Complex Functions (1B)

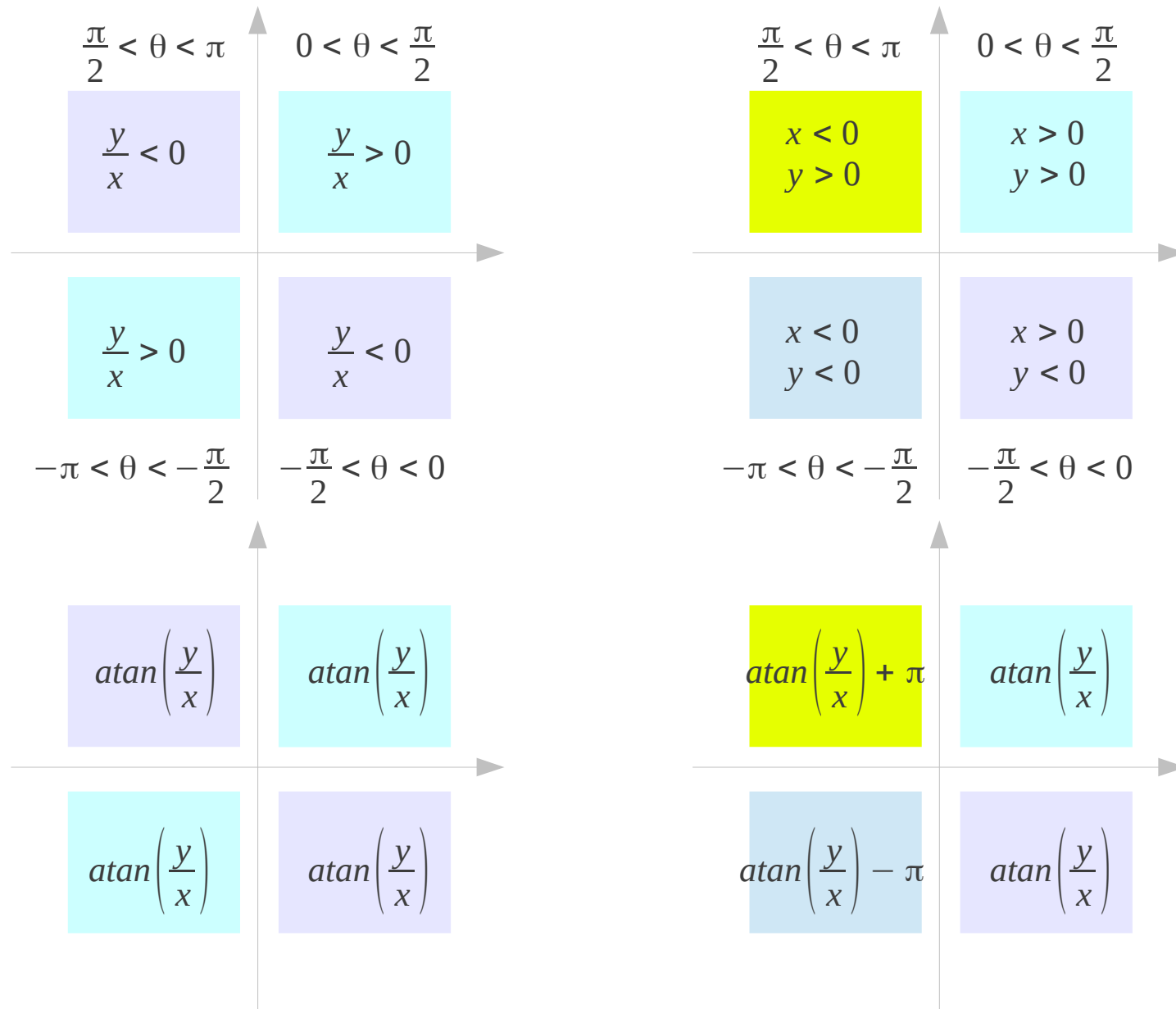Young Won Lim
1/11/14
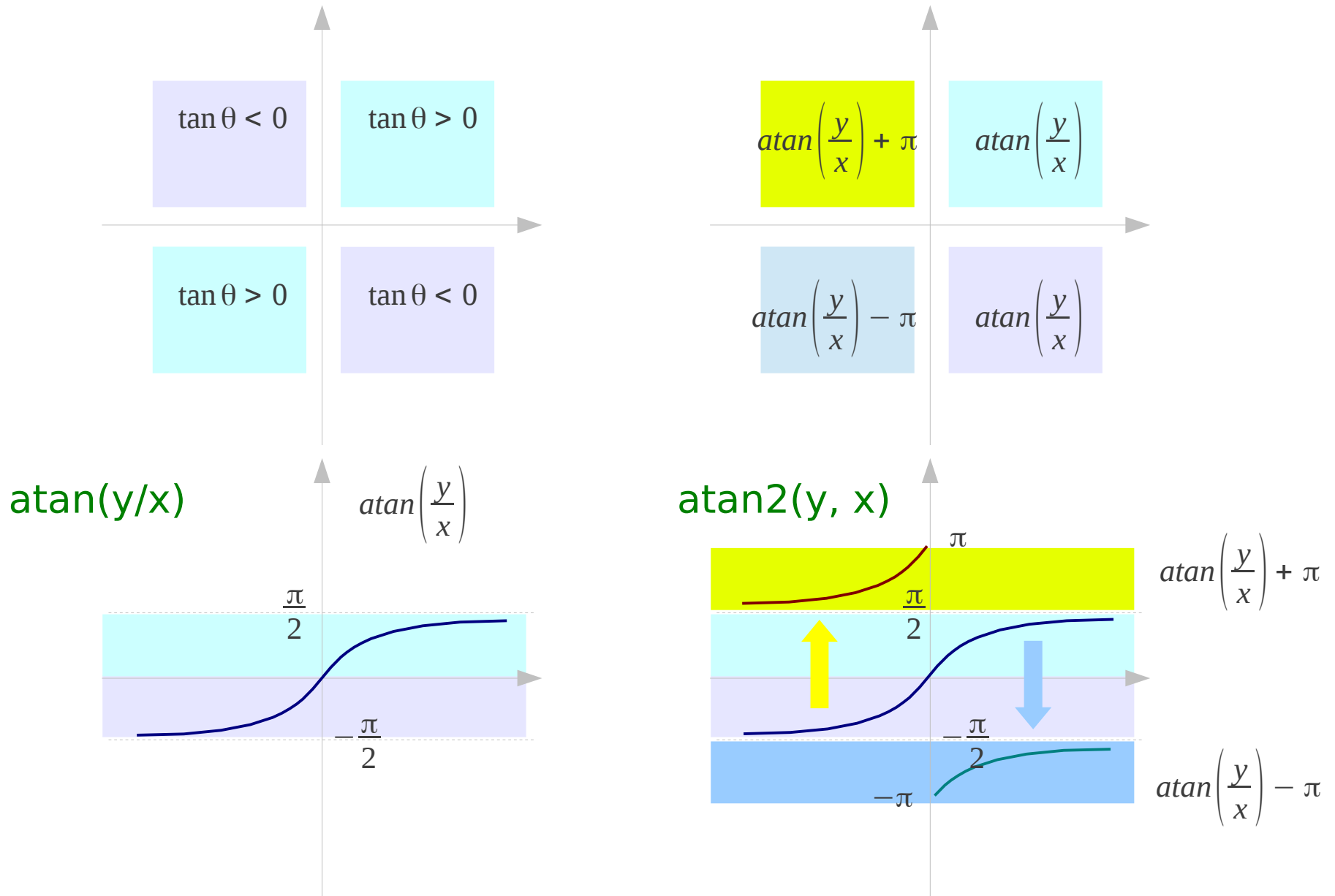
Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Argument