CT Correlation (2A)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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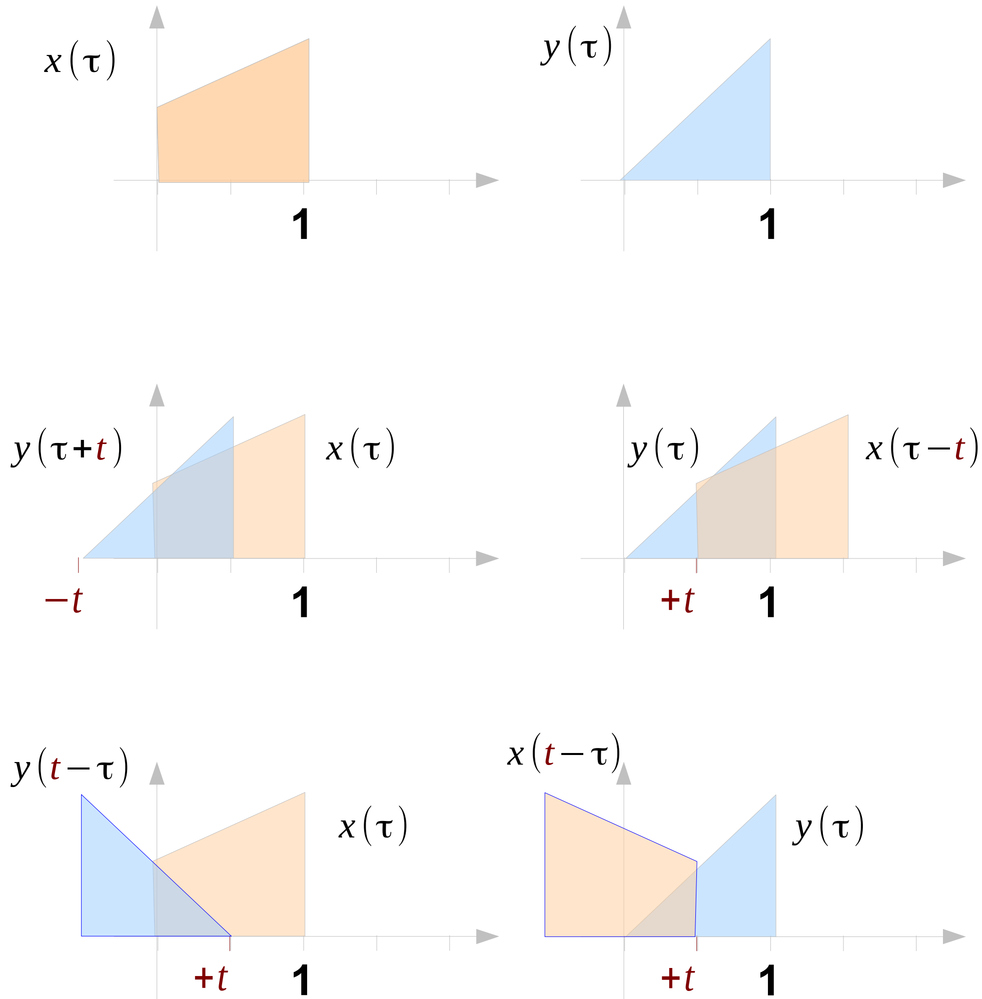
Two types of improper integrals

$$\int_{-\infty}^{+\infty} x(\tau)\, y(\tau+t)\, d\tau$$

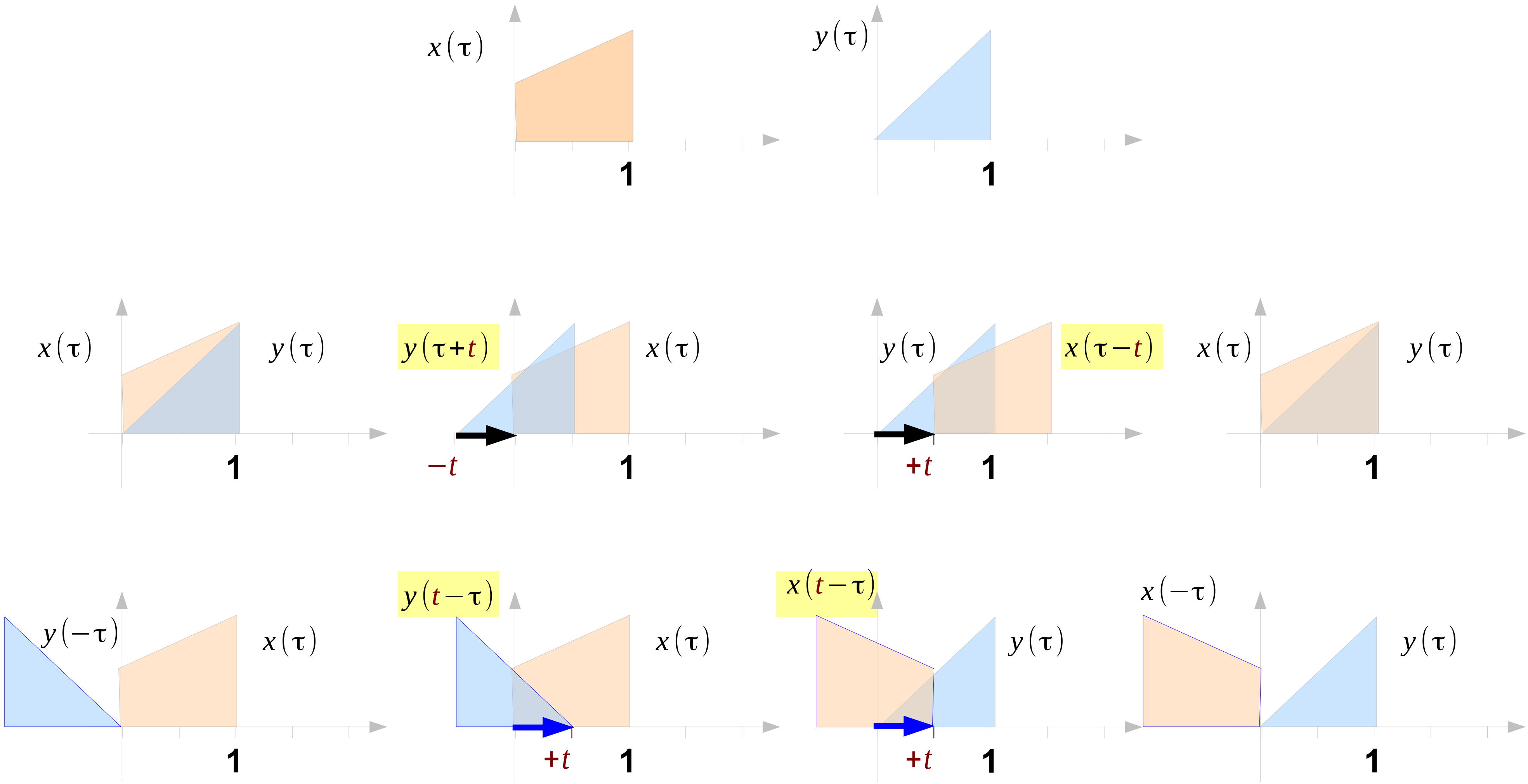
$$\int_{-\infty}^{+\infty} x(\tau-t)\, y(\tau)\, d\tau$$



$$\int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$\int_{-\infty}^{+\infty} x(t-\tau)\, y(\tau)\, d\tau$$

Shift only vs. Flip-Shift



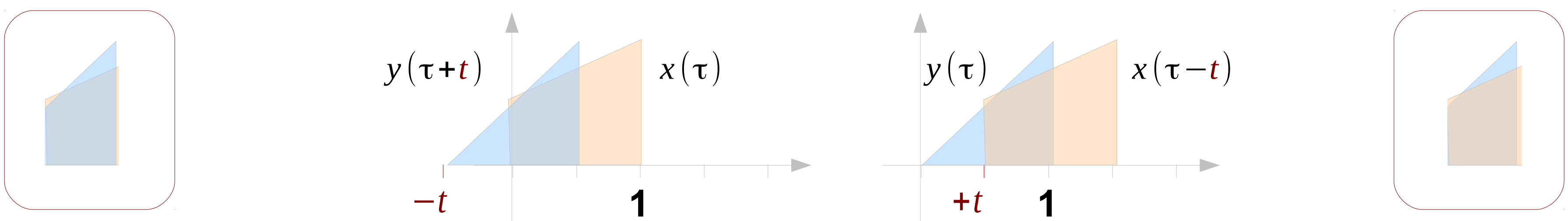
Correlation and Convolution Integrals

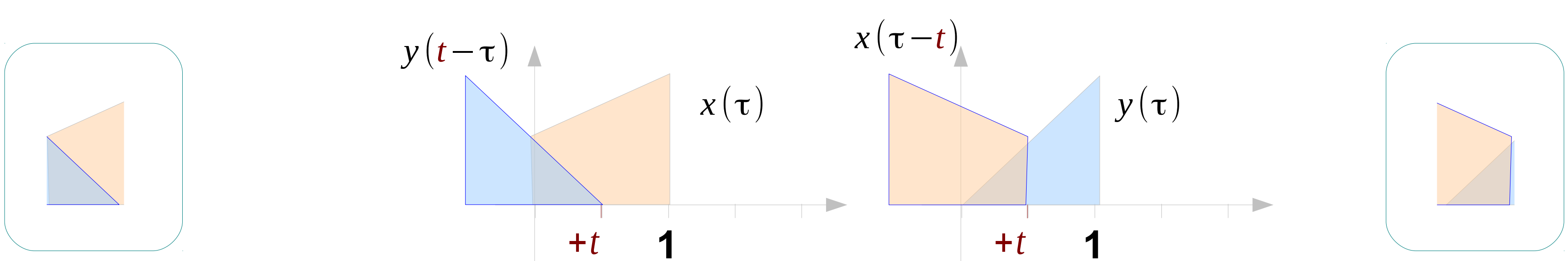
Correlation

$$\int_{-\infty}^{+\infty} x(\tau)\, y(\tau+t)\, d\tau = \int_{-\infty}^{+\infty} x(\tau-t)\, y(\tau)\, d\tau$$

Convolution

$$\int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} x(t-\tau)\, y(\tau)\, d\tau$$





Conventional Notations

Correlation

$$\int_{-\infty}^{+\infty} x(\tau)\, y(\tau+t)\, d\tau = \int_{-\infty}^{+\infty} x(\tau-t)\, y(\tau)\, d\tau$$

Convolution

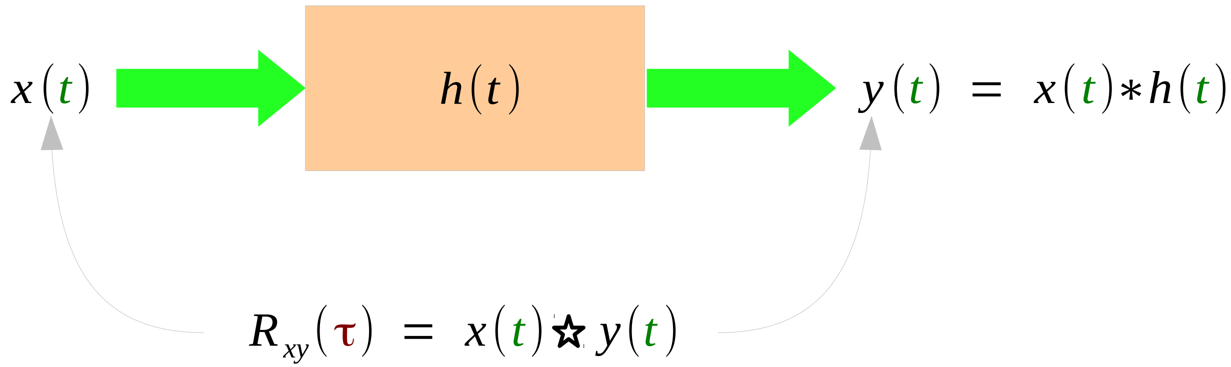
$$\int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} x(t-\tau)\, y(\tau)\, d\tau$$

real $x(t)$, $y(t)$

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt = \int_{-\infty}^{+\infty} x(t-\tau)\, y(t)\, dt = R_{yx}(\tau)$$

$$x(t) * y(t) = \int_{-\infty}^{+\infty} x(t-\tau)\, y(\tau)\, d\tau = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau = y(t) * x(t)$$

Conventional Notations



real $x(t), \; y(t)$

$$R_{xy}(\tau) = x(t) \star y(t) = \int_{-\infty}^{+\infty} x(t) \, y(t+\tau) \, dt = \int_{-\infty}^{+\infty} x(t-\tau) \, y(t) \, dt = R_{yx}(\tau)$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(t-\tau) \, h(\tau) \, d\tau = \int_{-\infty}^{+\infty} x(\tau) \, h(t-\tau) \, d\tau = h(t) * x(t)$$

