CLTI – Time Response (7A)

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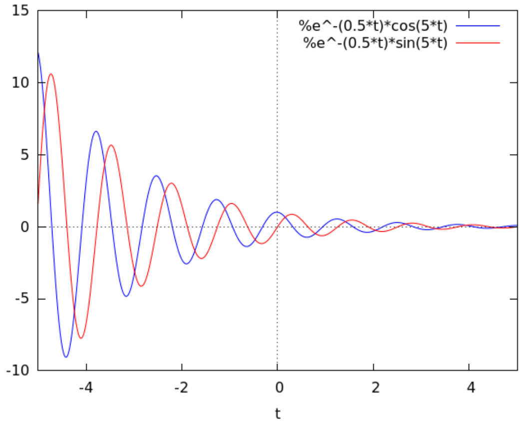
* *Everlasting Exponential Inputs*
* *Causal Exponential Inputs*

Exponential and Sinusoid Functions

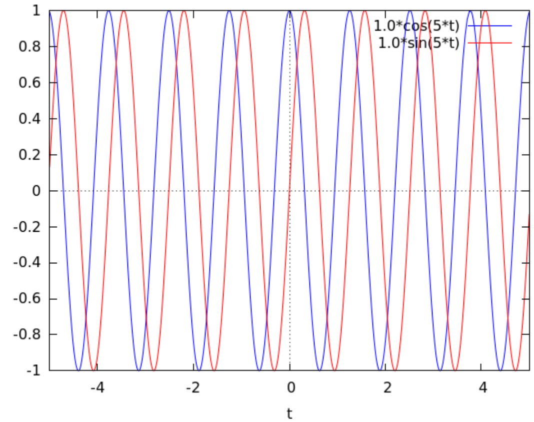
exponential function

$$e^{st} = e^{\sigma t + i\omega t} = e^{\sigma t}(\cos(\omega t) + i\sin(\omega t)) \qquad (s = \sigma + i\omega)$$

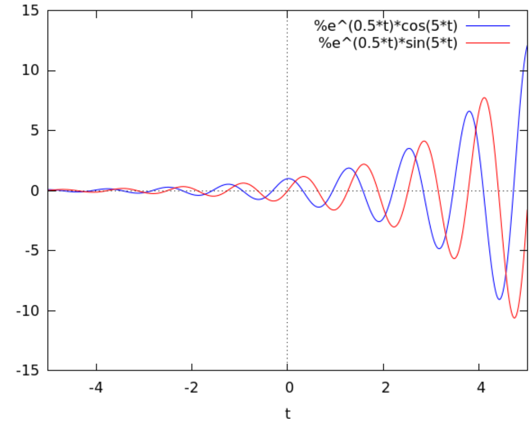
$\Re\{s\} < 0 \quad (\sigma < 0)$



$\Re\{s\} = 0 \quad (\sigma = 0)$



$\Re\{s\} > 0 \quad (\sigma > 0)$



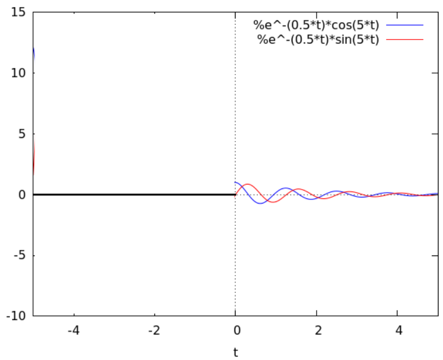
sinusoid function

$$e^{\zeta t} = e^{i\omega t} \qquad (\zeta = i\omega)$$

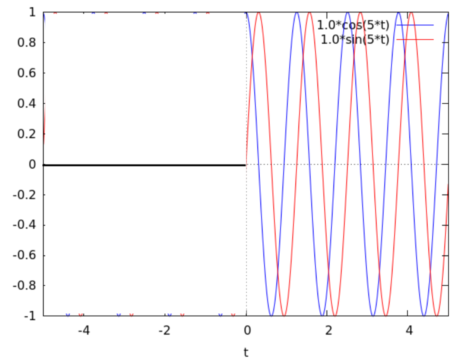
Causal & Everlasting Exponential Functions

