DT Correlation (1B)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Correlation of Energy Signals

Discrete Time LTI System

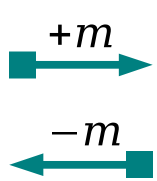
Energy Signals

$$R_{xy}[m] = \sum_{n=-\infty}^{+\infty} x[n]\, y^*[n+m]$$

$$= \sum_{n=-\infty}^{+\infty} x[n-m]\, y^*[n]$$

$$R_{xy}[m] = \sum_{n=-\infty}^{+\infty} x[n]\, y[n+m] \quad \text{(real)}$$

$$= \sum_{n=-\infty}^{+\infty} x[n-m]\, y[n] \quad \text{(real)}$$



Continuous Time LTI System

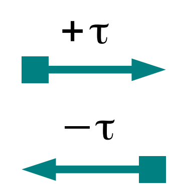
Energy Signals

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y^*(t+\tau)\, dt$$

$$= \int_{-\infty}^{+\infty} x(t-\tau)\, y^*(t)\, dt$$

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt \quad \text{(real)}$$

$$= \int_{-\infty}^{+\infty} x(t-\tau)\, y(t)\, dt \quad \text{(real)}$$



Correlation of Power Signals

Discrete Time LTI System

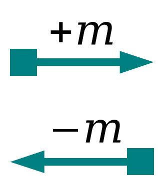
Power Signals

$$R_{xy}[m] = \lim_{N\to\infty} \frac{1}{N} \sum_{n=\langle N\rangle} x[n]\, y^*[n+m]$$

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{n=\langle N\rangle} x[n-m]\, y^*[n]$$

$$R_{xy}[m] = \lim_{N\to\infty} \frac{1}{N} \sum_{n=\langle N\rangle} x[n]\, y[n+m] \quad \text{(real)}$$

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{n=\langle N\rangle} x[n-m]\, y[n] \quad \text{(real)}$$



Continuous Time LTI System

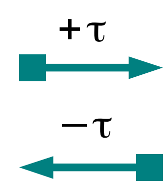
Power Signals

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{T} x(t)\, y^*(t+\tau)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{T} x(t-\tau)\, y^*(\tau)\, dt$$

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{T} x(t)\, y(t+\tau)\, dt \quad \text{(real)}$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{T} x(t-\tau)\, y(\tau)\, dt \quad \text{(real)}$$



Correlation of Periodic Power Signals

Discrete Time LTI System

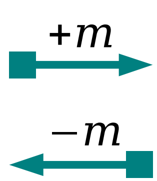
Periodic Power Signals

$$R_{xy}[m] = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, y^*[n+m]$$

$$= \frac{1}{N}\sum_{n=\langle N\rangle} x[n-m]\, y^*[n]$$

$$R_{xy}[m] = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, y[n+m]$$ (real)

$$= \frac{1}{N}\sum_{n=\langle N\rangle} x[n-m]\, y[n]$$ (real)



Continuous Time LTI System

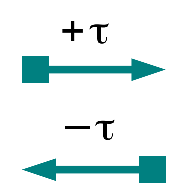
Periodic Power Signals

$$R_{xy}(\tau) = \frac{1}{T}\int_T x(t)\, y^*(t+\tau)\, dt$$

$$= \frac{1}{T}\int_T x(t-\tau)\, y^*(\tau)\, dt$$

$$R_{xy}(\tau) = \frac{1}{T}\int_T x(t)\, y(t+\tau)\, dt$$ (real)

$$= \frac{1}{T}\int_T x(t-\tau)\, y(t)\, dt$$ (real)



Correlation & Convolution : Energy Signals

Discrete Time LTI System

Energy Signals

$$R_{xy}[m] = \sum_{n=-\infty}^{+\infty} x[n]\,y[n+m] \quad \text{(real)}$$

$$x[n] * y[n] = \sum_{m=-\infty}^{+\infty} x[n-m]\,y[m]$$

$$R_{xy}[m] = x[-m] * y[m]$$

$$R_{xy}[m] \overset{\text{DTFT}}{\Longleftrightarrow} X^*(F)\,Y(F)$$

$$x[-n] * y[n] = \sum_{m=-\infty}^{+\infty} x[-n+m]\,y[m]$$

$$x[-m] * y[m] = \sum_{n=-\infty}^{+\infty} x[n-m]\,y[n]$$

$$x[-m] * y[m] = \sum_{n=-\infty}^{+\infty} x[n]\,y[n+m]$$

Continuous Time LTI System

Energy Signals

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\,y(t+\tau)\,dt \quad \text{(real)}$$