Correlation and Convolution

real $x(t),\ y(t),\ h(t)$

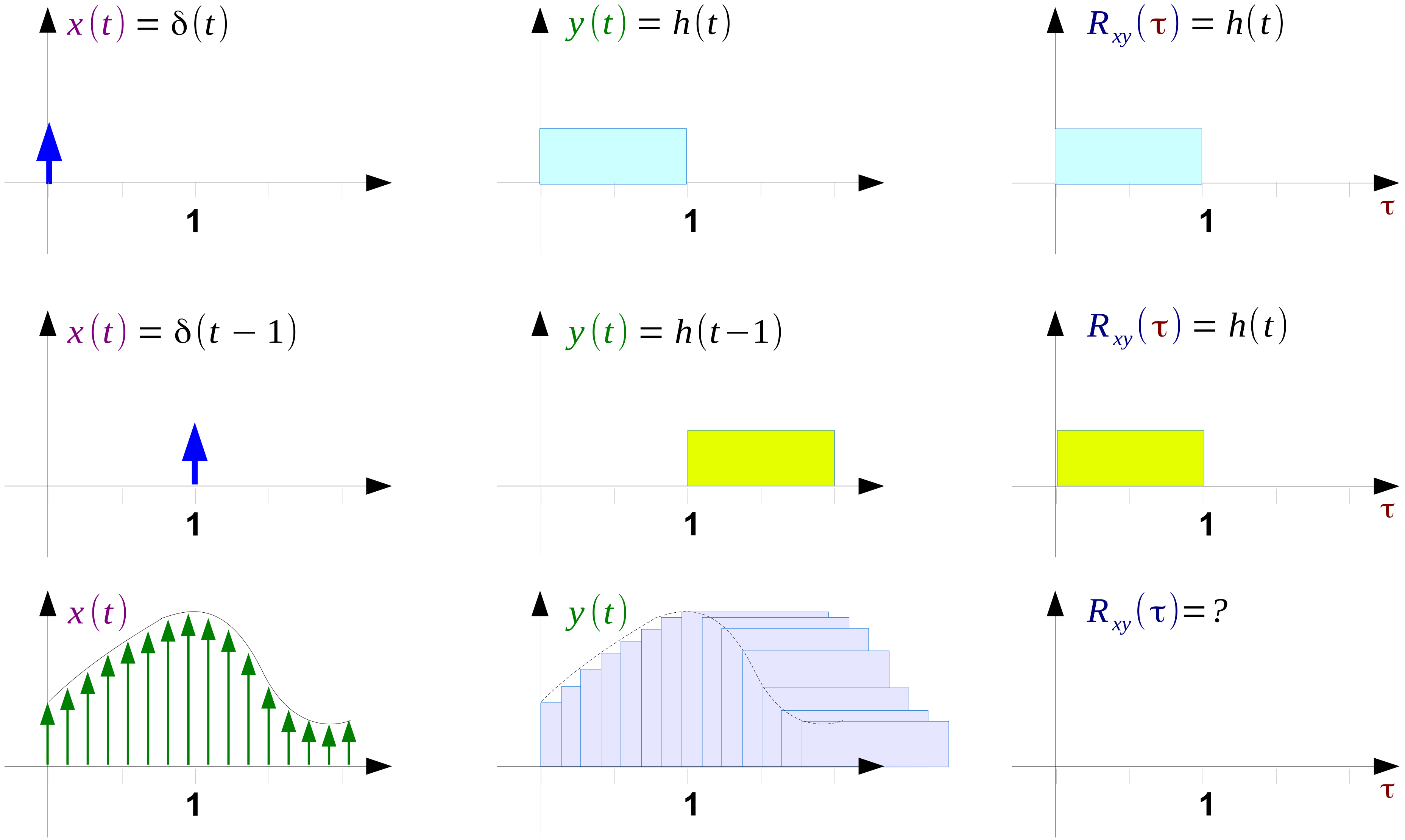
Correlation $$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\,y(t+\tau)\,dt = \int_{-\infty}^{+\infty} x(t-\tau)\,y(t)\,dt$$

Convolution $$x(t)*h(t) = \int_{-\infty}^{+\infty} x(t-\tau)\,h(\tau)\,d\tau = \int_{-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau$$

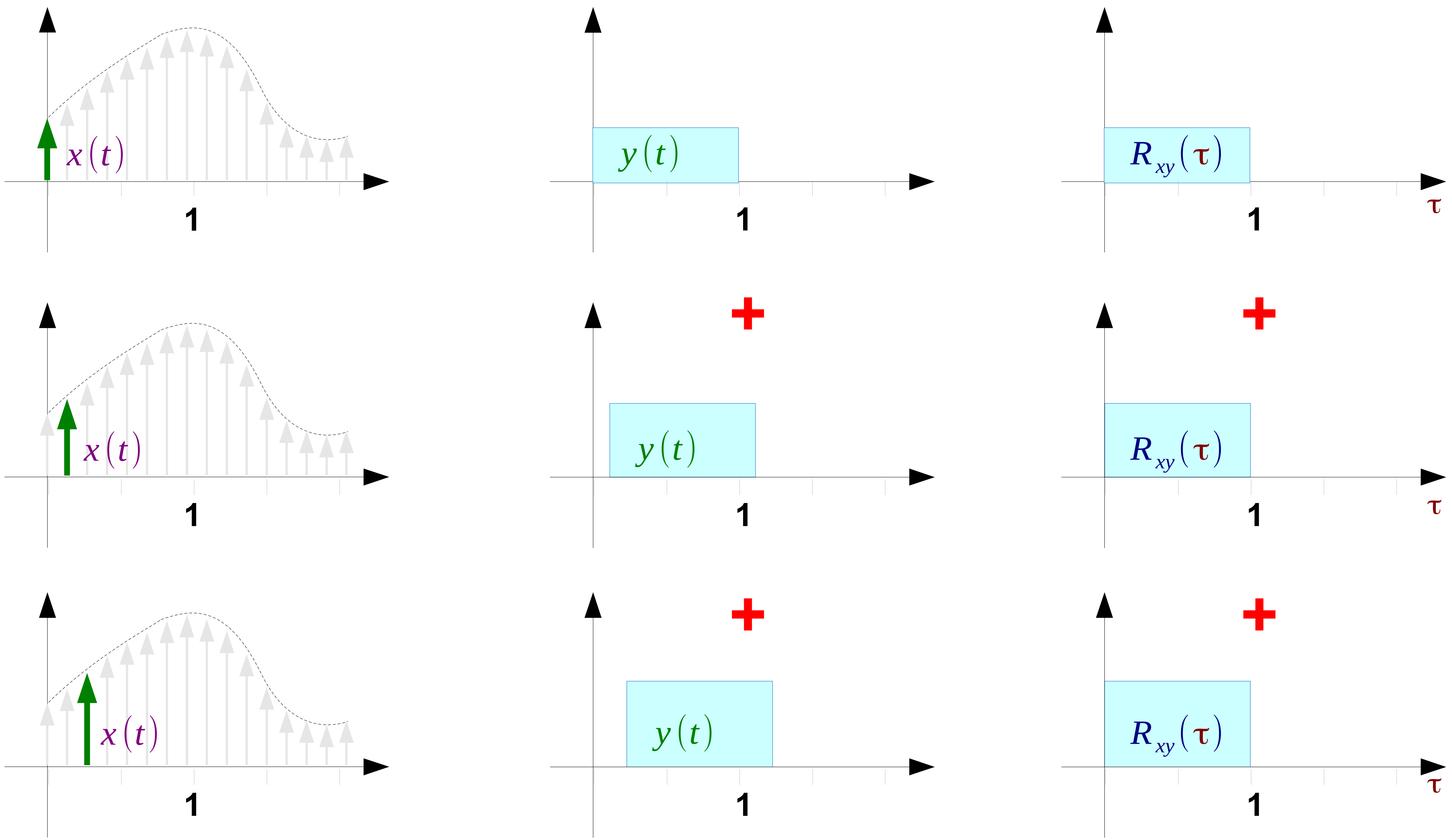
$$x(-t) \Leftrightarrow X^*(f)$$

$$R_{xy}(\tau) = x(-\tau)*y(\tau) \Rightarrow R_{xy}(\tau) \Leftrightarrow X^*(f)\,Y(f)$$

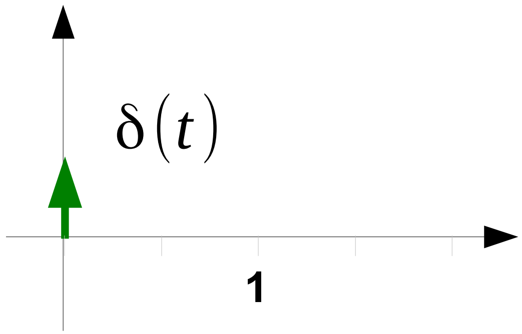
Correlation of y(t) with δ(t) (1)



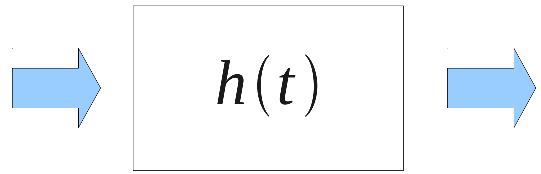
Correlation of y(t) with x(t) (2)

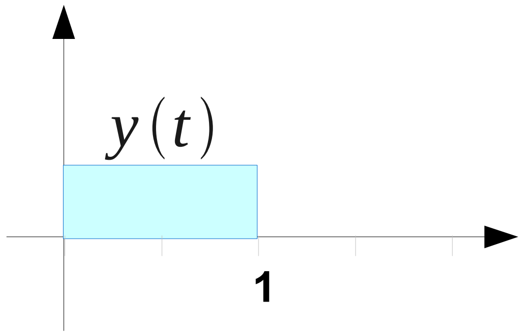


Convolution: delayed response of h(t) (3)



Linear System

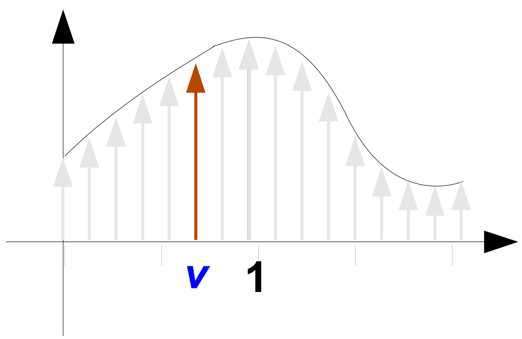


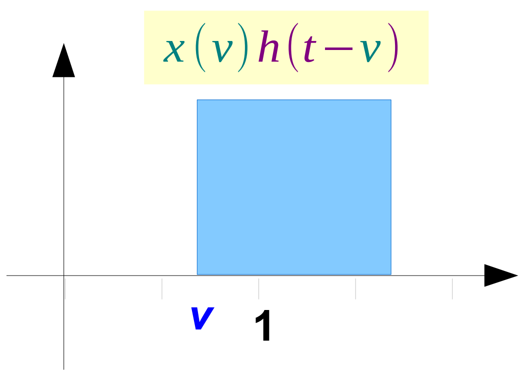


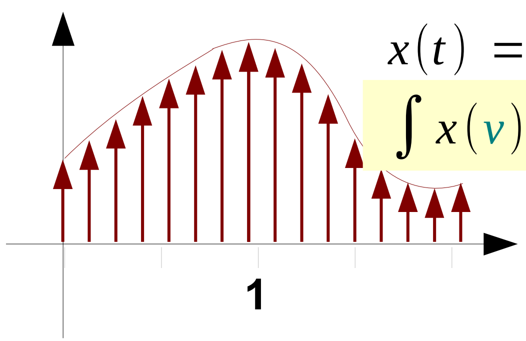
$\delta(t-v) \rightarrow h(t-v)$

$x(v)\,\delta(t-v) \rightarrow x(v)\,h(t-v)$

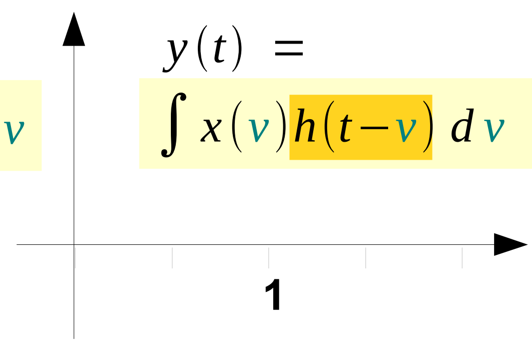
input value at time v → x(v)
delayed impulse response → x(v) h(t - v)



