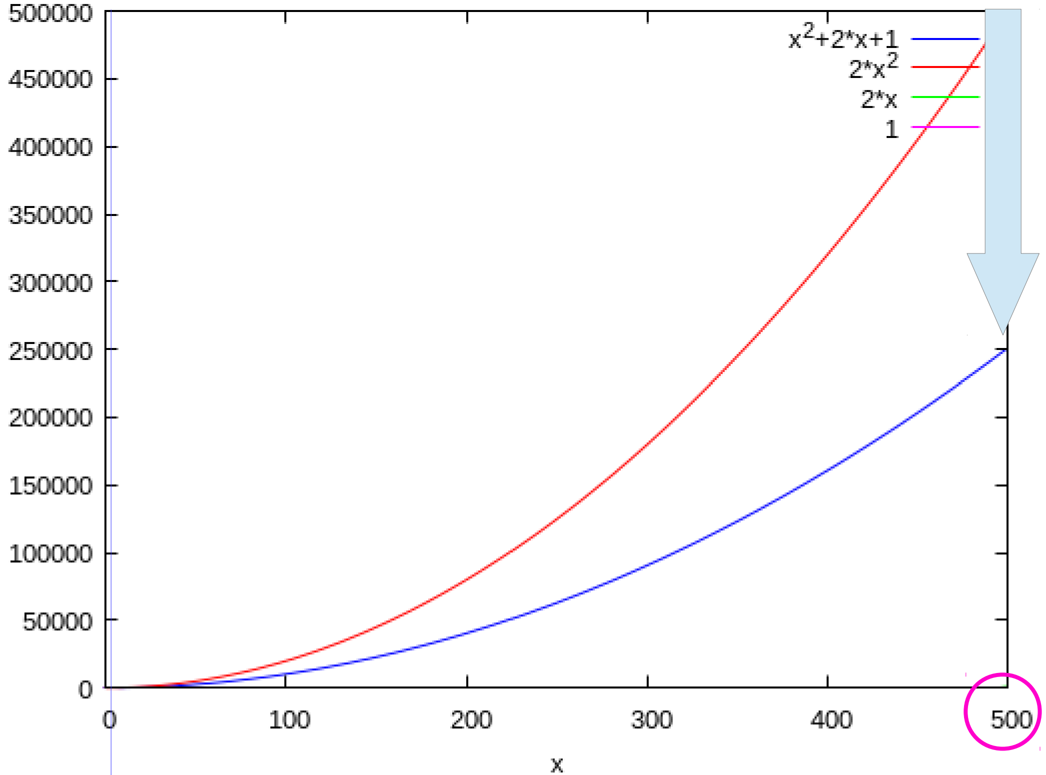
1

Zoom Out

for $x > 2.414$ $x^2+2x+1 < 2x^2$

for $x > k$ $f(n) < C g(n)$

Large Range, $2x^2$



distinguishable

$2x^2$ 500000

x^2+2x+1 $250000+1001$

$2x$

1

Zoom Out

$for \quad x > 2.414$ $\quad x^2+2x+1 < 2x^2$

$for \quad x > k$ $\quad f(n) < C g(n)$