- **causal exponential** function

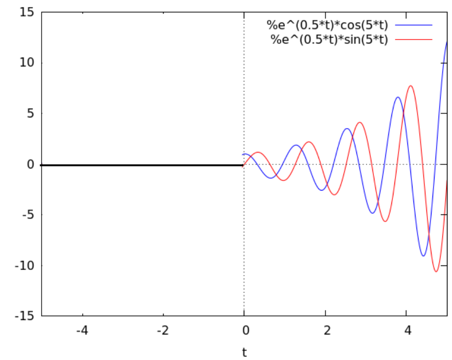
$\Re\{s\} < 0 \quad (\sigma < 0)$



$\Re\{s\} = 0 \quad (\sigma = 0)$



$\Re\{s\} > 0 \quad (\sigma > 0)$

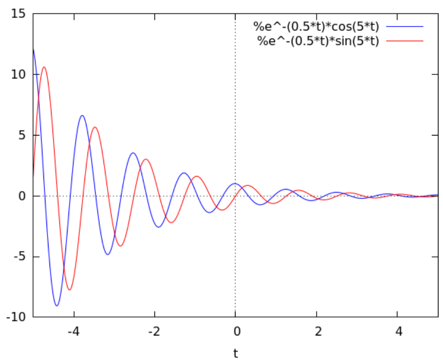


from $t = 0$

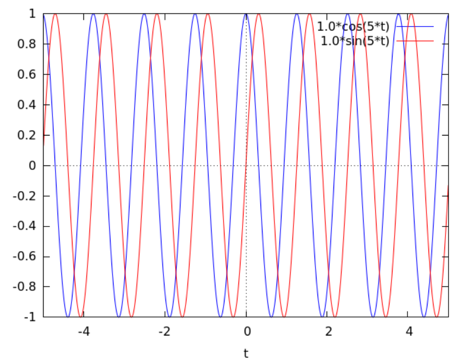
$$e^{st}\, u(t)$$

- **everlasting exponential** function

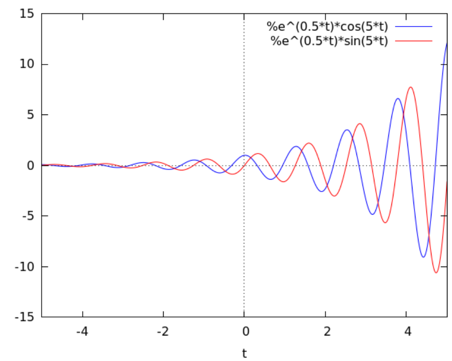
$\Re\{s\} < 0 \quad (\sigma < 0)$



$\Re\{s\} = 0 \quad (\sigma = 0)$



$\Re\{s\} > 0 \quad (\sigma > 0)$



from $t = -\infty$

$$e^{st}$$

Everlasting & Causal Functions

- **exponential** function

$$\left(s = \sigma + i\,\omega\right)$$

- **sinusoid** function

$$\left(\zeta = i\,\omega\right)$$

- **everlasting exponential** function

applied at $t = -\infty$

$$e^{st}$$

- **everlasting sinusoid** function

applied at $t = -\infty$

$$e^{\zeta t}$$

- **causal exponential** function

applied at $t = 0$

applied at $t = 0$

$$e^{st}\,u(t)$$

- **causal sinusoid** function

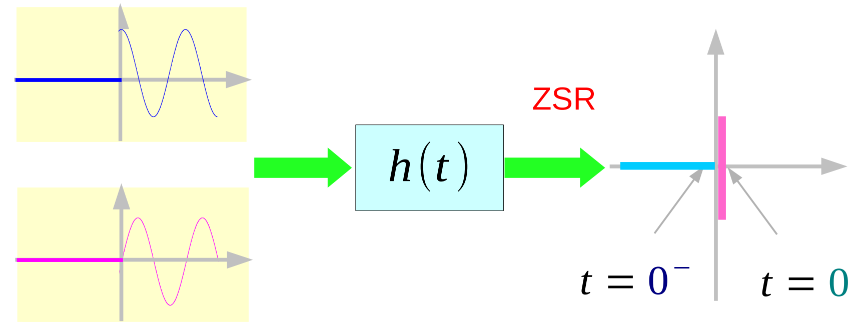
$$e^{\zeta t}\,u(t)$$

Sinusoidal Inputs and States

- **causal sinusoid function**

zero state at $t = 0-$

non-zero state at $t = 0+$



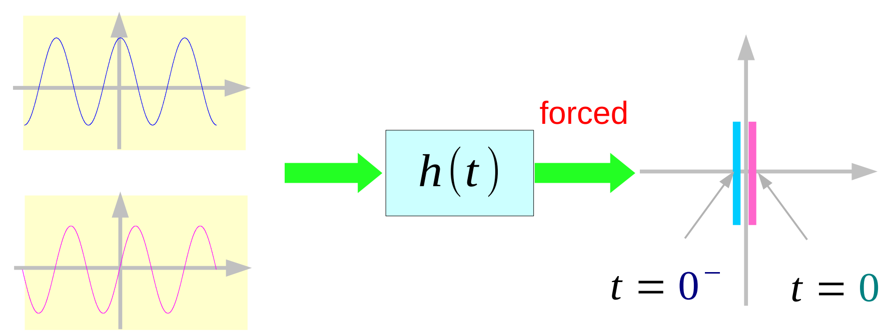
input applied
at $t = 0$

possible discontinuity

- **everlasting sinusoid function**

state at $t = 0-$ = state at $t = 0+$

continuous state between $t = 0-$ & $0+$

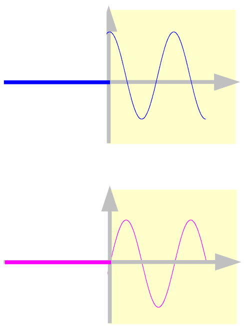


input applied
at $t = -\infty$

continuous states

States before t=0 and after t=0

- causal sinusoid function



input applied at t = 0

$e^{\zeta t} u(t)$

may be discontinuous

non-zero conditions at time 0^+

created by the causal signal

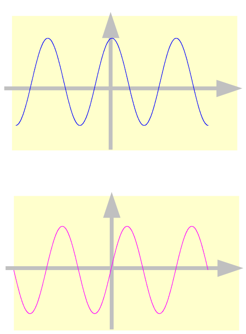
$h(t)$

0	$y^{(0)}(0^+)$
0	$y^{(1)}(0^+)$
0	$y^{(2)}(0^+)$
⋮	⋮

ZSR

Zero State Response

- everlasting sinusoid function



input applied at t = -∞

$e^{\zeta t}$

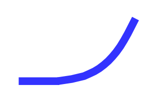
continuous initial conditions

$h(t)$

$y^{(0)}(0^-)$	$y^{(0)}(0^+)$
$y^{(1)}(0^-)$	$y^{(1)}(0^+)$
$y^{(2)}(0^-)$	$y^{(2)}(0^+)$
⋮	⋮