of a Complex Number (1)
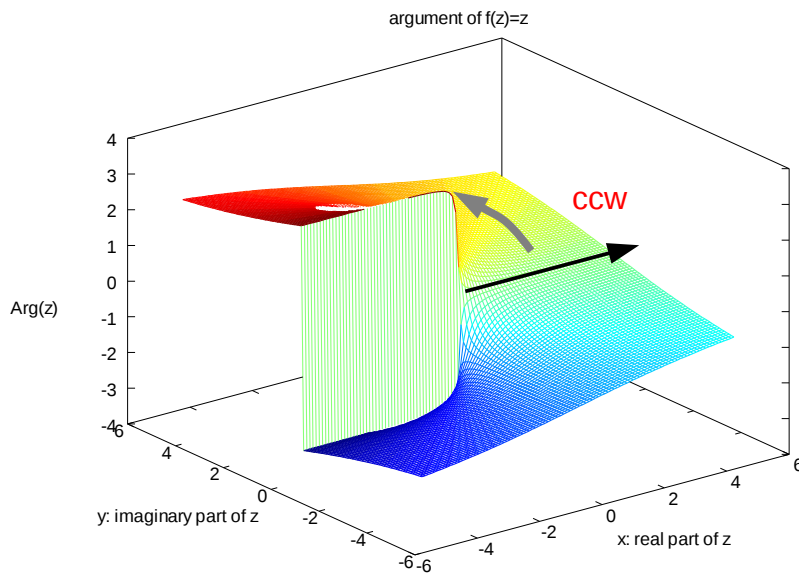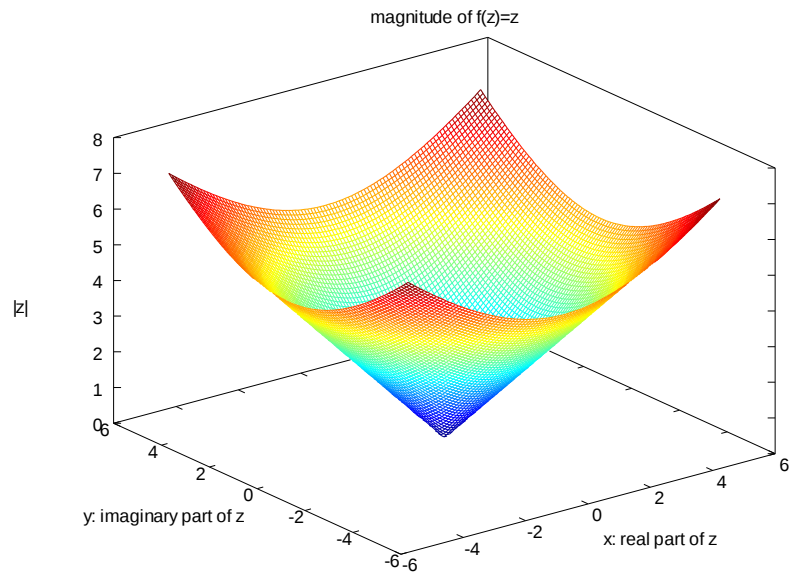
$$\frac{\pi}{2} < \theta < \pi \qquad 0 < \theta < \frac{\pi}{2}$$