Functions and Ranges

$$\begin{cases} 10 \cdot x^2 \\ x^2 + 2x + 1 \\ 2x \\ 1 \end{cases}$$

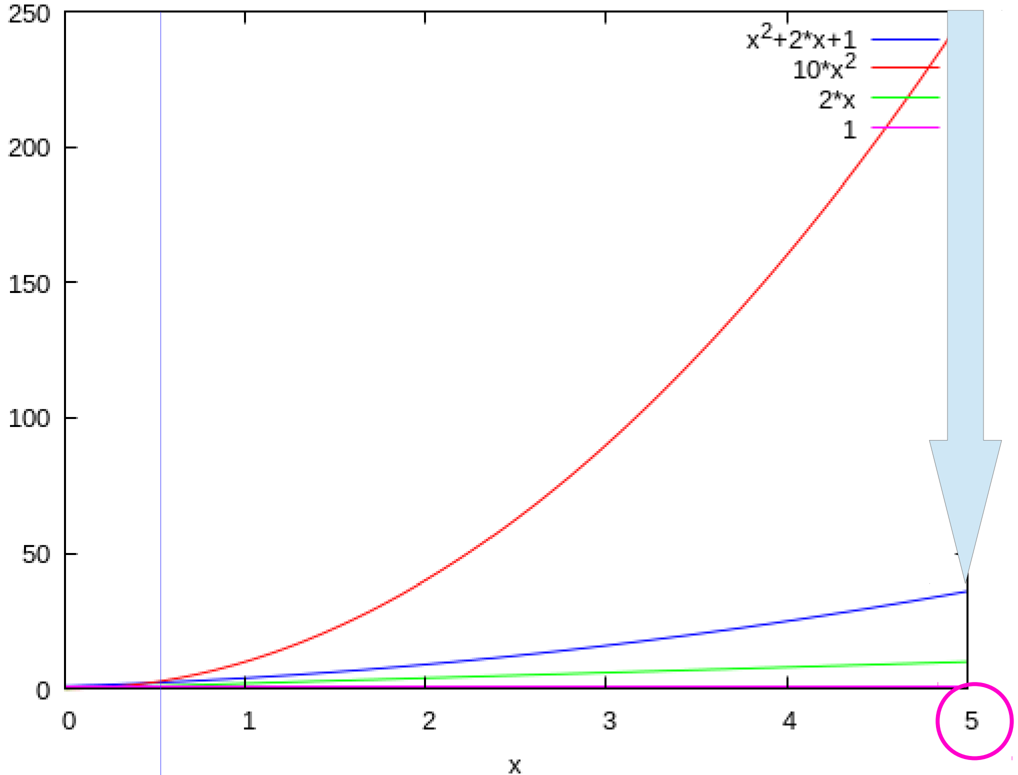
$$C_1 = [0, 5]$$

$$C_2 = [0, 100]$$

$$C_3 = [0, 500]$$

Small Range, $10x^2$

distinguishable



$10x^2$ 250

x^2+2x+1 25+11

$2x$

