DFT Analysis (5A)

Young Won Lim
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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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DFT

To determine the frequency content of
a windowed signal

continuous-time periodic signal
→ sampled windowed signal

DTFT is not suitable
- it applies to discrete-time aperiodic signals
- aperiodic signal requires
  - an infinite number of sines and cosines
    - take an incredibly long computation time

Orthogonal Signals

Consider sine and cosine waves
which has an integer number of cycles
in a window period

The correlation of the following waves are zero

- any two such sine waves
- any two such cosine waves
- any such sine and cosine waves

DFT – Correlating Process

The DFT determines the frequency content
gf the windowed discrete periodic signal
by correlating it with the discrete sinusoids

1. an integer number of cycles in the window
2. the lowest frequency (one cycle)
   the fundamental frequency
3. integer multiple of the fundamental frequency
   The harmonic frequencies

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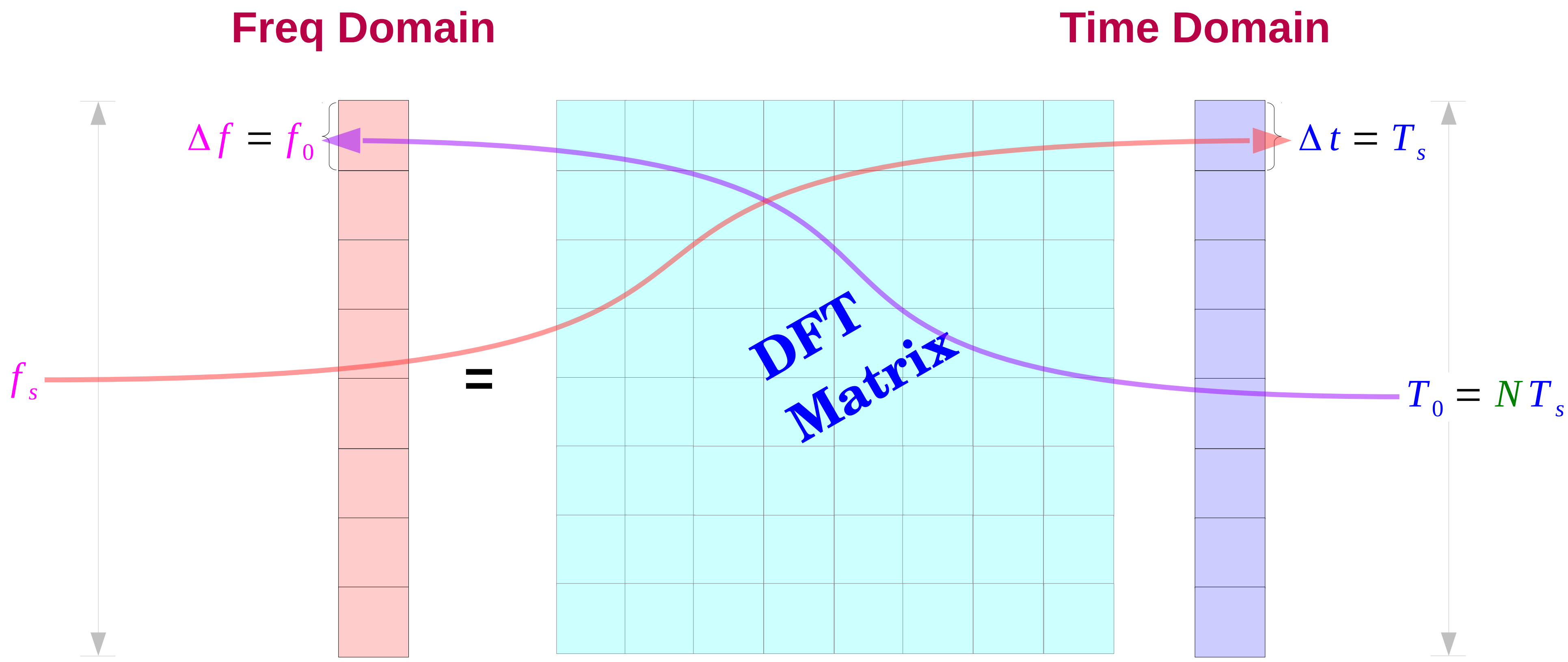
Relations between Sampling Frequency and Time



$$f_0 = \frac{1}{T_0} = \frac{1}{N T_s} \qquad f_s \geq \frac{2}{T_0} \qquad f_s \geq 2 f_0 \qquad T_s = \frac{1}{f_s}$$

Fundamental Frequency

$$f_0 = \frac{f_s}{N}$$

f_0 : *fundamental frequency*
f_s : *sampling frequency*
N : *number of samples*

$$f_k = k \cdot f_0 = k \cdot \frac{f_s}{N}$$

f_k : *harmonic frequency*

Frequency Resolution

$$\Delta f = f_0 = \frac{f_s}{N}$$

If the signal has two frequencies that are at least Δf apart, then the DFT is able to distinguish these.

To increase frequency resolution
1. reducing the sampling frequency f_s, keeping N the same
2. Increasing the number of samples N, keeping f_s the same

Increasing the window duration ($T_0 = NT_s$)
Increasing the period ($T_0 = NT_s$)
Decreasing the fundamental frequency ($f_0 = 1/T_0$)
Decreasing Δf

Spectral Leakage

1. non-integer number of cycles in a window

2. discontinuity (different starting and ending)

The correlation value is not zero

The energy in the one frequency
has leaked out to all other frequencies

Spectral Leakage – Correlation

1. non-integer number of cycles in a window

The highest correlation when the harmonic frequency is close to the frequency of the input

The correlation decreases when the harmonic frequency Is move away from the frequency of the input

A peak close to the actual frequency of the input signal
The amplitude gradually rolled off above and below

Spectral Leakage – discontinuities

2. discontinuity (different starting and ending)

Abrupt change in the repeated input signal
Involves a large number of frequencies

Results in spreading of energy

Special window is used to reduce the abrupt change
And thus to reduce the spectral leakage

References

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[4] Digital Signal Processing – A Hands-on Approach, C. Schuler and M. Chugani, McGraw-Hill, 2004