$$\frac{y}{x} < 0 \qquad \frac{y}{x} > 0$$

$$\frac{y}{x} > 0 \qquad \frac{y}{x} < 0$$

$$-\pi < \theta < -\frac{\pi}{2} \qquad -\frac{\pi}{2} < \theta < 0$$

$$\frac{\pi}{2} < \theta < \pi \qquad 0 < \theta < \frac{\pi}{2}$$

$$x < 0, \; y > 0 \qquad x > 0, \; y > 0$$

$$x < 0, \; y < 0 \qquad x > 0, \; y < 0$$

$$-\pi < \theta < -\frac{\pi}{2} \qquad -\frac{\pi}{2} < \theta < 0$$

$$atan\left(\frac{y}{x}\right) \qquad atan\left(\frac{y}{x}\right)$$

$$atan\left(\frac{y}{x}\right) \qquad atan\left(\frac{y}{x}\right)$$

$$atan\left(\frac{y}{x}\right) + \pi \qquad atan\left(\frac{y}{x}\right)$$

$$atan\left(\frac{y}{x}\right) - \pi \qquad atan\left(\frac{y}{x}\right)$$

Complex Functions (1A)

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Argument of a Complex Number (2)

$\tan\theta < 0$ | $\tan\theta > 0$

$\tan\theta > 0$ | $\tan\theta < 0$

$atan\left(\dfrac{y}{x}\right) + \pi$ | $atan\left(\dfrac{y}{x}\right)$

$atan\left(\dfrac{y}{x}\right) - \pi$ | $atan\left(\dfrac{y}{x}\right)$

atan(y/x)

$atan\left(\dfrac{y}{x}\right)$

$\dfrac{\pi}{2}$

$-\dfrac{\pi}{2}$

atan2(y, x)