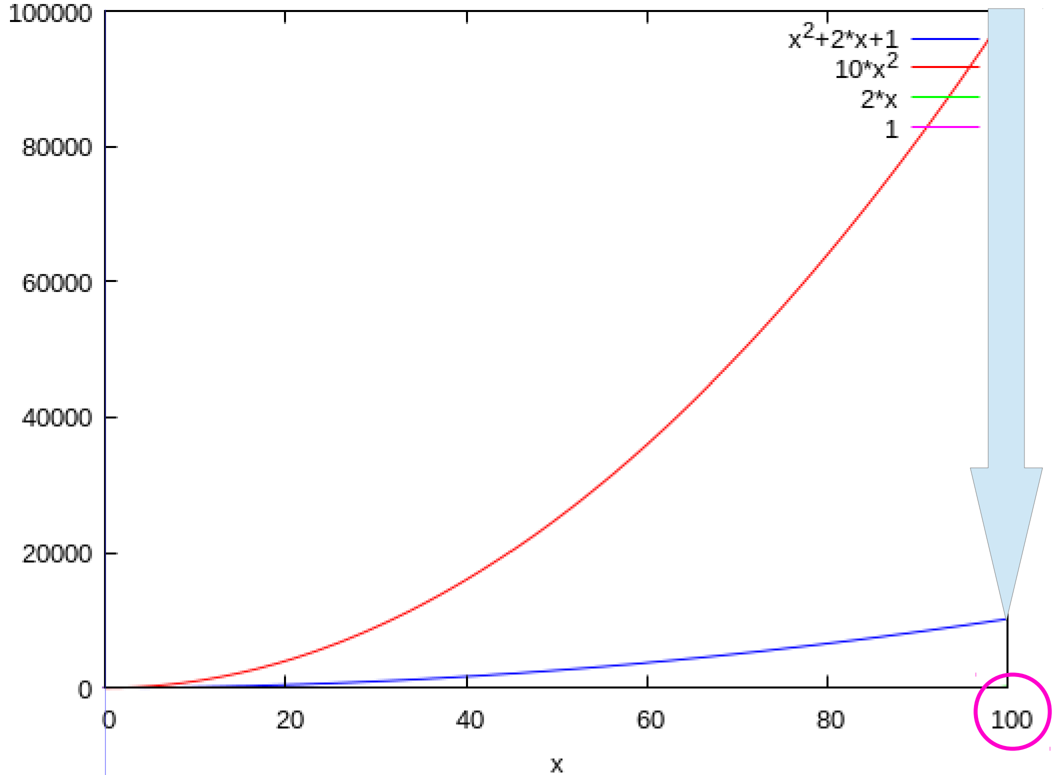
1

Zoom Out

for $x > 0.462$ $x^2+2x+1 < 10x^2$

for $x > k$ $f(n) < C g(n)$

Medium Range, $10x^2$



distinguishable

$10x^2$ 100000

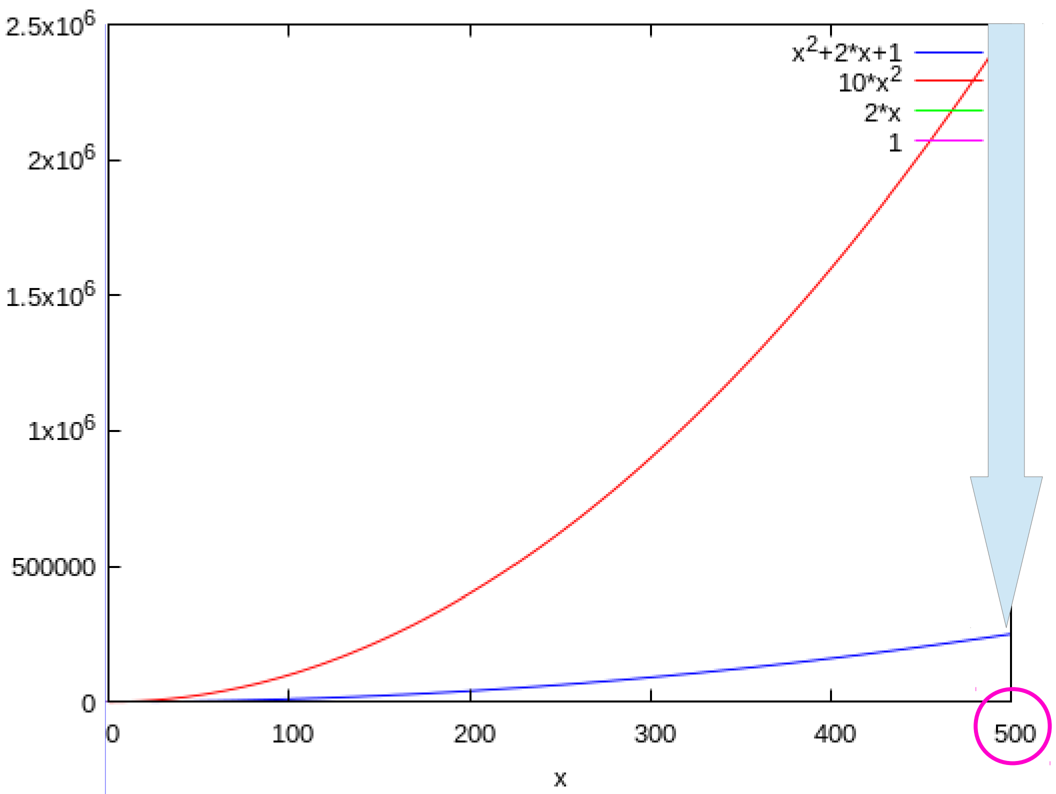
x^2+2x+1 $10000+201$

Zoom Out

for $x > 0.462$ $x^2+2x+1 < \mathbf{10}\,x^2$