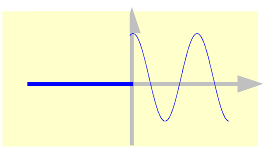
forced

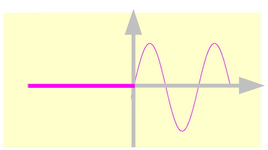
Steady State Response



Initial Conditions for Causal Sinusoidal Inputs

		$[-\infty,\ 0^-]$		$[0^+,\ +\infty]$	
zero input response	⇨	$y_{zi}(0^-) \neq 0$	$=$	$y_{zi}(0^+)$	continuous
+		+		+	
zero state response	⇨	$y_{zs}(0^-) = 0$	$\neq$	$y_{zs}(0^+)$	
		$\parallel$		$\parallel$	
causal sinusoid		$y(0^-)$	$\neq$	$y(0^+)$	
		$\nparallel$		$\parallel$	
natural response	⇨	$y_h(0^-)$	$=$	$y_h(0^+)$	
+		+		+	
forced response	⇨	$y_p(0^-)$	$=$	$y_p(0^+)$	





Initial Conditions for Everlasting Sinusoidal Inputs

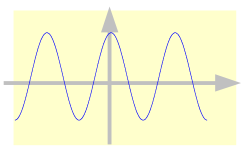
$[-\infty,\ 0^-]$ $[0^+,\ +\infty]$

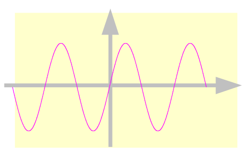
zero input response ⇨ $y_{zi}(0^-) \neq 0$ $=$ $y_{zi}(0^+)$ continuous

+ $\quad$ + $\quad$ +

zero state response ⇨ $y_{zs}(0^-) = 0$ $\neq$ $y_{zs}(0^+)$

$\parallel$ $\quad$ $\parallel$





everlasting sinusoid

$y(0^-)$ $=$ $y(0^+)$

$\parallel$ $\quad$ $\parallel$

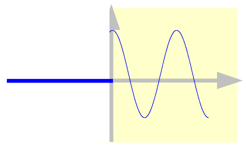
natural response ⇨ $y_h(0^-) = 0$ $=$ $y_h(0^+) = 0$ continuous

+ $\quad$ + $\quad$ +

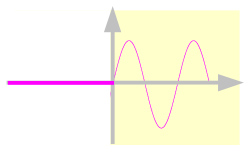
forced response ⇨ $y_p(0^-) \neq 0$ $=$ $y_p(0^+)$ continuous

Sinusoidal Inputs and System Responses

Causal Inputs



input applied
at t = 0



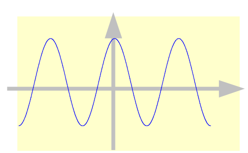
ZSR

$\rightarrow h(t) \rightarrow$

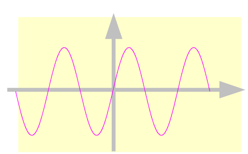
ZIR + ZSR **(O)**

$y_n + y_p$ **(Δ)**

Everlasting Inputs



input applied
at t = -∞



forced

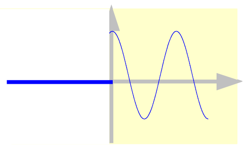
$\rightarrow h(t) \rightarrow$

ZIR + ZSR **(X)**

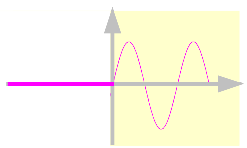
$y_n + y_p$ **(O)**

System Responses and Valid Intervals

Causal Inputs

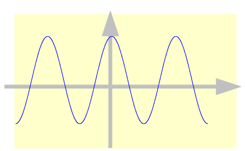


input applied
at t = 0

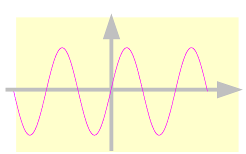


ZIR + ZSR	expression assumes $-\infty < t < +\infty$ already suppressed $t < 0$ *part* **valid interval** $[-\infty, \ +\infty]$
$y_n + y_p$	expression assumes $-\infty < t < +\infty$ must disregard $t < 0$ *part* **valid interval** $[0, \ +\infty]$

Non-causal Inputs



input applied
at t = -∞



ZIR + ZSR	expression assumes $-\infty < t < +\infty$ not valid $t < 0$ *part* **valid interval** $[0, \ +\infty]$
$y_n + y_p$	expression assumes $-\infty < t < +\infty$ valid also for $t < 0$ *part* **valid interval** $[-\infty, \ +\infty]$