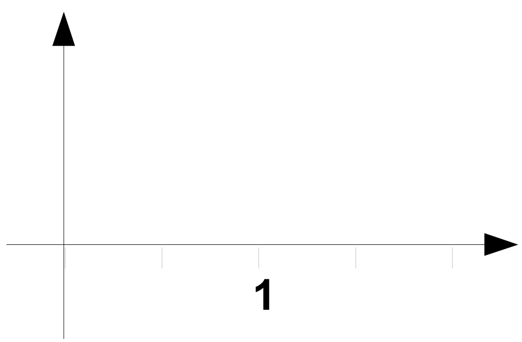




$x(t) = \int x(v)\delta(t-v)\,dv$



$y(t) = \int x(v)h(t-v)\,dv$



Correlation and Convolution

real $x(t),\ y(t),\ h(t)$

Correlation $$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt = \int_{-\infty}^{+\infty} x(t-\tau)\, y(t)\, dt$$

$$y(t) = \int x(v)\, h(t-v)\, dv$$
$$= \int x(t-v)\, h(v)\, dv$$

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t-\tau)\, y(t)\, dt$$
$$= \int_{-\infty}^{+\infty} x(t-\tau) \left[\int x(t-\xi) h(\xi)\, d\xi \right] dt$$
$$= \int h(\xi) \left[\int_{-\infty}^{+\infty} x(t-\tau) x(t-\xi)\, dt \right] d\xi$$
$$= \int h(\xi)\, R_{xx}(\tau-\xi)\, d\xi$$
$$= h(\tau) * R_{xx}(\tau)$$

Correlation and Convolution

real $x(t),\ y(t),\ h(t)$

Correlation $$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt = \int_{-\infty}^{+\infty} x(t-\tau)\, y(t)\, dt$$

$$x(t) = \int x(v)\,\delta(t-v)\, dv \qquad y(t) = \int x(v)\, h(t-v)\, dv$$

$$x(t-\tau) = \int x(v)\,\delta(t-\tau-v)\, dv \qquad y(t+\tau) = \int x(v)\, h(t+\tau-v)\, dv$$

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt$$

$$= \int_{-\infty}^{+\infty} \left[\int x(v)\,\delta(t-v)\, dv\right] \left[\int x(\xi)\, h(t+\tau-\xi)\, d\xi\right] dt$$

$$= \int_{-\infty}^{+\infty} \left[\int\int x(v)\, x(\xi)\,\delta(t-v)\, h(t+\tau-\xi)\, d\xi\, dv\right] dt$$

$$= \int\int x(v)\, x(\xi) \left[\int_{-\infty}^{+\infty} \delta(t-v)\, h(t+\tau-\xi)\, dt\right] d\xi\, dv$$

$$= \int\int x(v)\, x(\xi) \left[\int_{-\infty}^{+\infty} \delta(t-v)\, h(v+\tau-\xi)\, dt\right] d\xi\, dv$$

$$= \int\int x(v)\, x(\xi)\, h(v+\tau-\xi)\, d\xi\, dv$$

Correlation and Convolution

$$
\begin{aligned}
R_{xy}(\tau) &= \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt \\
&= \int \int x(v)\, x(\xi)\, h(v+\tau-\xi)\, d\xi\, dv \\
&= \int \int x(v)\, x(\xi)\, h(v-\xi+\tau)\, d\xi\, dv \\
&= \int R_{xx}(v-\xi)\, h(v-\xi+\tau)\, dv
\end{aligned}
$$

Convolution: Commutative Law

$x(t) \rightarrow$ [$h(t)$] $\rightarrow y(t)$ LTI

$x(v)$ $h(v)$ $\xrightarrow{\text{Flip}}$ $h(-v)$ $\xrightarrow{\text{Shift}}$ $h(t-v)$

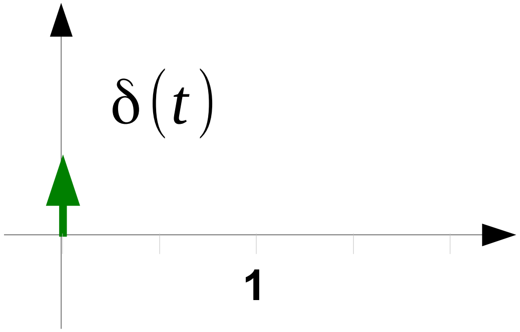
$$\int x(v)h(t-v)\,dv = y(t)$$

$$\int h(v)x(t-v)\,dv = y(t)$$

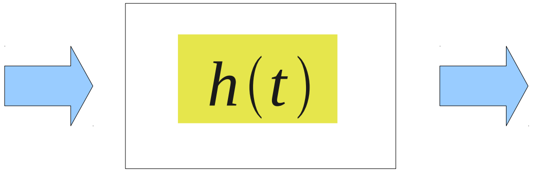
$h(v)$ $x(v)$ $\xrightarrow{\text{Flip}}$ $x(-v)$ $\xrightarrow{\text{Shift}}$ $x(t-v)$

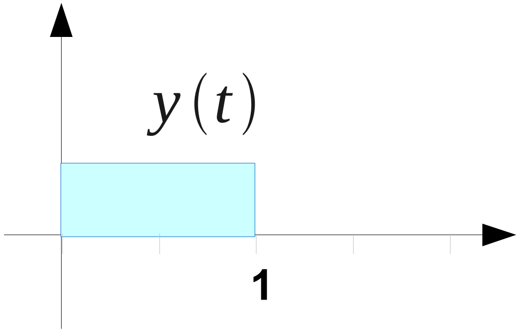
$h(t) \rightarrow$ [$x(t)$] $\rightarrow y(t)$ LTI

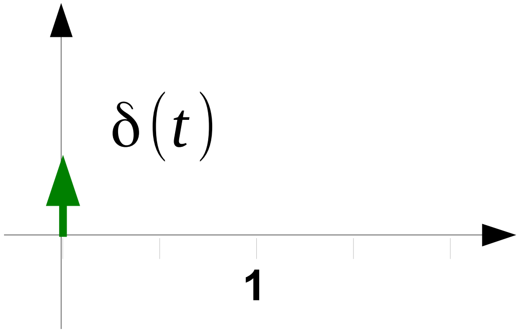
Convolution: delayed response of x(t) (1)



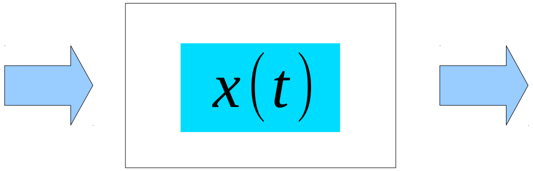
Impulse response *h(t)*

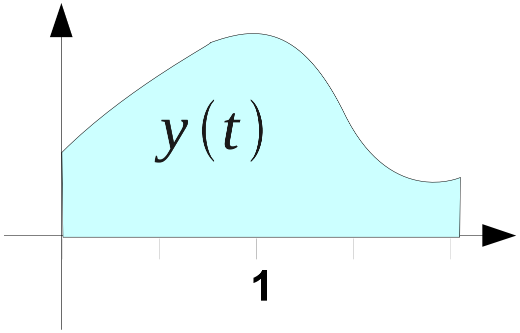


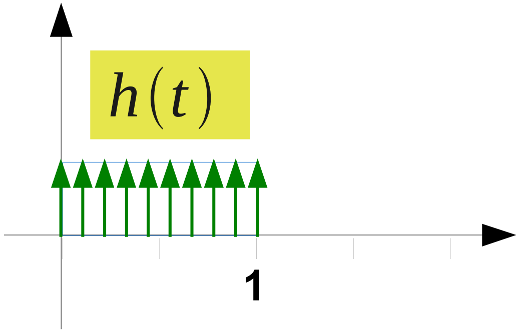




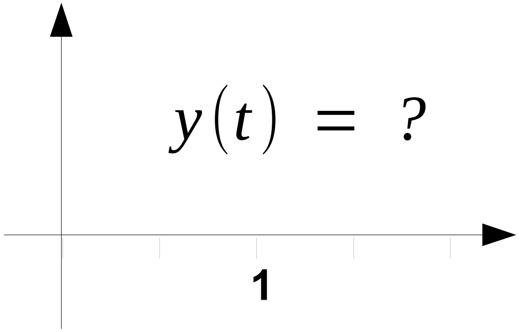
Impulse response *x(t)*



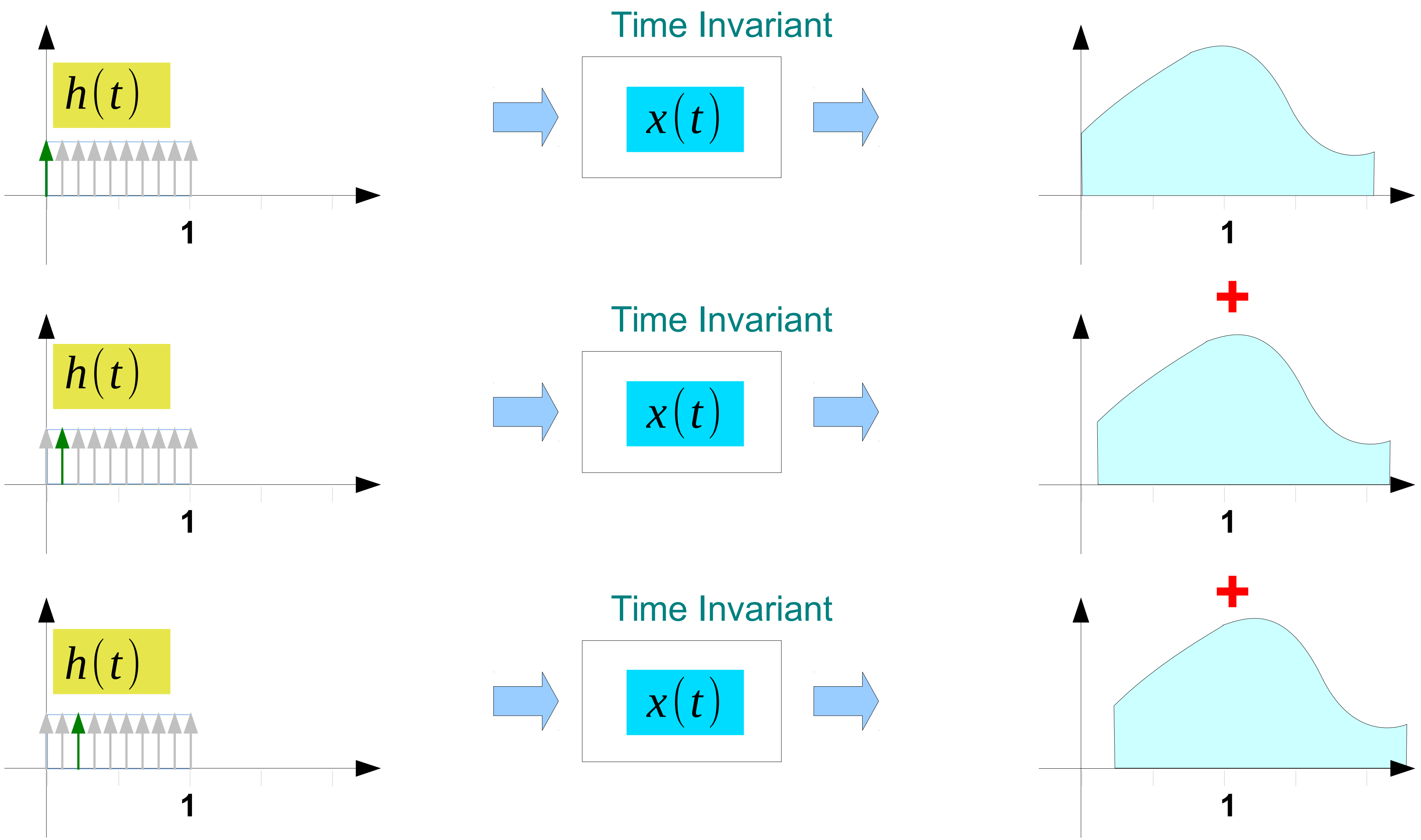


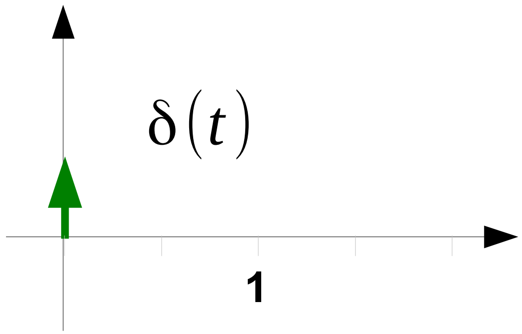


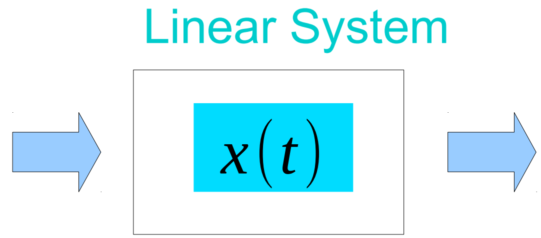
delayed response
by 1

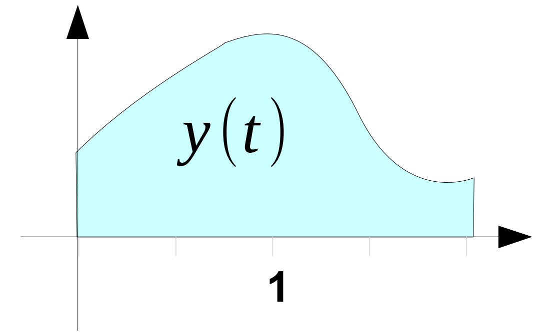


Convolution: delayed response of x(t) (2)