for $x > k$ $f(n) < \boldsymbol{C}\,g(n)$

distinguishable



$10x^2$ 2500000

x^2+2x+1 $250000+1001$

for $x > 0.462$ $x^2+2x+1 < 10x^2$

for $x > k$ $f(n) < C\,g(n)$

Functions and Ranges

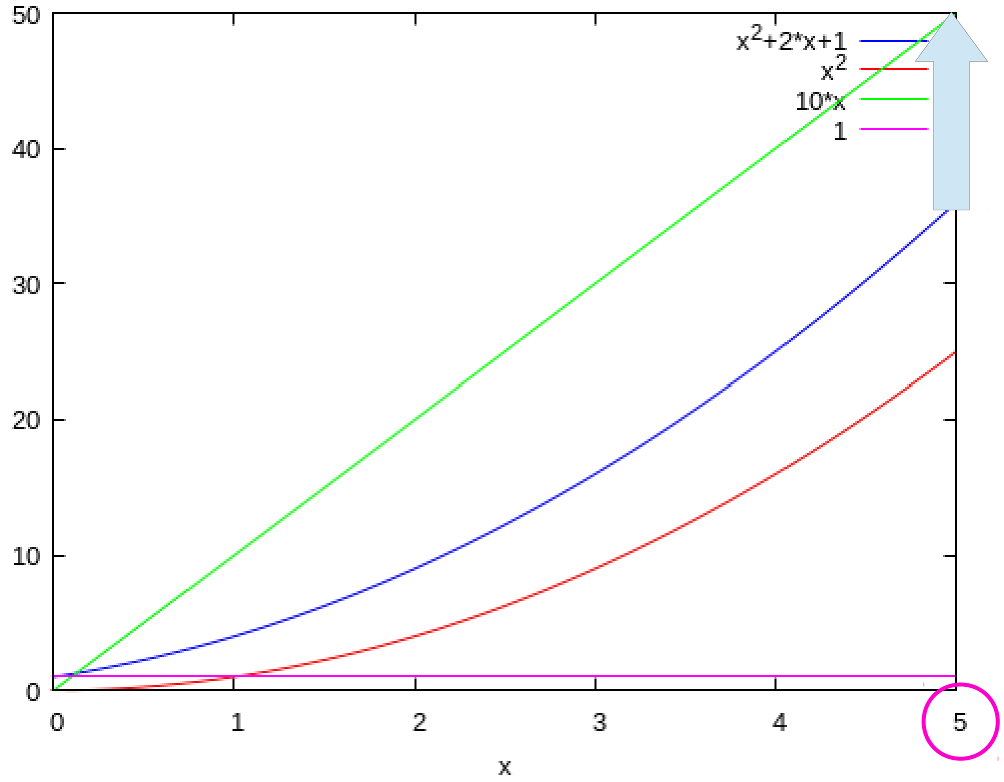
$$\begin{cases} 10 \cdot x \\ x^2 + 2x + 1 \\ x^2 \\ 1 \end{cases}$$