Sinusoidal Inputs and ZSR & SSR

zero conditions

Zero State Response

input applied
at t = 0

$e^{\zeta t} u(t)$

$h(t)$

0
0
0
⋮

$h(t)$ can contain *mode* terms
which approach to zero

ZIR ZSR

$$\cancel{\sum_i c_i e^{\lambda_i t}} + \boxed{h(t) * \{e^{\zeta t} u(t)\}}$$

$$\boxed{\sum_i K_i e^{\lambda_i t} + \beta e^{\zeta t}}$$

natural forced

non-causal

$e^{\zeta t}$

$h(t)$

input applied
at t = -∞

$$h(t) * \{e^{\zeta t}\} \Rightarrow \boxed{\lim_{t \to \infty} [h(t) * \{e^{\zeta t}\}]}$$

$$\cancel{\sum_i K_i e^{\lambda_i t}} \to 0$$

$$\sum_i K_i e^{\lambda_i t} + \beta e^{\zeta t} \Rightarrow \boxed{\lim_{t \to \infty} y_p = \lim_{t \to \infty} \beta e^{\zeta t}}$$

natural forced

Steady State Response

Steady State Response of a ZSR

zero conditions

Zero State Response

Steady State Response

ZIR ZSR

$$\cancel{\sum_i c_i e^{\lambda_i t}} + \boxed{h(t) * \{e^{\zeta t} u(t)\}} \xrightarrow{t \to \infty} \boxed{\lim_{t \to \infty} \left[h(t) * \{e^{\zeta t}\}\right]}$$

$$\boxed{\sum_i K_i e^{\lambda_i t} + \beta e^{\zeta t}} \xrightarrow{t \to \infty} \boxed{\lim_{t \to \infty} y_p = \lim_{t \to \infty} \beta e^{\zeta t}}$$

natural forced

$$h(t) * \{e^{\zeta t}\} \Rightarrow \boxed{\lim_{t \to \infty} \left[h(t) * \{e^{\zeta t}\}\right]}$$

0

$$\cancel{\sum_i K_i e^{\lambda_i t}} + \boxed{\beta e^{\zeta t}} \Rightarrow \boxed{\lim_{t \to \infty} y_p = \lim_{t \to \infty} \beta e^{\zeta t}}$$

natural forced

Steady State Response

1st Order System

input applied
at t = -∞

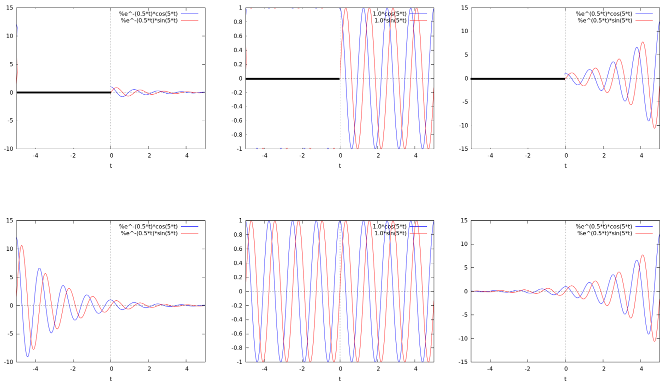
e^{st} → $h(t)$ → $H(s)\, e^{st}$

$e^{\mu t}$ $\quad$ $e^{\lambda t}$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s-\lambda)}$$

1st order
→ single mode
→ complex conjugate
→ real λ

complex exponential
→ complex $\mu = \sigma + j\omega$



causal $x(t)$

non-causal $x(t)$

1^{st} Order System – Convolution Expression

input applied
at $t = -\infty$

1^{st} order → real λ
complex μ

$$e^{st} \longrightarrow \boxed{h(t)} \longrightarrow H(s)\, e^{st}$$

$$e^{\mu t} \qquad e^{\lambda t} \qquad \frac{1}{(\mu-\lambda)} e^{\mu t}$$

$$\boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s-\lambda)}}$$

$$\begin{aligned}
y(t) &= e^{\lambda t} * e^{\mu t} \\
&= \int_{-\infty}^{+\infty} e^{\lambda \tau} e^{\mu (t-\tau)} d\tau \\
&= e^{\mu t} \cdot \int_{-\infty}^{+\infty} e^{\lambda \tau} e^{-\mu \tau} d\tau \\
&= e^{\mu t} \cdot \frac{1}{(\lambda-\mu)} \left[e^{(\lambda-\mu)\tau} \right]_{-\infty}^{+\infty} \\
&= e^{\mu t} \cdot \boxed{H(\mu)}
\end{aligned}$$

Integration Interval

$e^{\mu t} \cdot \int_0^t e^{(\lambda-\mu)\tau} d\tau$	causal $x(t)$ $t-\tau > 0$	causal $h(t)$ $\tau > 0$
$e^{\mu t} \cdot \int_0^{+\infty} e^{(\lambda-\mu)\tau} d\tau$	Non-causal $x(t)$	causal $h(t)$ $\tau > 0$
$e^{\mu t} \cdot \int_{-\infty}^{t} e^{(\lambda-\mu)\tau} d\tau$	causal $x(t)$ $t-\tau > 0$	Non-causal $h(t)$
$e^{\mu t} \cdot \int_{-\infty}^{+\infty} e^{(\lambda-\mu)\tau} d\tau$	Non-causal $x(t)$	Non-causal $h(t)$

Improper Integrations and ROC's

causal $x(t)$ $t-\tau > 0$	causal $h(t)$ $\tau > 0$	$e^{\mu t} \cdot \int_0^t e^{(\lambda-\mu)\tau} d\tau$	$\rightarrow$ $[e^{+(\lambda-\mu)t} - 1]$
Non-causal $x(t)$	causal $h(t)$ $\tau > 0$	$e^{\mu t} \cdot \int_0^{+\infty} e^{(\lambda-\mu)\tau} d\tau$	$\rightarrow$ $[e^{+(\lambda-\mu)\infty} - 1]$
causal $x(t)$ $t-\tau > 0$	Non-causal $h(t)$	$e^{\mu t} \cdot \int_{-\infty}^t e^{(\lambda-\mu)\tau} d\tau$	$\rightarrow$ $[e^{+(\lambda-\mu)t} - e^{-(\lambda-\mu)\infty}]$
Non-causal $x(t)$	Non-causal $h(t)$	$e^{\mu t} \cdot \int_{-\infty}^{+\infty} e^{(\lambda-\mu)\tau} d\tau$	$\rightarrow$ $[e^{+(\lambda-\mu)\infty} - e^{-(\lambda-\mu)\infty}]$

$$e^{\mu t} \cdot \frac{1}{(\lambda-\mu)} \cdot [e^{+(\lambda-\mu)t} - 1] \rightarrow \frac{1}{(\mu-\lambda)} \cdot [e^{\mu t} - e^{\lambda t}]$$