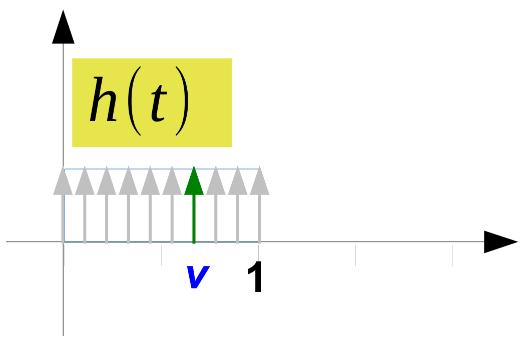




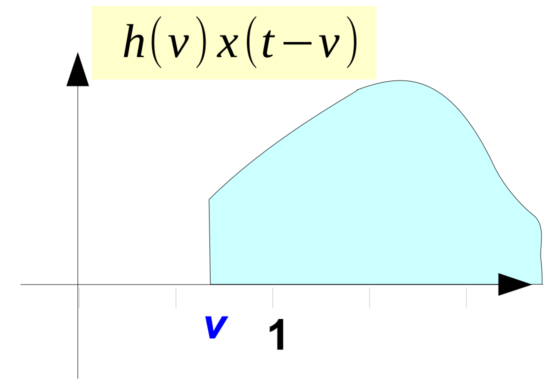


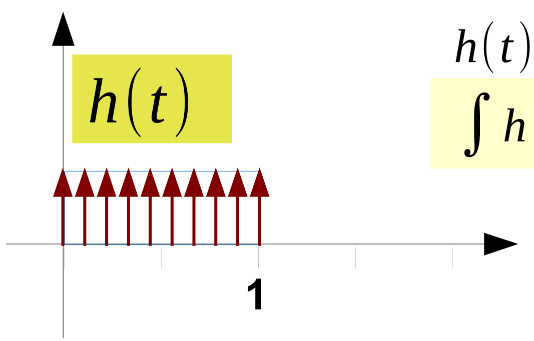
$\delta(t-v)$ → $x(t-v)$

$h(v)\,\delta(t-v)$ → $h(v)\,x(t-v)$



input value at time v
→ h(v)
delayed impulse response
→ h(v) x(t - v)

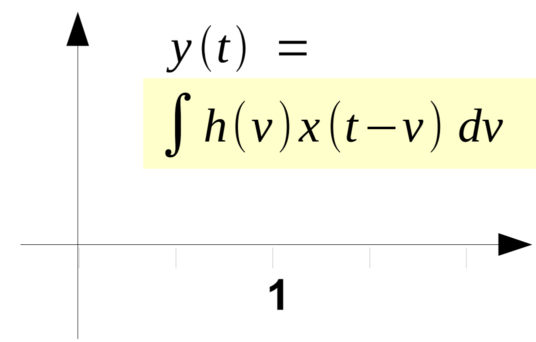




$$h(t) = \int h(v)\,\delta(t-v)\,dv$$

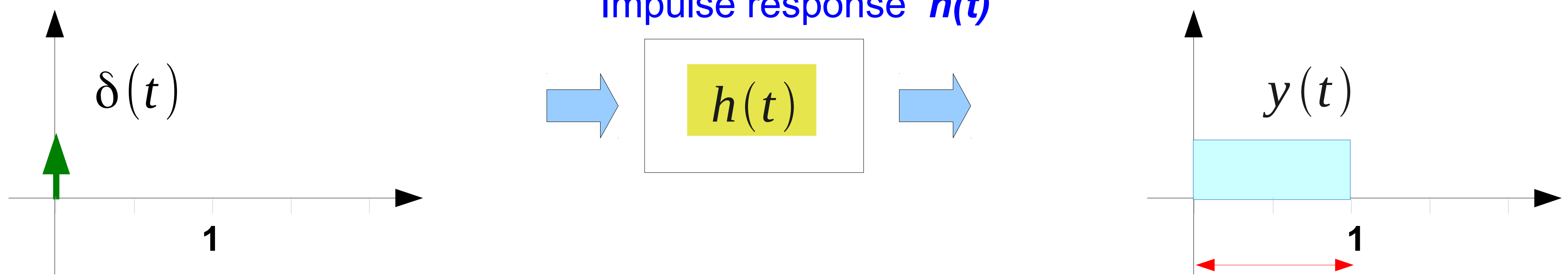
→

$$y(t) = \int h(v)\,x(t-v)\,dv$$



FIR and IIR

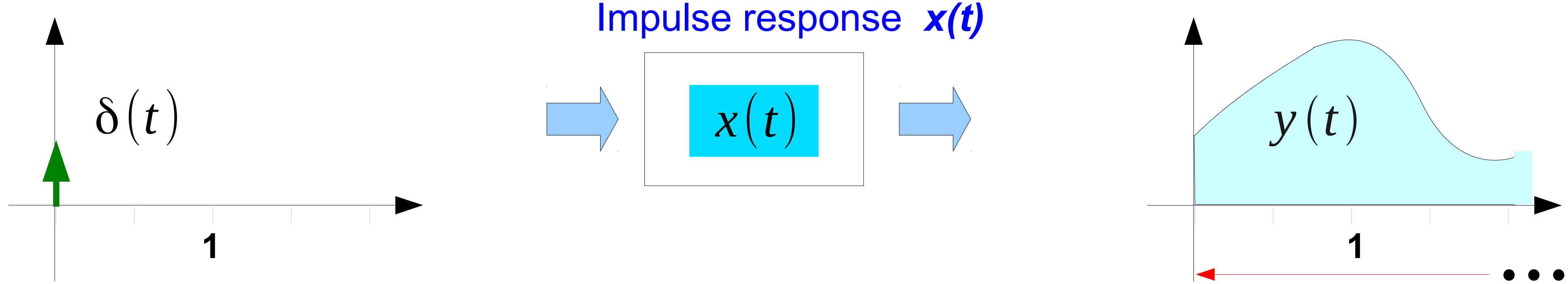
Impulse response ***h(t)***



Finite Impulse Response

Finite Duration

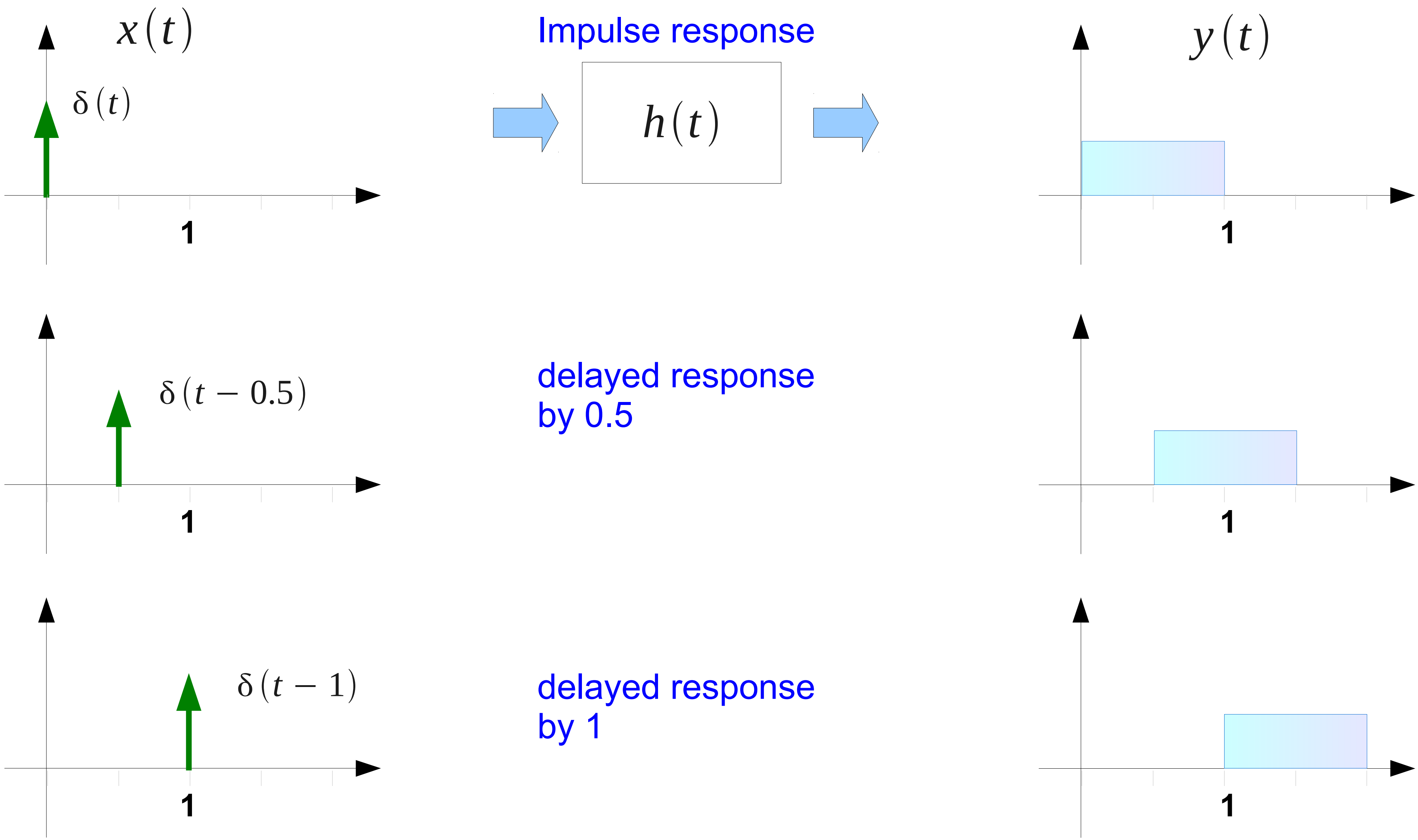
Impulse response ***x(t)***



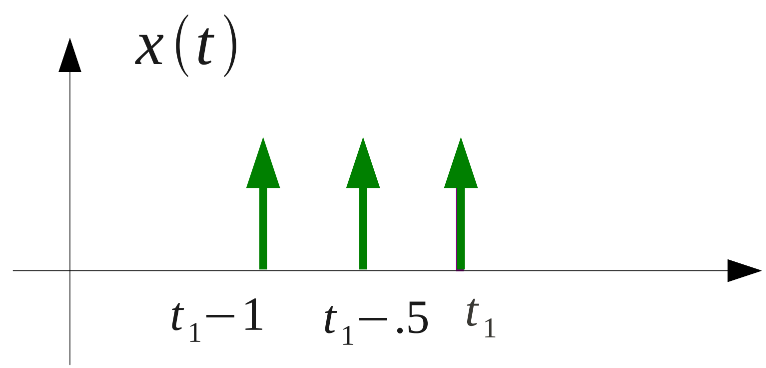
Infinite Impulse Response

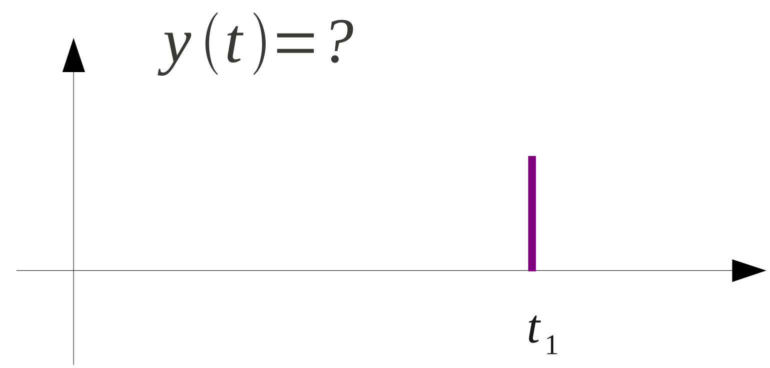
Infinite Duration

Impulse Response



LTI System





$\delta(t-t_1+1) \rightarrow h(t) \rightarrow h(t-t_1+1)$

$\delta(t-t_1+.5) \rightarrow h(t) \rightarrow h(t-t_1+.5)$

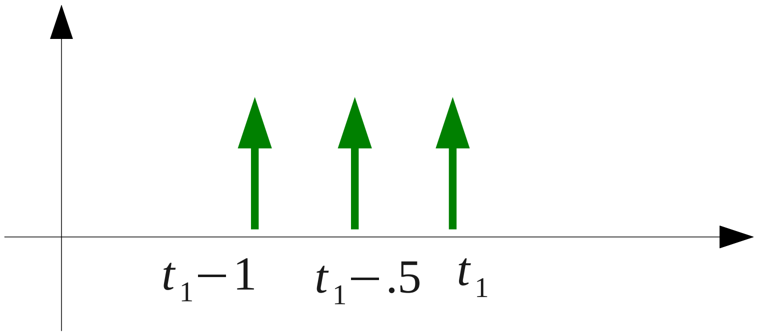
$\delta(t-t_1) \rightarrow h(t) \rightarrow h(t-t_1)$