$$x(t) * y(t) = \int_{-\infty}^{+\infty} x(t-\tau)\,y(\tau)\,d\tau$$

$$R_{xy}(\tau) = x(-\tau) * y(\tau)$$

$$R_{xy}(\tau) \overset{\text{CTFT}}{\Longleftrightarrow} X^*(f)\,Y(f)$$

$$x(-t) * y(t) = \int_{-\infty}^{+\infty} x(-t+\tau)\,y(\tau)\,d\tau$$

$$x(-\tau) * y(\tau) = \int_{-\infty}^{+\infty} x(t-\tau)\,y(t)\,dt$$

$$x(-\tau) * y(\tau) = \int_{-\infty}^{+\infty} x(t)\,y(t+\tau)\,dt$$

Time Reversal Fourier Transforms

Discrete Time LTI System

$$x[m] = \int_{1} X(F) e^{+j2\pi F n}\, dF$$

$$x[-m] = \int_{1} X(F) e^{-j2\pi F n}\, dF$$

$$x[-m] = -\int^{1} X(-\nu) e^{+j2\pi \nu m}\, d\nu$$

$x[-m]$ DTFT $X(-F)$

$-m$ $-F$

Continuous Time LTI System

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{+j2\pi f t}\, df$$

$$x(-t) = \int_{-\infty}^{+\infty} X(f) e^{-j2\pi f t}\, df$$

$$x(-t) = -\int_{+\infty}^{-\infty} X(-\nu) e^{+j2\pi \nu t}\, d\nu$$

$x(-t)$ CTFT $X(-f)$

$-t$ $-f$

Conjugate Fourier Transforms

Discrete Time LTI System

$$x[m] = \int_{1} X(F) e^{+j2\pi F n} \, dF$$

$$x^*[m] = \int_{1} X^*(F) e^{-j2\pi F m} \, dF$$

$$x^*[m] = -\int^{1} X^*(-\nu) e^{+j2\pi \nu m} \, d\nu$$

$x^*[m]$ DTFT $X^*(-F)$

$* \; m$ $\qquad * -F$

$x^*[-m]$ DTFT $X^*(F)$

$*(-m)$ $\qquad * -(-F)$

Continuous Time LTI System

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{+j2\pi f t} \, df$$

$$x^*(t) = \int_{-\infty}^{+\infty} X^*(f) e^{-j2\pi f t} \, df$$

$$x^*(t) = -\int_{+\infty}^{-\infty} X^*(-\nu) e^{+j2\pi \nu t} \, d\nu$$

$x^*(t)$ CTFT $X^*(-f)$

$* \; t$ $\qquad * -f$

$x^*(-t)$ CTFT $X^*(f)$

$*(-t)$ $\qquad * -(-f)$

Fourier Transforms of Real Signals

Discrete Time LTI System

$$x[m] = \int_{1} X(F) e^{+j2\pi F n}\, dF$$

$x^*[m]$ DTFT $X^*(-F)$

$x^*[-m]$ DTFT $X^*(F)$

|| ||

$x[-m]$ DTFT $X(-F)$

A real signal — Hermitian Symmetry

$x[-m]$ DTFT $X^*(F)$ (real)

Continuous Time LTI System

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{+j2\pi f t}\, df$$

$x^*(t)$ CTFT $X^*(-f)$

$x^*(-t)$ CTFT $X^*(f)$

|| ||

$x(-t)$ CTFT $X(-f)$

A real signal — Hermitian Symmetry

(real) $x(-t)$ CTFT $X^*(f)$

Correlation & Convolution : Energy Signals

Discrete Time LTI System

Energy Signals

Correlation Definition A

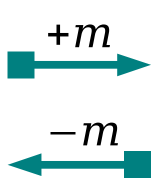
$$R_{xy}[m] = \sum_{n=-\infty}^{+\infty} x[n]\, y^*[n+m]$$

conjugate the second

Correlation Definition B

$$R_{xy}[m] = \sum_{n=-\infty}^{+\infty} x^*[n]\, y[n+m]$$

conjugate the first



Continuous Time LTI System

Energy Signals

Correlation Definition A

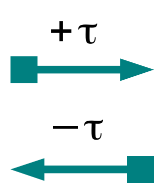
$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y^*(t+\tau)\, dt$$

conjugate the second

Correlation Definition B

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x^*(t)\, y(t+\tau)\, dt$$

conjugate the first



Correlation & Convolution : Energy Signals

Discrete Time LTI System

Energy Signals

Correlation Definition A

$$R_{xy}[m] = \sum_{n=-\infty}^{+\infty} x[n]\, y^*[n+m]$$

Convolution

$$x[n] * y^*[n] = \sum_{m=-\infty}^{+\infty} x[n-m]\, y^*[m]$$

$$x[-m] * y^*[m] = \sum_{n=-\infty}^{+\infty} x[n-m]\, y^*[n]$$

$$R_{xy}[m] = x[-m] * y^*[m]$$

$$R_{xy}[m] \overset{\text{DTFT}}{\Longleftrightarrow} X(-F)\,Y^*(-F)$$

$$R_{xy}[m] \overset{\text{DTFT}}{\Longleftrightarrow} X(F)\,Y^*(F)$$

(even)