ROC $(\lambda < \mu)$

$$\lim_{t \to \infty} e^{+(\lambda-\mu)t} = 0$$

$$e^{\mu t} \cdot \frac{1}{(\lambda-\mu)} \cdot [-1] \rightarrow \frac{1}{(\mu-\lambda)} \cdot e^{\mu t}$$

$$e^{\mu t} \cdot \frac{1}{(\lambda-\mu)} \cdot [e^{+(\lambda-\mu)t}] \rightarrow \frac{-1}{(\mu-\lambda)} \cdot e^{\lambda t}$$

$$e^{\mu t} \cdot \frac{1}{(\lambda-\mu)} \cdot [-\infty] \rightarrow \infty$$

1^{st} Order System – Laplace Transform

input applied at t = -∞

non-causal

$$e^{st} \longrightarrow \boxed{h(t)} \longrightarrow H(s)\, e^{st}$$

$$e^{\mu t} \qquad e^{\lambda t} \qquad \frac{1}{(\mu-\lambda)} e^{\mu t}$$

$$\frac{d}{dt} y(t) - \lambda\, y(t) \;=\; x(t)$$

$$s\,Y(s) - y(0^-) - \lambda\, Y(s) = X(s)$$

$$Y(s) \;=\; \frac{X(s)}{(s-\lambda)} + \frac{y(0^-)}{(s-\lambda)}$$

Zero State $\quad y(0^-) = 0$

$$X(s) = \frac{1}{(s-\mu)}$$

Zero State Response

$$Y(s) \;=\; \frac{1}{(s-\lambda)(s-\mu)}$$

$$= \frac{1}{(\mu-\lambda)}\left(\frac{1}{(s-\mu)} - \frac{1}{(s-\lambda)}\right)$$

$$\boxed{y(t) \;=\; \frac{1}{(\mu-\lambda)}\left(e^{\mu t} - e^{\lambda t}\right)}$$

Steady State Response

$$\boxed{y(t) \;=\; \lim_{t\to\infty} \frac{1}{(\mu-\lambda)} e^{\mu t}}$$

Characteristic Mode inputs

input applied
at t = -∞

non-causal

$$e^{st} \rightarrow \boxed{h(t)} \rightarrow H(s)\, e^{st}$$

$$e^{\lambda t} \qquad e^{\lambda t} \qquad \cancel{\frac{1}{(\mu-\lambda)} e^{\mu t}}$$

$$h(t) \longleftrightarrow H(s)$$

$$\boxed{H(s) = \int_0^{+\infty} \boxed{h(\tau)}\, e^{-s\tau}\, d\tau}$$

$$y(t) = e^{\lambda t} * e^{\lambda t}$$

$$= \int_{-\infty}^{+\infty} e^{\lambda \tau} e^{\lambda(t-\tau)}\, d\tau$$

$$= e^{\lambda t} \cdot \int_{-\infty}^{+\infty} e^{\lambda \tau} e^{-\lambda \tau}\, d\tau$$

$$= e^{\lambda t} \cdot t$$

$$= e^{\lambda t} \cdot \boxed{H(s)}$$

$e^{st} \cdot \int_0^t 1\, d\tau$	causal *x(t)* $t-\tau > 0$	causal *h(t)* $\tau > 0$
$e^{st} \cdot \int_0^{+\infty} 1\, d\tau$	Non-causal *x(t)*	causal *h(t)* $\tau > 0$
$e^{st} \cdot \int_{-\infty}^t 1\, d\tau$	causal *x(t)* $t-\tau > 0$	Non-causal *h(t)*
$e^{st} \cdot \int_{-\infty}^{+\infty} 1\, d\tau$	Non-causal *x(t)*	Non-causal *h(t)*

Eigenvalue H(s) to an everlasting exponential input

$\boldsymbol{A}\boldsymbol{x} = \lambda\boldsymbol{x}$	$\boldsymbol{A}$	matrix	$\boldsymbol{x}$	eigenvector	λ	eigenvalue
$\frac{d}{dt}y(t) = \lambda y(t)$	$\frac{d}{dt}$	linear map	$y(t)$	eigenfunction	λ	eigenvalue
$L x(t) = \lambda x(t)$	L	linear map	$x(t)$	eigenfunction	λ	eigenvalue

$$\underbrace{\int_{-\infty}^{+\infty} h(\tau)e^{s(t-\tau)}d\tau}_{L\,x} = e^{st}\cdot\boxed{\int_{-\infty}^{+\infty} h(\tau)e^{-s\tau}d\tau} = \underbrace{e^{st}\cdot\boxed{H(s)}}_{x\,\lambda}$$