$\rightarrow h(t) \rightarrow$

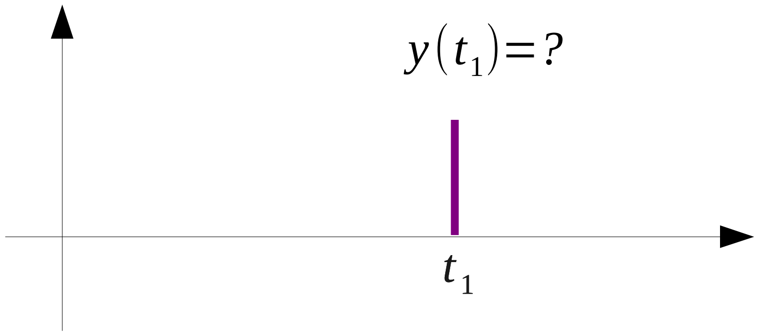
$x(t) =$
$\delta(t-t_1+1) +$
$\delta(t-t_1+.5) +$
$\delta(t-t_1)$

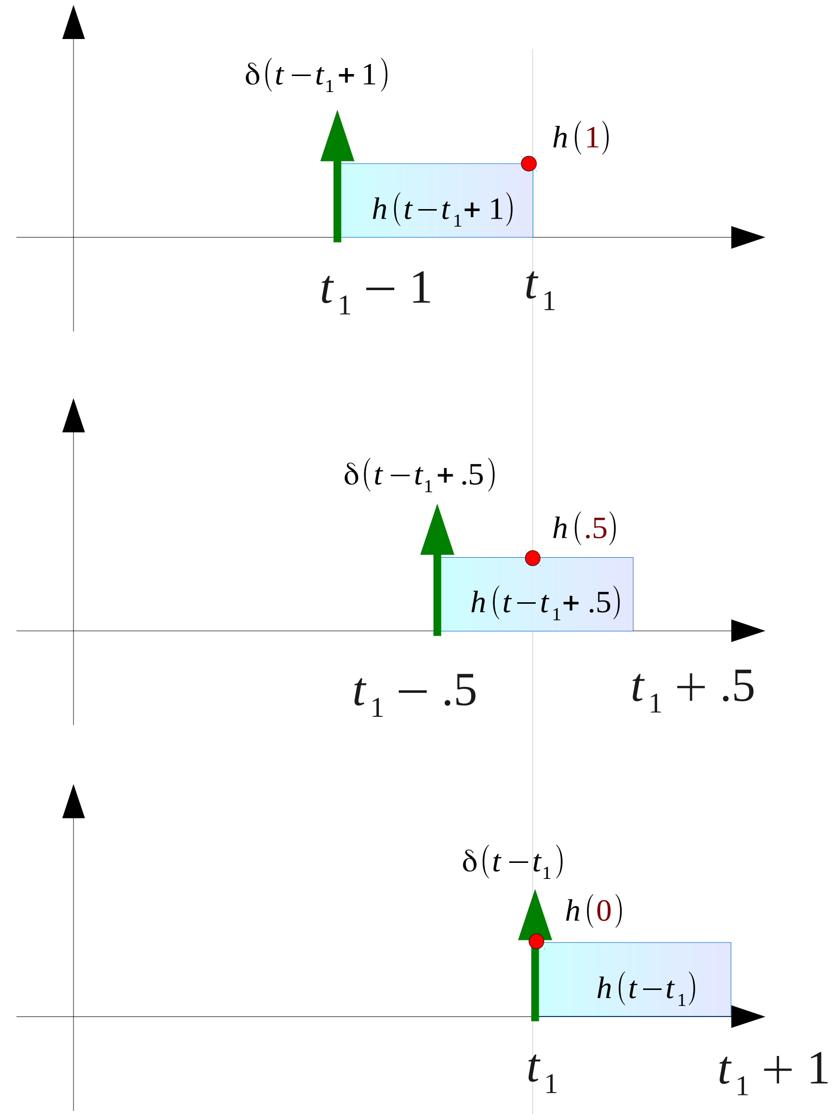
$y(t) =$
$h(t-t_1+1) +$
$h(t-t_1+.5) +$
$h(t-t_1)$

Computing $y(t_1)$: Output at $t = t_1$



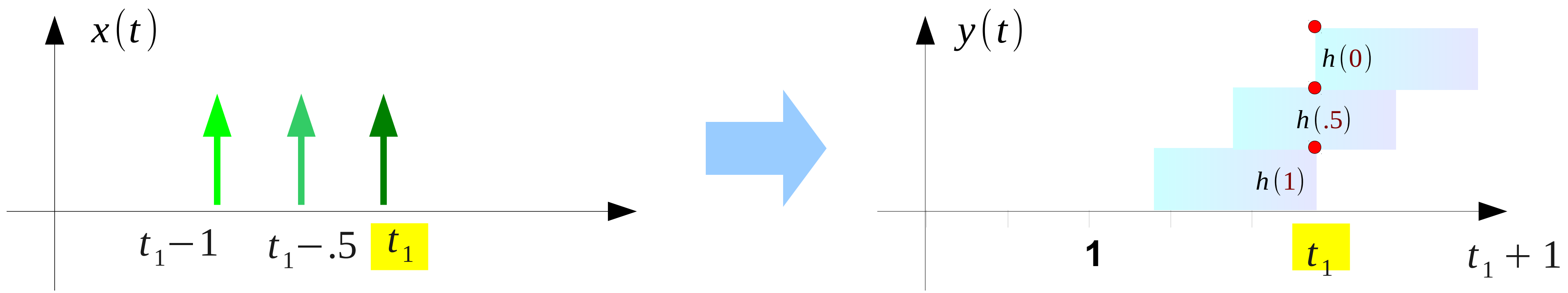
$$x(t) = \delta(t - t_1 + 1) + \delta(t - t_1 + .5) + \delta(t - t_1)$$





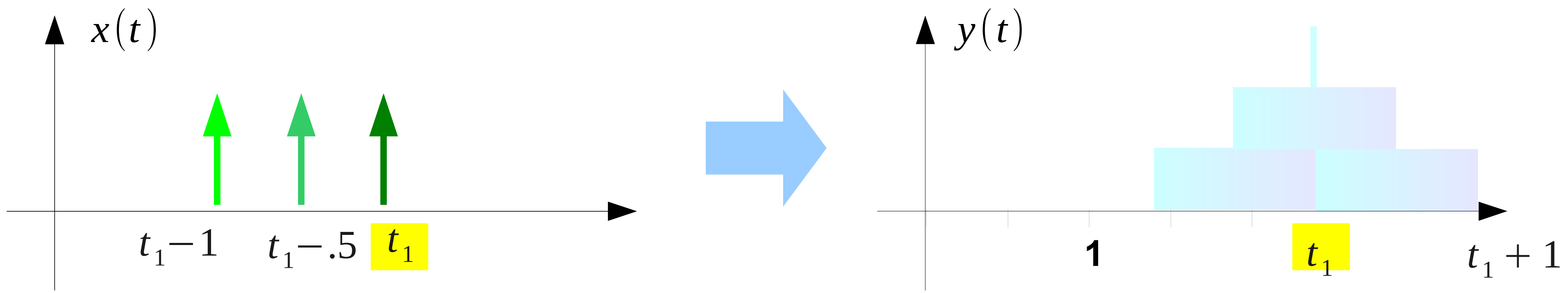
Computing $y(t_1)$: delayed impulse response h(t)

delayed impulse response – h(t)



$x(t) \;=\; \delta(t - t_1 + 1) + \delta(t - t_1 + .5) + \delta(t - t_1)$

$y(t_1) \;=\; h(1) + h(.5) + h(0)$



$x(t) \;=\; \delta(t - t_1 + 1) + \delta(t - t_1 + .5) + \delta(t - t_1)$

$y(t) \;=\; h(t - t_1 + 1) + h(t - t_1 + .5) + h(t - t_1)$

Computing $y(t_1)$: flip and shift $x(t)$

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

Change of variables $t \rightarrow v$

$$x(v) = \delta(v-t_1+1) + \delta(v-t_1+.5) + \delta(v-t_1)$$

Flip around y axis and then shift to the right by t $v \rightarrow t-v$

$$x(t-v) = \delta(t-v-t_1+1) + \delta(t-v-t_1+.5) + \delta(t-v-t_1)$$

$$y(t) = \int x(t-v)\, h(v)\, dv$$

$$= \int \delta(t-v-t_1+1)\, h(v)\, dv$$ $\rightarrow h(t-t_1+1)$ impulse response delayed by t_1-1

$$+ \int \delta(t-v-t_1+.5)\, h(v)\, dv$$ $\rightarrow h(t-t_1+.5)$ impulse response delayed by $t_1-.5$

$$+ \int \delta(t-v-t_1)\, h(v)\, dv$$ $\rightarrow h(t-t_1)$ impulse response delayed by t_1

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$y(t_1) = h(1) + h(.5) + h(0)$$

Computing $y(t_1)$: flip and shift $h(t)$

$h(t)$

Change of variables $t \rightarrow v$

$h(v)$

Flip around y axis and then shift to the right by t $v \rightarrow t-v$

$h(t-v)$

$$y(t) = \int x(v) h(t-v)\, dv$$

$$= \int \delta(v-t_1+1) h(t-v)\, dv \rightarrow h(t-t_1+1)$$ impulse response delayed by t_1-1

$$+ \int \delta(v-t_1+.5) h(t-v)\, dv \rightarrow h(t-t_1+.5)$$ impulse response delayed by $t_1-.5$

$$+ \int \delta(v-t_1) h(t-v)\, dv \rightarrow h(t-t_1)$$ impulse response delayed by t_1

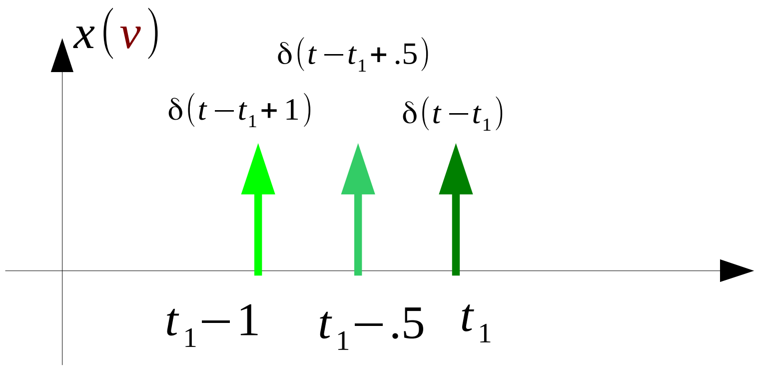
$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

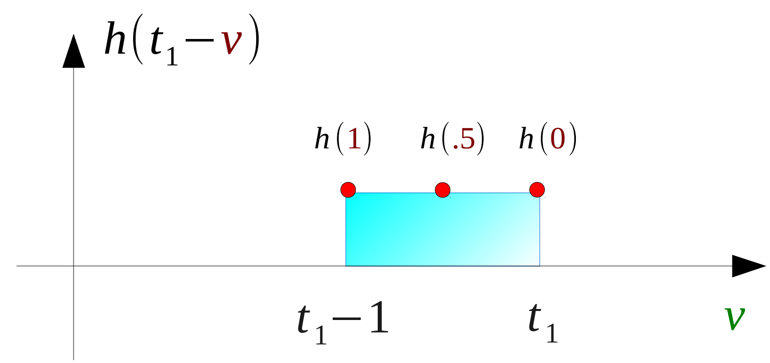
$$\Rightarrow \quad y(t_1) = h(1) + h(.5) + h(0)$$

Computing $y(t_1)$: commutativity (1)

$y(t) = \int x(v)h(t-v)\,dv$		$y(t) = \int x(t-v)h(v)\,dv$
$= \int \delta(v-t_1+1)h(t-v)\,dv$	➡ $h(t-t_1+1)$ ⬅	$= \int \delta(t-v-t_1+1)h(v)\,dv$
$+ \int \delta(v-t_1+.5)h(t-v)\,dv$	➡ $h(t-t_1+.5)$ ⬅	$+ \int \delta(t-v-t_1+.5)h(v)\,dv$
$+ \int \delta(v-t_1)h(t-v)\,dv$	➡ $h(t-t_1)$ ⬅	$+ \int \delta(t-v-t_1)h(v)\,dv$

Flip and Shift h(t)