$$D_1 = [0, 5]$$

$$D_2 = [0, 100]$$

$$D_3 = [0, 500]$$

Small Range, *10x*



$10x$

distinguishable

x^2+2x+1

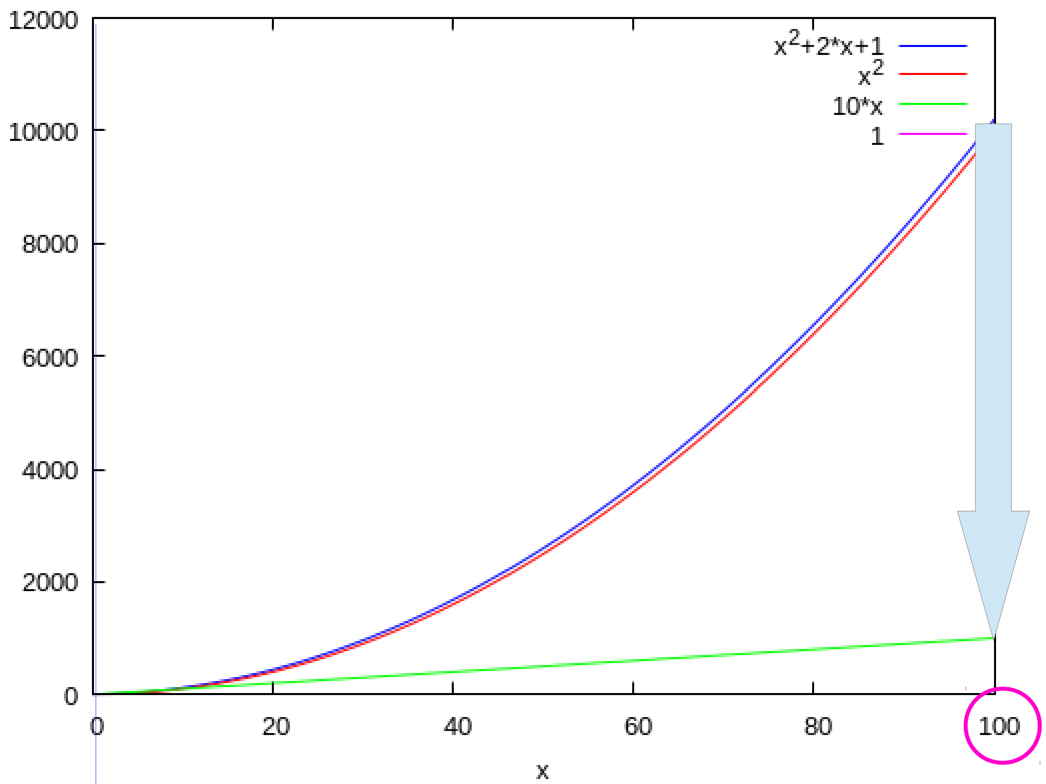
x^2

1

Zoom Out

for $0.127 < x < 7.873$ $\quad x^2+2x+1 < 10x$

Medium Range, *10x*



x^2+2x+1

distinguishable

x^2

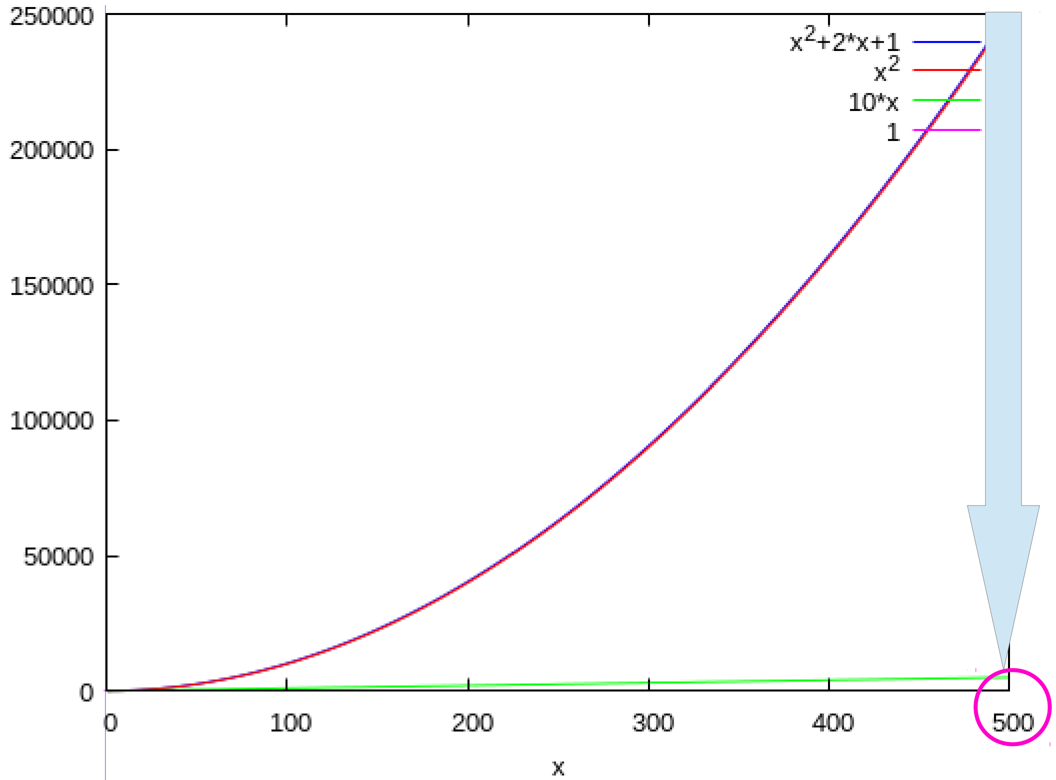
$\mathbf{10}\,x$

Zoom Out

for $x > 7.873$ $\mathbf{10}\,x < x^2+2x+1$

for $x > k$ $\mathbf{C}\,g(n) < f(n)$

Large Range, *10x*



x^2+2x+1

x^2

distinguishable

$\mathbf{10}\,x$

$for \quad x > 7.873 \qquad \mathbf{10}\,x < x^2+2x+1$

$for \quad x > k \qquad \mathbf{C}\,g(n) < f(n)$

Big-O

Let f and g be functions   (Z→R or R→R)
from the set of integers or
the set of real numbers
to the set of real numbers.