$$\int_{-\infty}^{+\infty} h(t-\tau)e^{s\tau}d\tau$$

Eigenvalue
A constant with
a parameter s

$y_p(t)$ to an everlasting exponential input

$$\frac{d^N y(t)}{dt^N} + a_1 \frac{d^{N-1} y(t)}{dt^{N-1}} + \cdots + a_{N-1} \frac{d\,y(t)}{dt} + a_N\, y(t) \;=\; b_0 \frac{d^M x(t)}{dt^M} + b_1 \frac{d^{M-1} x(t)}{dt^{M-1}} + \cdots + b_{N-1} \frac{d\,x(t)}{dt} + b_N\, x(t)$$

$$y_p(t) = \boxed{\beta e^{st}} \qquad\qquad x(t) = \boxed{e^{st}}$$

$$\boxed{(D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N)} \cdot y(t) \;=\; \boxed{(b_0 D^M + b_1 D^{M-1} + \cdots + b_{N-1} D + b_N)} \cdot x(t)$$

$$(s^N + a_1 s^{N-1} + \cdots + a_{N-1} s + a_N) \cdot \beta e^{st} \;=\; (b_0 s^M + b_1 s^{M-1} + \cdots + b_{N-1} s + b_N) \cdot e^{st}$$

$$\beta \;=\; \frac{(b_0 s^M + b_1 s^{M-1} + \cdots + b_{N-1} s + b_N)}{(s^N + a_1 s^{N-1} + \cdots + a_{N-1} s + a_N)} \;=\; \frac{P(s)}{Q(s)} \;=\; H(s)$$

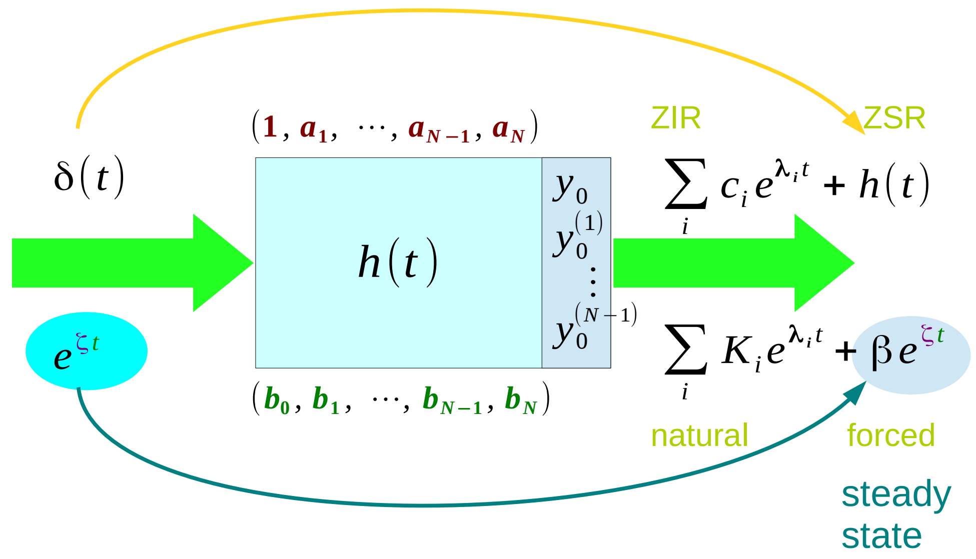
$$D^r e^{st} \;=\; \frac{d^r}{dt^r} e^{st} \;=\; s^r e^{st}$$

$$Q(D) e^{st} \;=\; Q(s) e^{st}$$

$$P(D) e^{st} \;=\; P(s) e^{st}$$

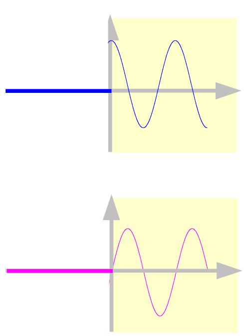
$$\boxed{H(s) \;=\; \frac{P(s)}{Q(s)}}$$

<u>Causal</u> Exponential Total Response



input applied
at t = 0

$for\ a\ given\ s = \zeta$ (pure imaginary)

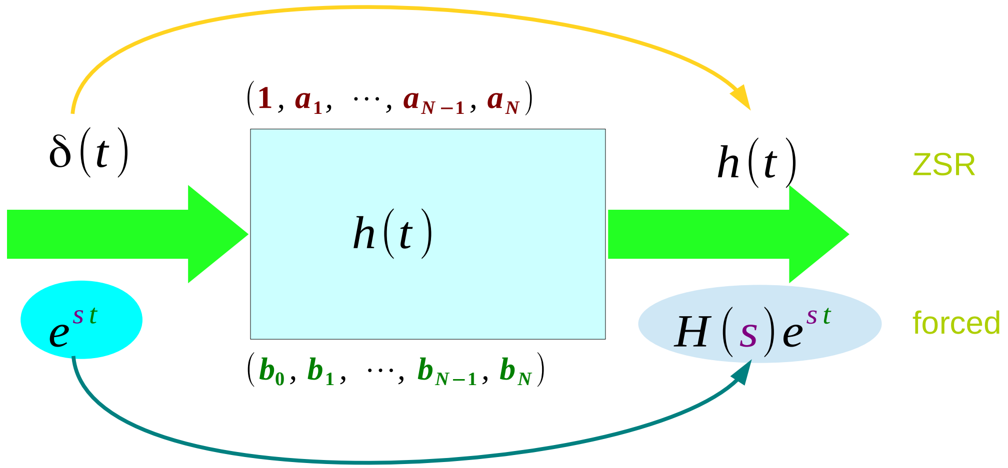


natural forced

$$y(t) = \sum_i K_i e^{\lambda_i t} + \boxed{H(\zeta)} e^{\zeta t} \qquad t \geq 0$$