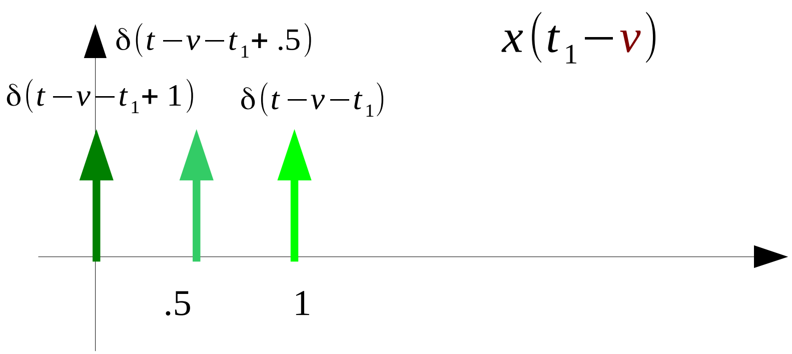


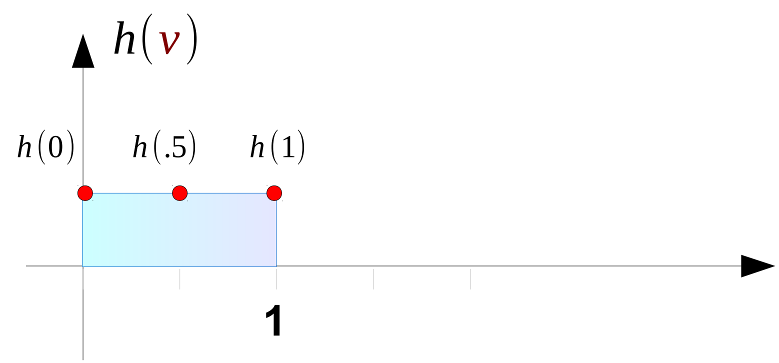


Computing $y(t_1)$: commutativity (2)

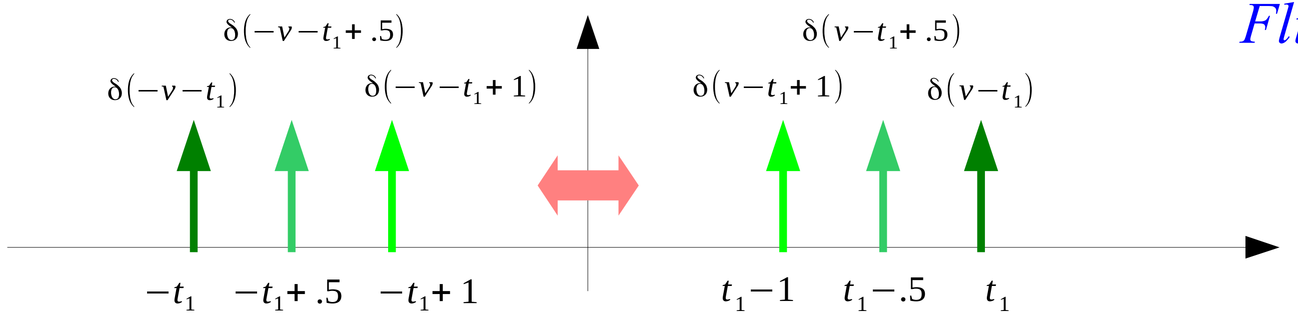
$y(t) = \int x(v) h(t-v)\, dv$		$y(t) = \int x(t-v) h(v)\, dv$
$= \int \delta(v-t_1+1) h(t-v)\, dv$	$h(t-t_1+1)$	$= \int \delta(t-v-t_1+1) h(v)\, dv$
$+ \int \delta(v-t_1+.5) h(t-v)\, dv$	$h(t-t_1+.5)$	$+ \int \delta(t-v-t_1+.5) h(v)\, dv$
$+ \int \delta(v-t_1) h(t-v)\, dv$	$h(t-t_1)$	$+ \int \delta(t-v-t_1) h(v)\, dv$

Flip and shift input x(t)

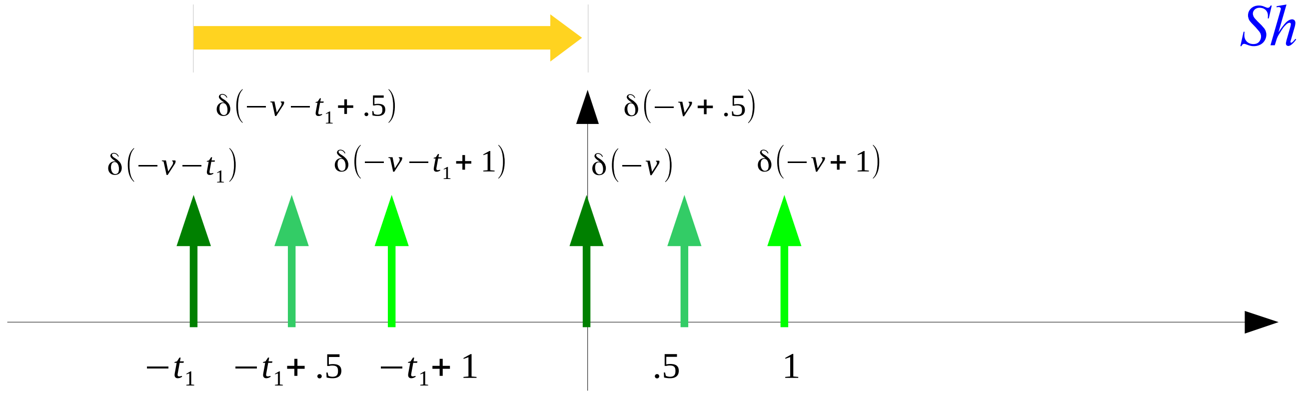




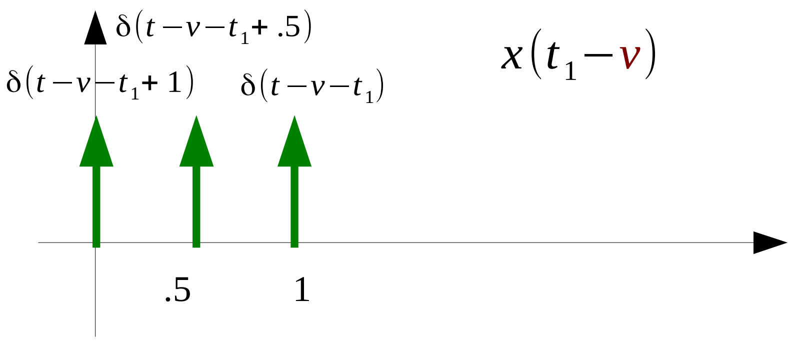
Computing $y(t_1)$: commutativity (3)

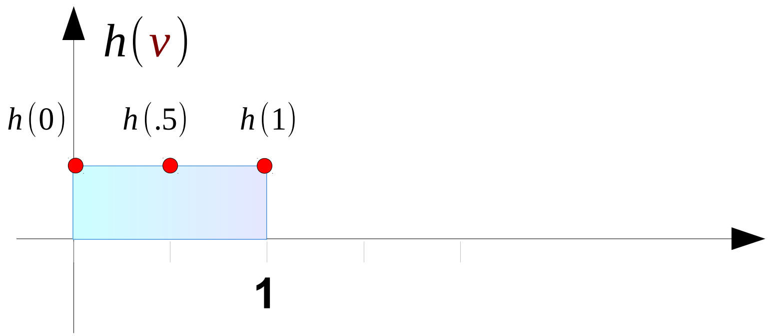


Flip around y-axis

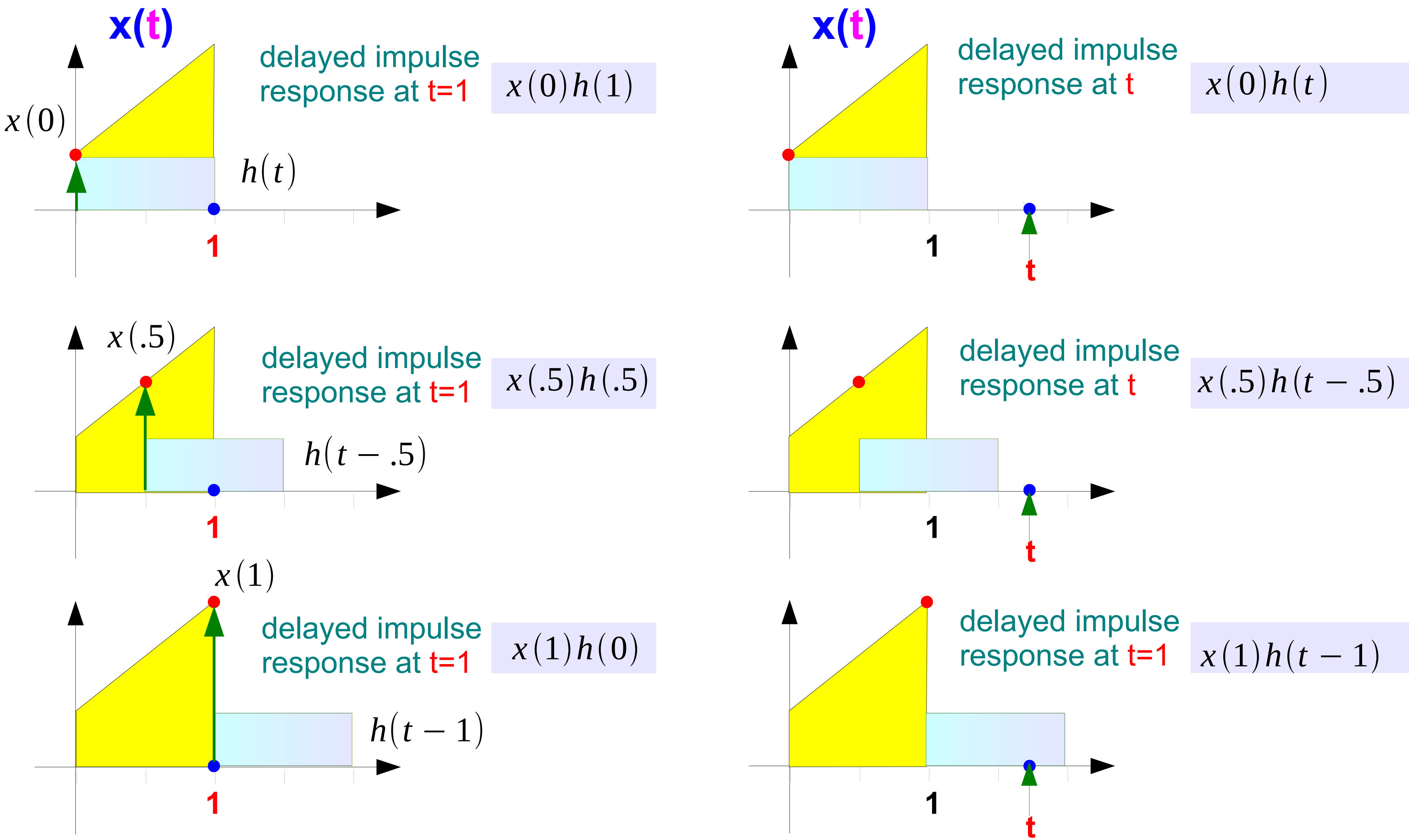


Shift to the right by t_1

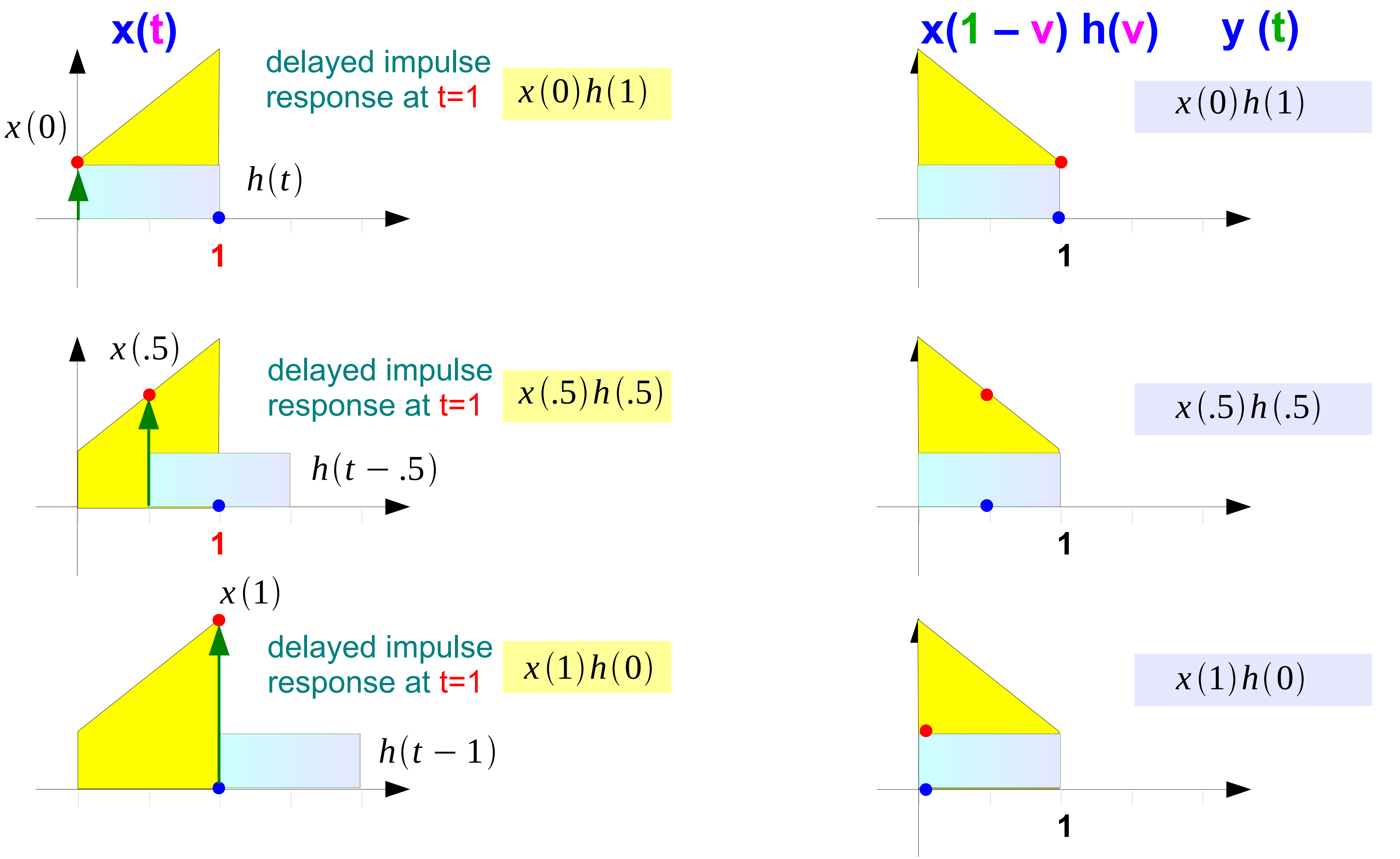




Computing y(1), y(t)