Continuous Time LTI System

Energy Signals

Correlation Definition A

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y^*(t+\tau)\, dt$$

Convolution

$$x(t) * y^*(t) = \int_{-\infty}^{+\infty} x(t-\tau)\, y^*(\tau)\, d\tau$$

$$x(-\tau) * y^*(\tau) = \int_{-\infty}^{+\infty} x(t-\tau)\, y^*(t)\, dt$$

$$R_{xy}(\tau) = x(-\tau) * y^*(\tau)$$

$$R_{xy}(\tau) \overset{\text{CTFT}}{\Longleftrightarrow} X(-f)\,Y^*(-f)$$

(even)

$$R_{xy}(\tau) \overset{\text{CTFT}}{\Longleftrightarrow} X(f)\,Y^*(f)$$

Correlation & Convolution : Energy Signals

Discrete Time LTI System

Energy Signals

Correlation Definition B

$$R_{xy}[m] = \sum_{n=-\infty}^{+\infty} x^*[n]\,y[n+m]$$

Convolution

$$x^*[n] * y[n] = \sum_{m=-\infty}^{+\infty} x^*[n-m]\,y[m]$$

$$x^*[-m] * y[m] = \sum_{n=-\infty}^{+\infty} x^*[n-m]\,y[n]$$

$$R_{xy}[m] = \left[x^*[-m] * y[m]\right]$$

$$R_{xy}[m] \overset{\text{DTFT}}{\Longleftrightarrow} X^*(F)\,Y(F)$$

Continuous Time LTI System

Energy Signals

Correlation Definition B

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x^*(t)\,y(t+\tau)\,dt$$

Convolution

$$x^*(t) * y(t) = \int_{-\infty}^{+\infty} x^*(t-\tau)\,y(\tau)\,d\tau$$

$$x^*(-\tau) * y(\tau) = \int_{-\infty}^{+\infty} x^*(t-\tau)\,y(t)\,dt$$

$$R_{xy}(\tau) = \left[x^*(-\tau) * y(\tau)\right]$$

$$R_{xy}(\tau) \overset{\text{CTFT}}{\Longleftrightarrow} X^*(f)\,Y(f)$$

Correlation Functions

Discrete Time LTI System

Energy Signals

$$R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]\, y[n+m]$$

Power Signals

Power Signal + Energy Signal

$$R_{xy}[m] = \lim_{N\to\infty} \frac{1}{N} \sum_{n=\langle N\rangle} x[n]\, y[n+m]$$

Continuous Time LTI System

Energy Signals

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt$$

Power Signals

Power Signal + Energy Signal

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{T} x(t)\, y(t+\tau)\, dt$$

AutoCorrelation of Energy Signals

Discrete Time LTI System

Energy Signals

$$R_{xx}[m] = \sum_{n=-\infty}^{+\infty} x[n]\, x^*[n+m]$$

$$= \sum_{n=-\infty}^{+\infty} x[n-m] x^*[n]$$

$$R_{xx}[0] = \sum_{n=-\infty}^{+\infty} x^2[n]$$ total energy

Continuous Time LTI System

Energy Signals

$$R_{xx}(\tau) = \int_{-\infty}^{+\infty} x(t)\, x^*(t+\tau)\, dt$$

$$= \int_{-\infty}^{+\infty} x(t)\, x^*(t+\tau)\, dt$$

$$R_{xx}(0) = \int_{-\infty}^{+\infty} x^2(t)\, dt$$ total energy

AutoCorrelation of Power Signals

Discrete Time LTI System

Power Signals

$$R_{xx}[m] = \lim_{N \to \infty} \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, x^*[n+m]$$

$$= \lim_{N \to \infty} \frac{1}{N} \sum_{n=\langle N \rangle} x[n-m]\, x^*[n]$$

$$R_{xx}[0] = \lim_{N \to \infty} \frac{1}{N} \sum_{n=\langle N \rangle} x^2[n]$$ total energy

Continuous Time LTI System

Power Signals

$$R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_T x(t)\, x^*(t+\tau)\, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_T x(t-\tau)\, x^*(\tau)\, dt$$

$$R_{xx}(0) = \lim_{T \to \infty} \frac{1}{T} \int_T x^2(t)\, dt$$ total energy

AutoCorrelation of Periodic Power Signals

Discrete Time LTI System

Periodic Power Signals

$$R_{xx}[m] = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, x^*[n+m]$$

$$= \frac{1}{N}\sum_{n=\langle N\rangle} x[n-m]\, x^*[n]$$

$$R_{xx}[m] = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, x^*[n+m]$$

Continuous Time LTI System

Periodic Power Signals

$$R_{xx}(\tau) = \frac{1}{T}\int_T x(t)\, x^*(t+\tau)\, dt$$

$$= \frac{1}{T}\int_T x(t-\tau)\, x^*(\tau)\, dt$$

$$R_{xx}(\tau) = \frac{1}{T}\int_T x(t)\, x^*(t+\tau)\, dt$$

References

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