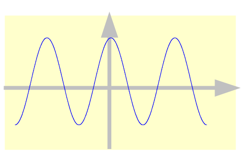
$$Y(s) = \sum_i \frac{K_i}{(s-\lambda_i)} + \frac{H(s)}{(s-\zeta)}$$

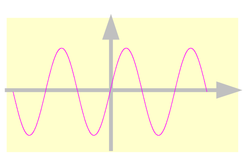
Everlasting Exponential Total Response



input applied
at t = -∞

for a given $s = \zeta$ (pure imaginary)





forced

$$y(t) = H(\zeta)e^{\zeta t} \qquad -\infty < t < +\infty$$

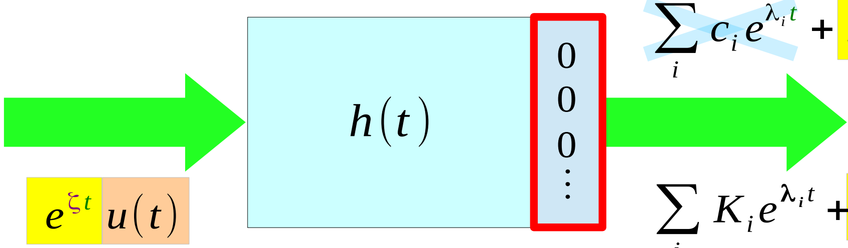
$$Y(s) = H(s)X(s)$$

Forced Response to a causal exponential input

input applied
at t = 0

ZIR ZSR

$$\cancel{\sum_i c_i e^{\lambda_i t}} + h(t) * \{e^{\zeta t} u(t)\}$$



$$\sum_i K_i e^{\lambda_i t} + \beta e^{\zeta t}$$

natural forced

natural forced

$$y(t) = \sum_i K_i e^{\lambda_i t} + \beta e^{\zeta t} \qquad (t > 0)$$

$$Y(s) = \sum_i \frac{K_i}{(s - \lambda_i)} + H(\zeta) X(\zeta)$$

For a forced response	*any complex* ζ
For a steady response	*a pure imaginary* ζ

Steady State Response

$$\lim_{t \to \infty} y_p(t) = \beta e^{\zeta t}$$

Transient Response

$$\{y(t) - \lim_{t \to \infty} y_p(t)\} = \sum_i K_i e^{\lambda_i t}$$

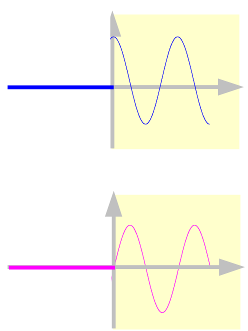
Forced Response to a causal exponential input

$$\left(b_0,\ b_1,\ \cdots,\ b_{N-1},\ b_N\right)$$

$$\left(1,\ a_1,\ \cdots,\ a_{N-1},\ a_N\right)$$

$$Q(D) = \left(D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N\right)$$

$$P(D) = \left(b_0 D^M + b_1 D^{M-1} + \cdots + b_{N-1} D + b_N\right)$$



$$Q(D)\,y(t) = P(D)\,x(t)$$

$$Q(D)[\beta e^{\zeta t}] = P(D)\,e^{\zeta t}$$

$$\beta Q(D)\,e^{\zeta t} = P(D)\,e^{\zeta t}$$

$$D^r e^{\zeta t} = \frac{d^r}{dt^r} e^{\zeta t} = \zeta^r e^{\zeta t}$$

$$Q(D)\,e^{\zeta t} = Q(\zeta)\,e^{\zeta t}$$

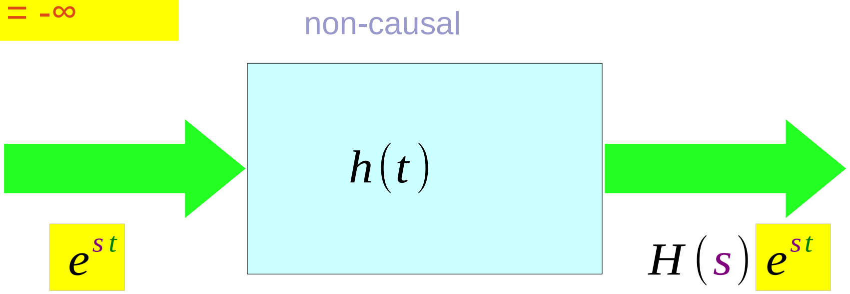
$$P(D)\,e^{\zeta t} = P(\zeta)\,e^{\zeta t}$$

ζ : **NOT** a characteristic mode

$$\boxed{\beta = \frac{P(\zeta)}{Q(\zeta)}}$$

Total Response to an everlasting exponential input

input applied
at t = -∞



$$h(t) \longleftrightarrow H(s)$$

$$\boxed{H(s) = \int_0^{+\infty} \boxed{h(\tau)} e^{-s\tau}\, d\tau}$$

$$\begin{aligned}
y(t) &= h(t) * e^{st} \\
&= \int_{-\infty}^{+\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau \\
&= e^{st} \cdot \boxed{\int_{-\infty}^{+\infty} h(\tau)\, e^{-s\tau}\, d\tau} \qquad \begin{matrix} h(t) = 0 \\ (t < 0) \end{matrix} \\
&= e^{st} \cdot \boxed{H(s)}
\end{aligned}$$

convolution works for
non-causal input x(t) also

But if a causal system is assumed

$h(t) = 0 \quad (t < 0)$

$$\int_{-\infty}^{+\infty} h(\tau)\, e^{-s\tau}\, d\tau = \int_{0}^{+\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