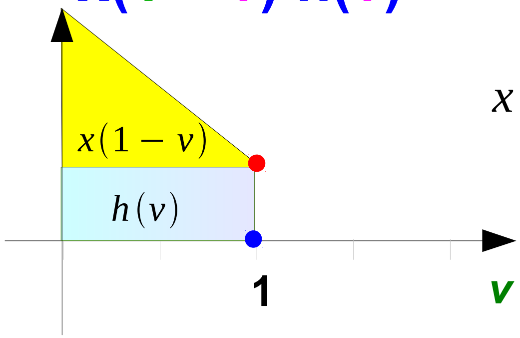


Computing y(1) : shift & flip x(t)

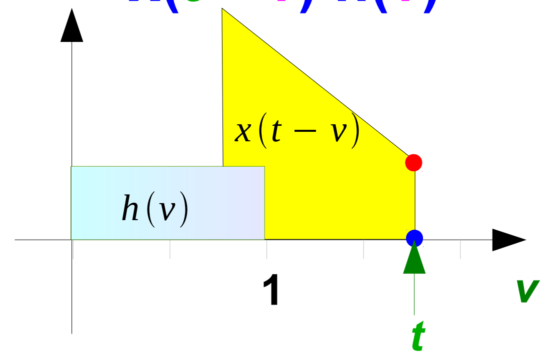


Computing y(t) : shift & flip x(t)

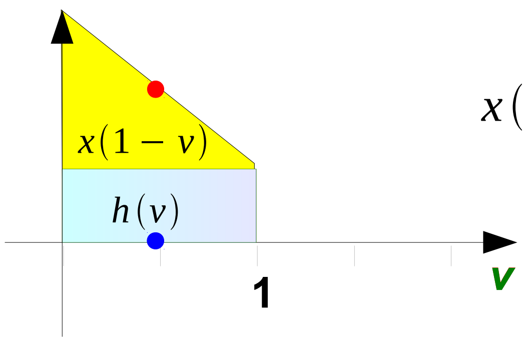
$x(1 - v)\, h(v)$ **$y\,(1)$** **$x(t - v)\, h(v)$** **$y\,(t)$**



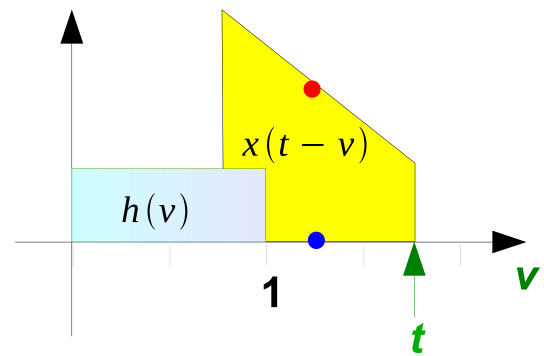
$$x(0)\,h(1)$$



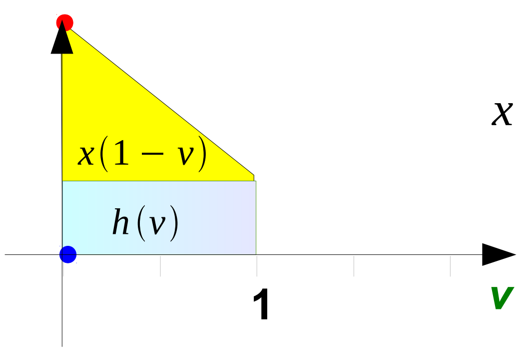
$$x(t-t)\,h(t) = x(0)\,h(t)$$



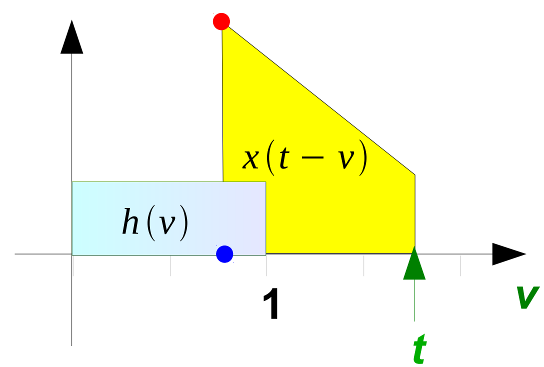
$$x(.5)\,h(.5)$$



$$x(t-t+.5)\,h(t-.5) = x(.5)\,h(t-.5)$$

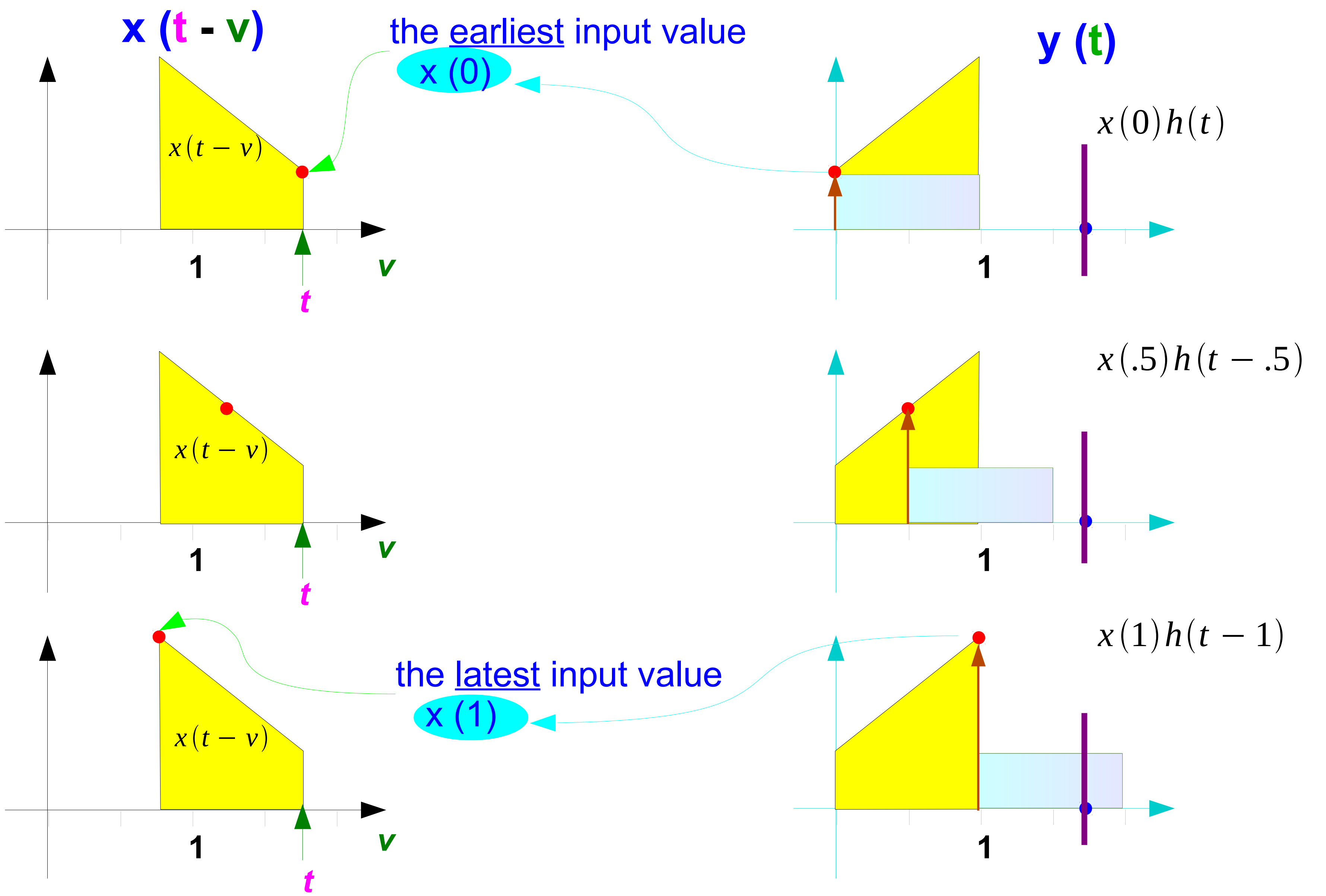


$$x(1)\,h(0)$$



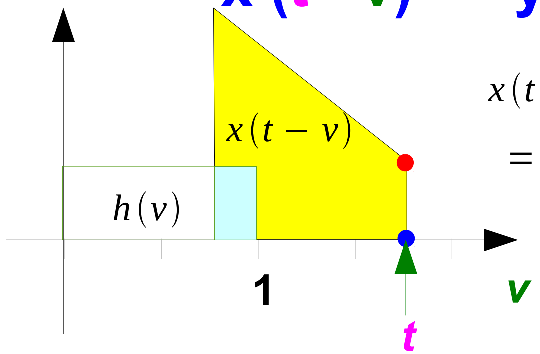
$$x(t-t+1)\,h(t-1) = x(1)\,h(t-1)$$

Computing y(t) : the earliest and latest inputs

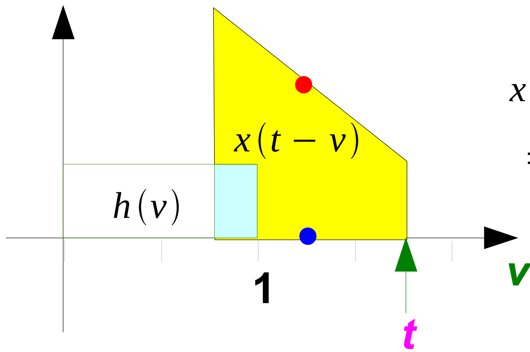


Computing y(t), y(s)