We say f(x) is **O(**g(x)**)**   "f(x) is **big-oh** of g(x)"

If there are constants C and k such that

$$|f(x)| \le C|g(x)| \quad \text{whenever } x > k.$$

g(x) : **upper bound** of f(x)

Big-Ω

Let f and g be functions (Z → R or R → R)
from the set of integers or
the set of real numbers
to the set of real numbers.

We say f(x) is **Ω(**g(x)**)** "f(x) is **big-omega** of g(x)"

If there are constants C and k such that

$C|g(x)| \leq |f(x)|$ whenever $x > k$.

↑

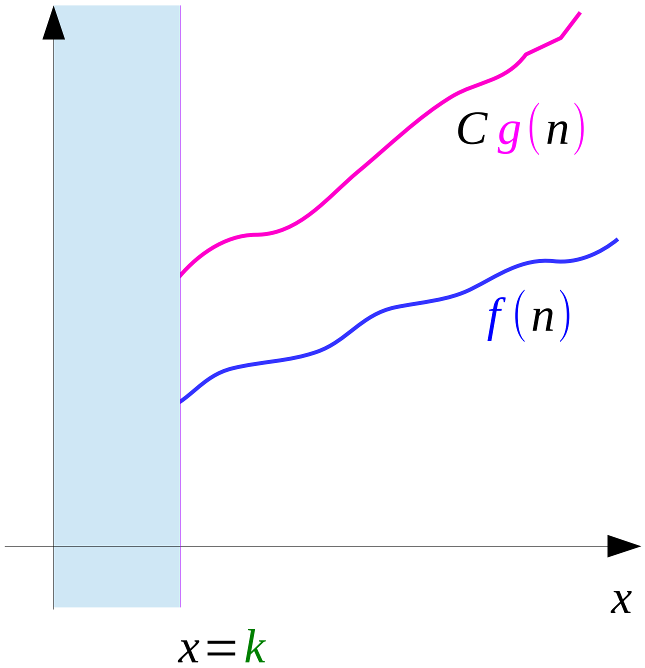
g(x) : **lower bound** of f(x)

Big-O

$$\text{for} \quad k < x$$

$$f(x) \leq C|g(x)|$$

$$f(x) \text{ is } O(g(x))$$



g(x) : **upper bound** of f(x)

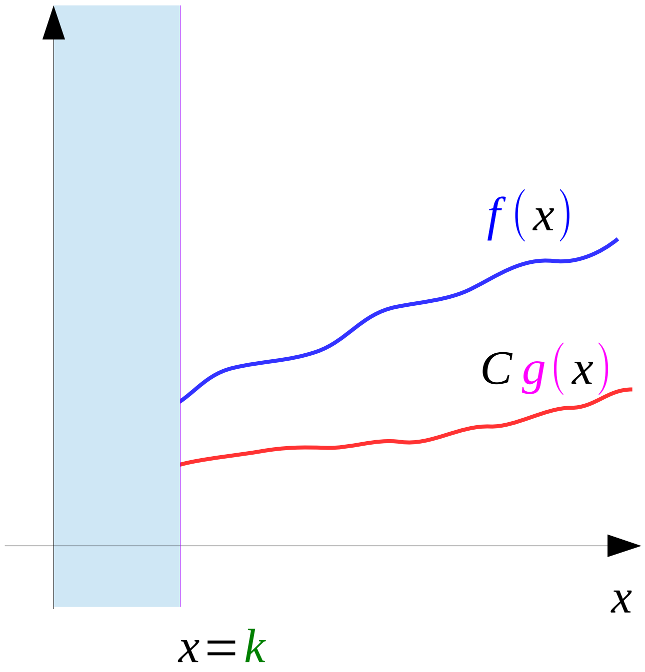
g(x) has a simpler form than f(x)
usually a single term

Big-Ω

$$\text{for} \quad k < x$$

$$f(x) \geq C|g(x)|$$

$$f(x) \;\; \text{is} \;\; \Omega(g(x))$$



g(x) : **lower bound** of f(x)

g(x) has a simpler form than f(x)
usually a single term

References

[1] http://en.wikipedia.org/
[2]