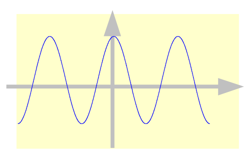
Unilateral
Laplace Transform

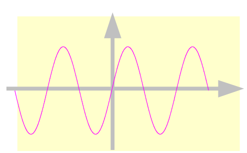
Total Response to an <u>everlasting</u> exponential input

$$\left(b_0,\ b_1,\ \cdots,\ b_{N-1},\ b_N\right)$$
$$\left(1,\ a_1,\ \cdots,\ a_{N-1},\ a_N\right)$$

$$Q(D) = \left(D^N + a_1 D^{N-1} + \cdots + a_{N-1} D + a_N\right)$$
$$P(D) = \left(b_0 D^M + b_1 D^{M-1} + \cdots + b_{N-1} D + b_N\right)$$

$$Q(D)\,y(t) = P(D)\,x(t)$$
$$Q(D)[h(t) * x(t)] = P(D)\,x(t)$$
$$Q(D)[e^{st} H(s)] = P(D)\,e^{st}$$
$$H(s)\,Q(D)\,e^{st} = P(D)\,e^{st}$$





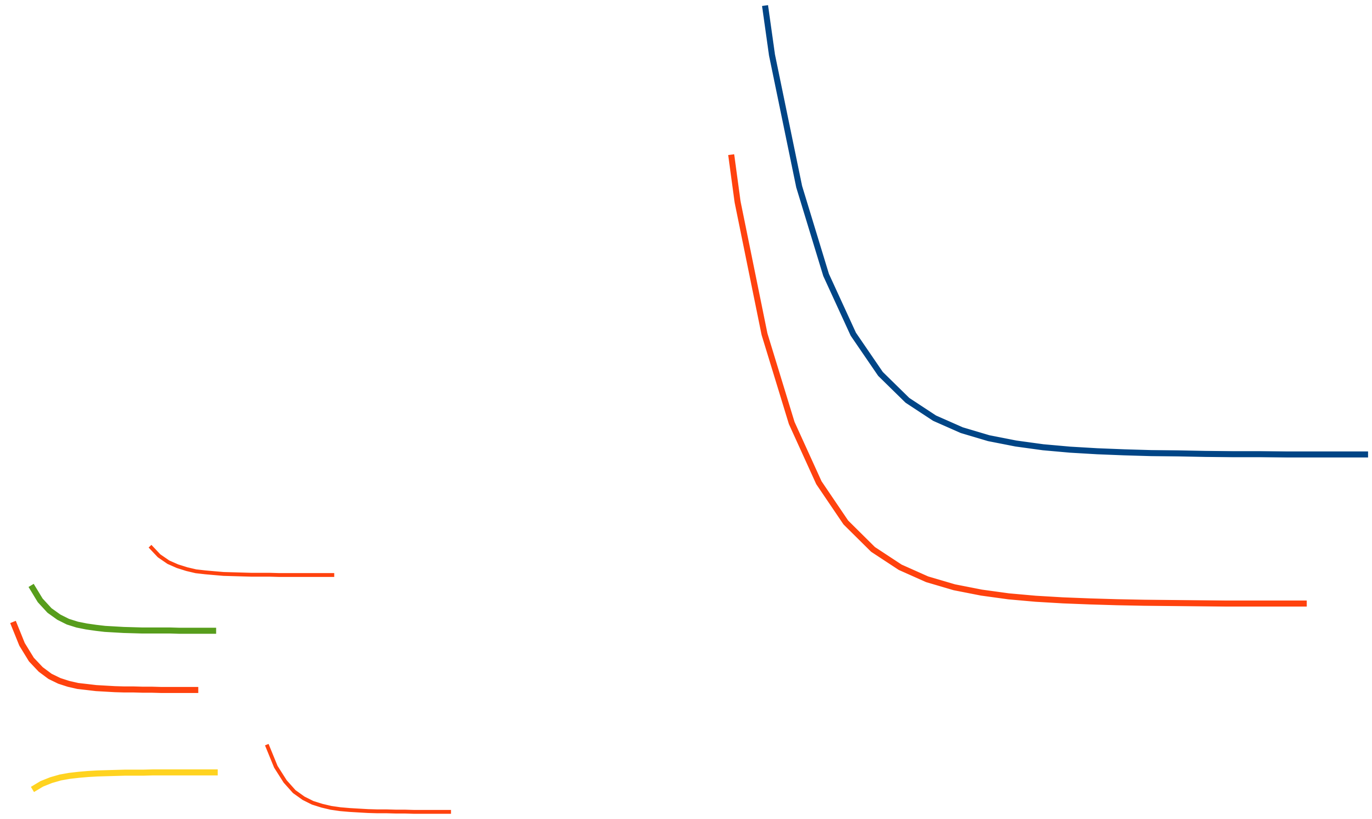
$$D^r e^{st} = \frac{d^r}{dt^r} e^{st} = s^r e^{st}$$

$$Q(D)\,e^{st} = Q(s)\,e^{st}$$
$$P(D)\,e^{st} = P(s)\,e^{st}$$

$$H(s)\,Q(s)\,e^{st} = P(s)\,e^{st}$$

$$\boxed{H(s) = \frac{P(s)}{Q(s)}}$$

Impulse Response h(t)



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