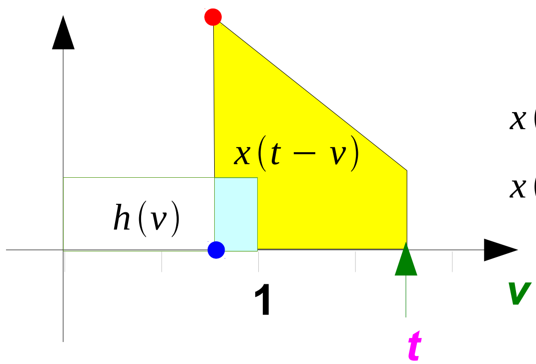
x (t - v) **y (t)**



$$x(t-t)h(t)$$
$$= x(0)h(t)$$

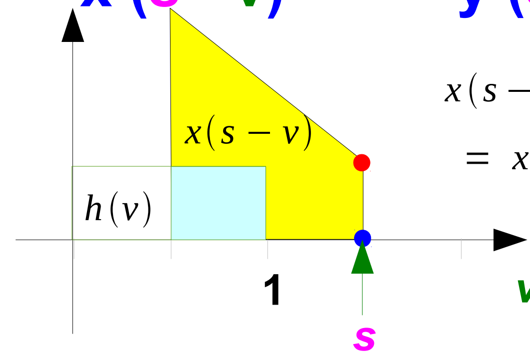


$$x(t-t+.5)h(t-.5)$$
$$= x(.5)h(t-.5)$$

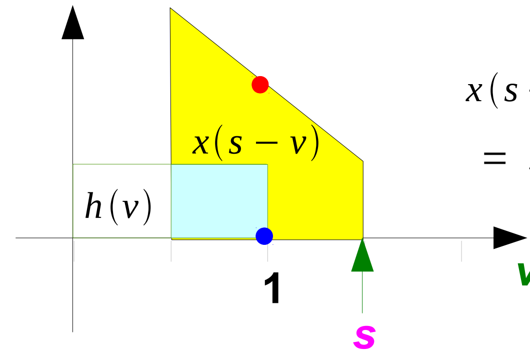


$$x(t-t+1)h(t-1)$$
$$x(1)h(t-1)$$

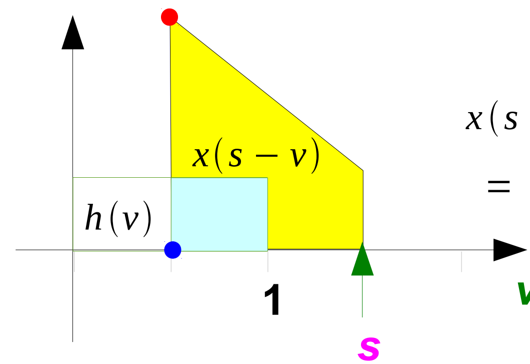
x (s - v) **y (s)**



$$x(s-s)h(s)$$
$$= x(0)h(s)$$

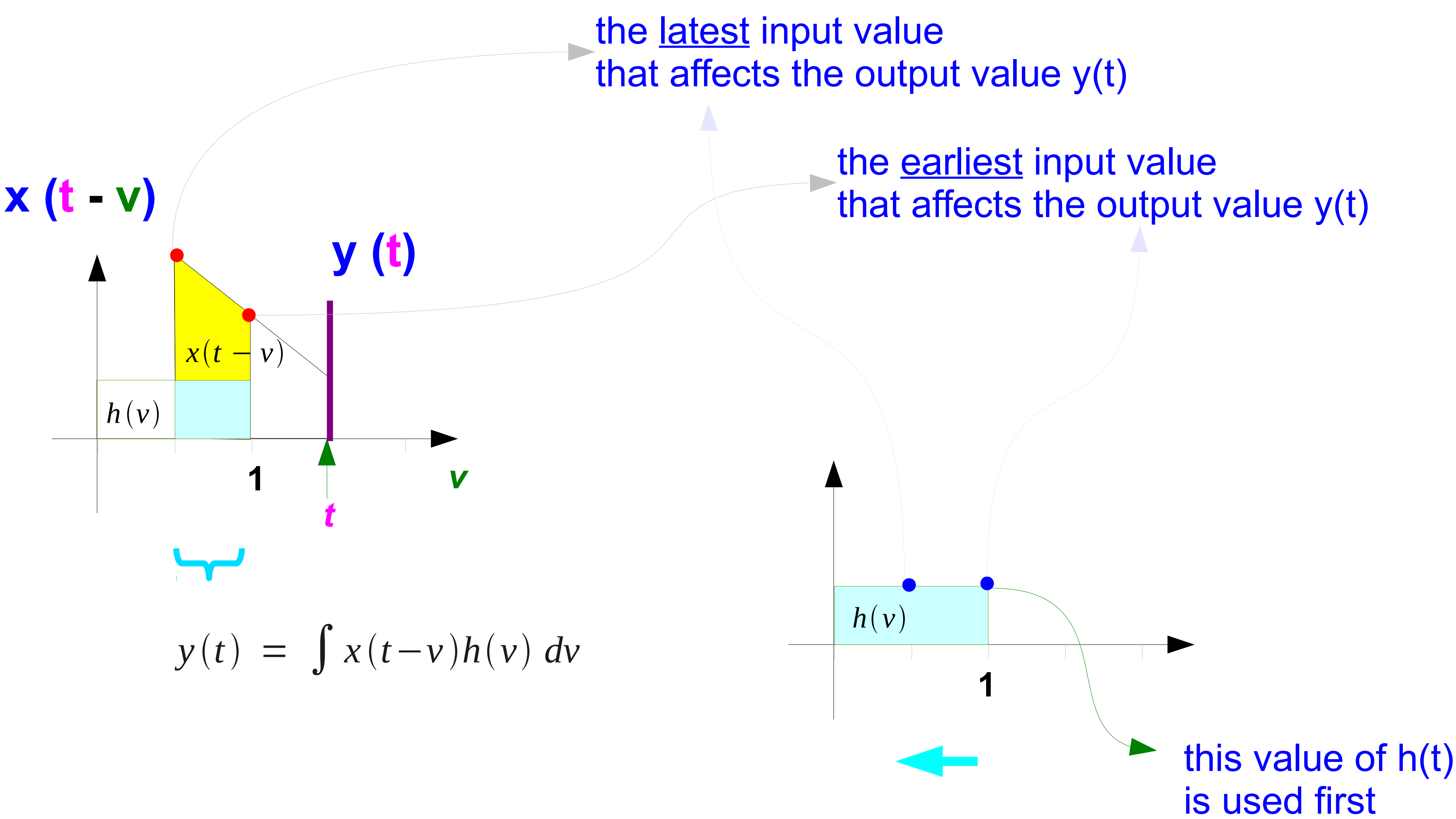


$$x(s-s+.5)h(s-.5)$$
$$= x(.5)h(s-.5)$$



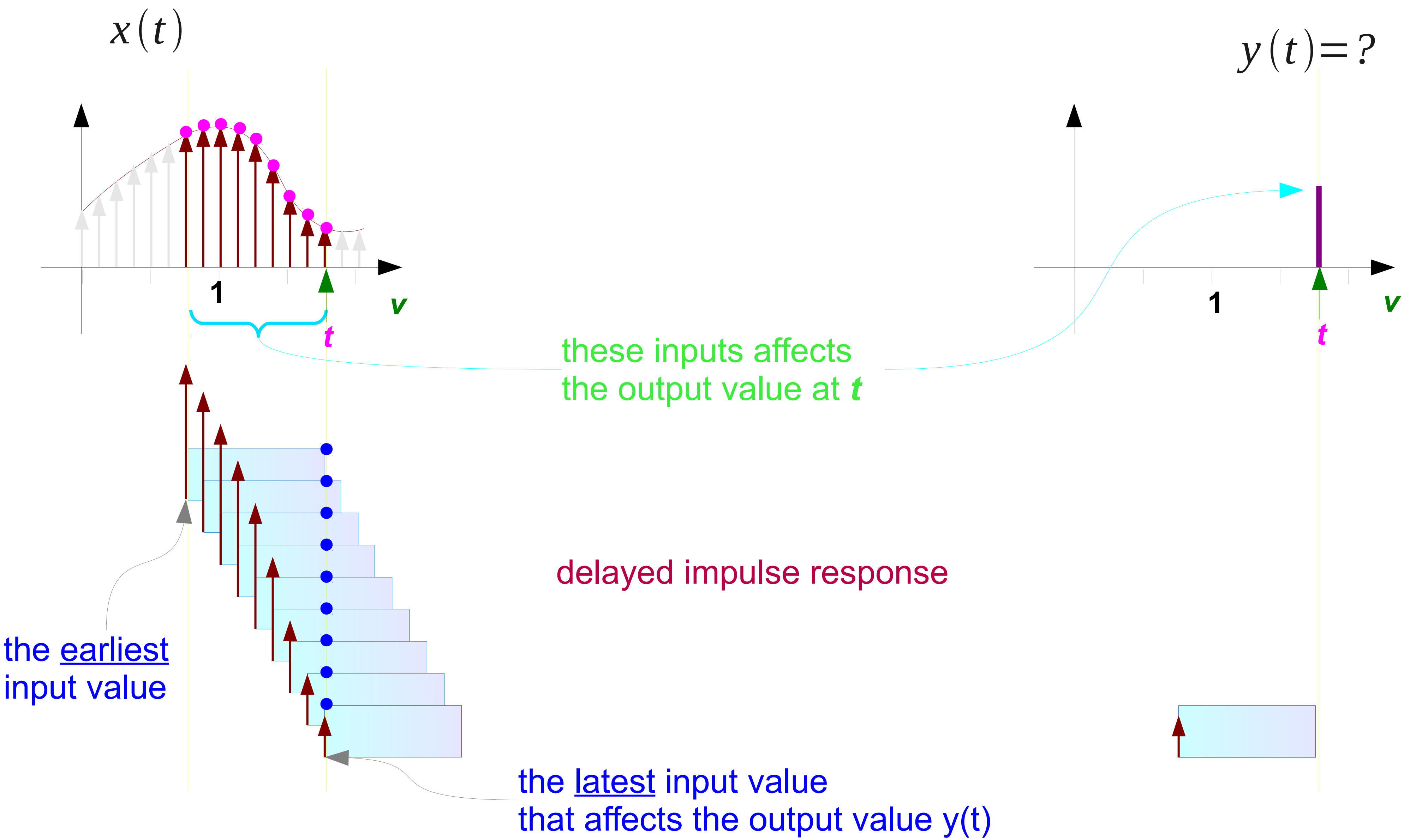
$$x(s-s+1)h(s-1)$$
$$= x(1)h(s-1)$$

Computing y(t) : earliest and latest inputs

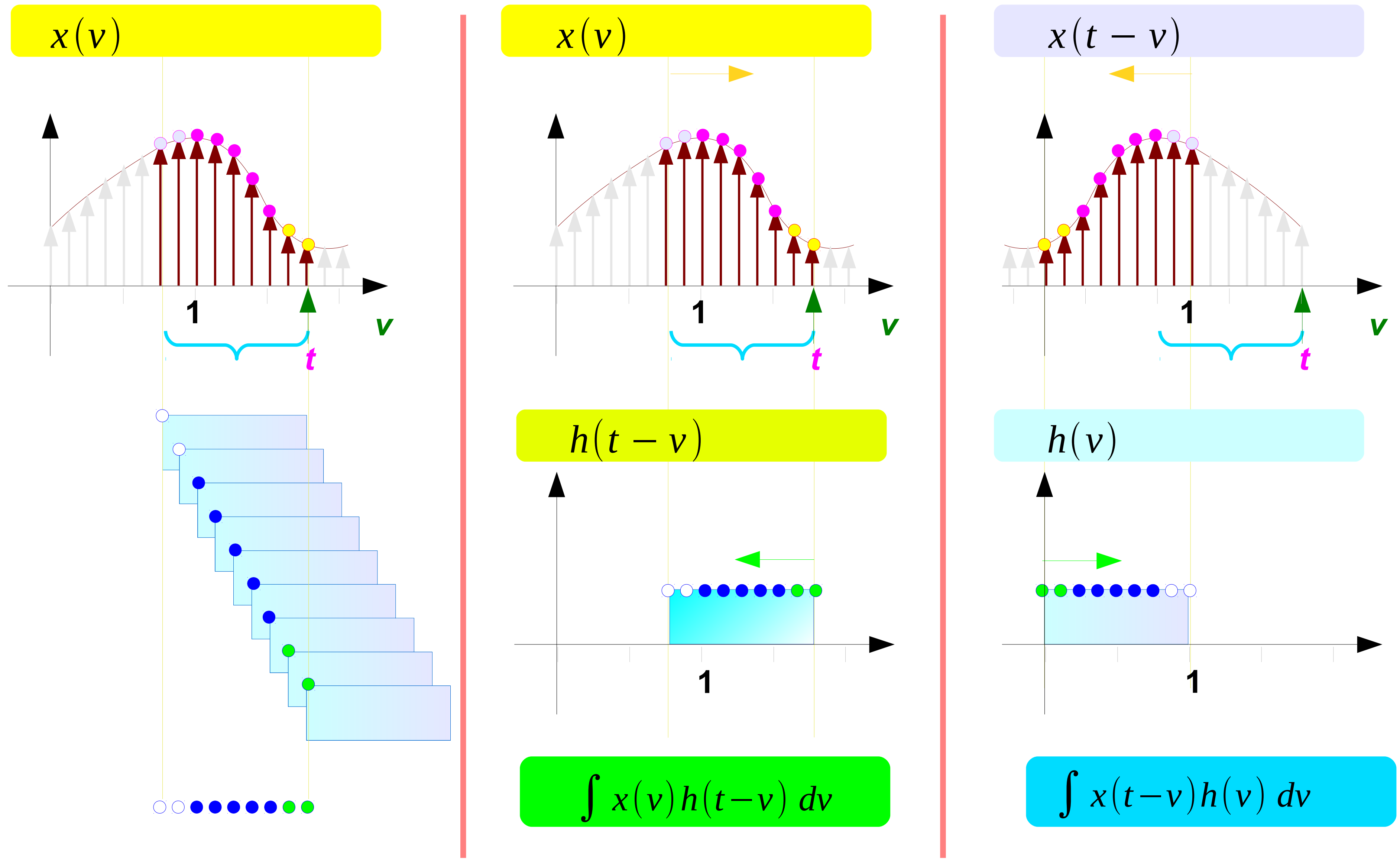


$$y(t) = \int x(t-v)h(v)\,dv$$

Computing y(t) : delayed impulse response



Computing y(t) : multiplication sequence



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003