π

$atan\left(\dfrac{y}{x}\right) + \pi$

$\dfrac{\pi}{2}$

$-\dfrac{\pi}{2}$

$-\pi$

$atan\left(\dfrac{y}{x}\right) - \pi$

Young Won Lim
1/11/14

f(z)=z

magnitude of f(z)=z

argument of f(z)=z

```
%-------------------------------------------------------------------------
% Plot f(z) = z^2
% Licensing: This code is distributed under the GNU LGPL license.
% Modified: 2012.11.23
% Author: Young W. Lim
%-------------------------------------------------------------------------

x = linspace(-5, +5, 100);
y = linspace(-5, +5, 100);
[xx yy] = meshgrid(x, y);

z = xx + i* yy;


mesh(xx, yy, abs(z))
title("magnitude of f(z)=z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("|z|");
print -demf z.mag.emf

pause

mesh(xx, yy, arg(z))
title("argument of f(z)=z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("Arg(z)");
print -demf z.arg.emf
```
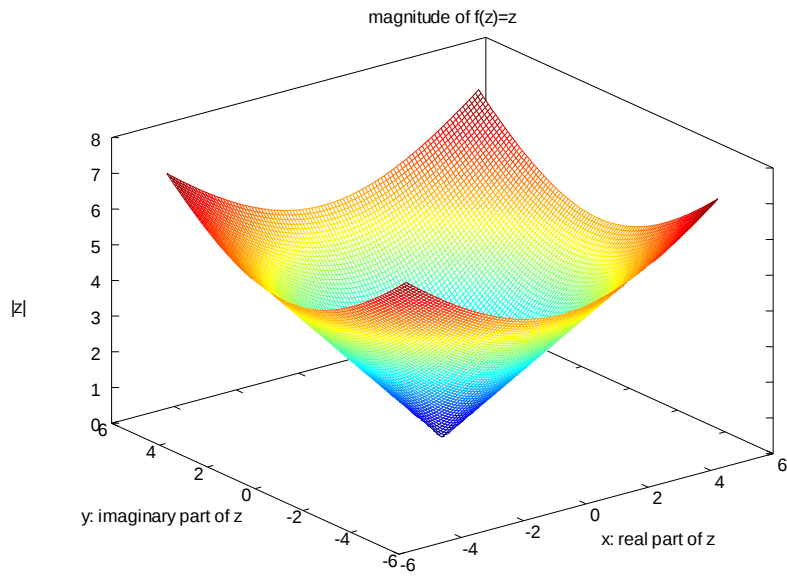
f(z)=z

magnitude of f(z)=z

real part of f(z)=z

argument of f(z)=z

Imaginary part of f(z)=z

CCW
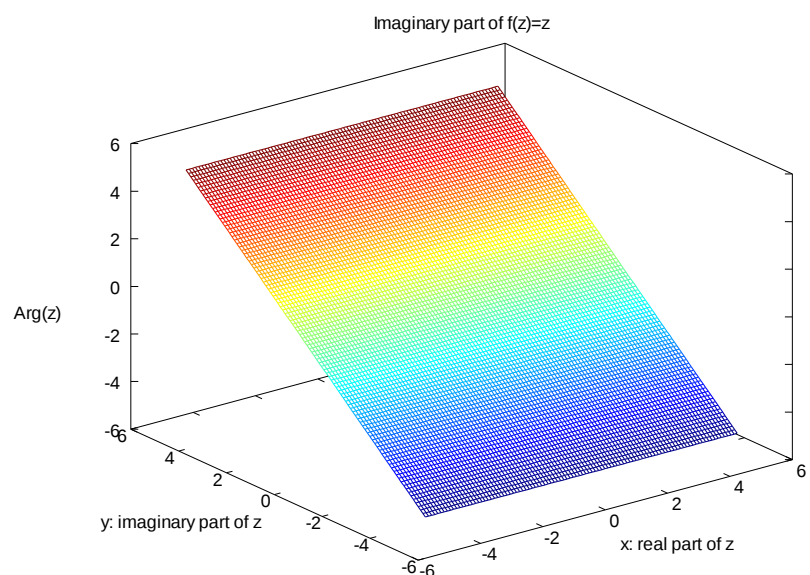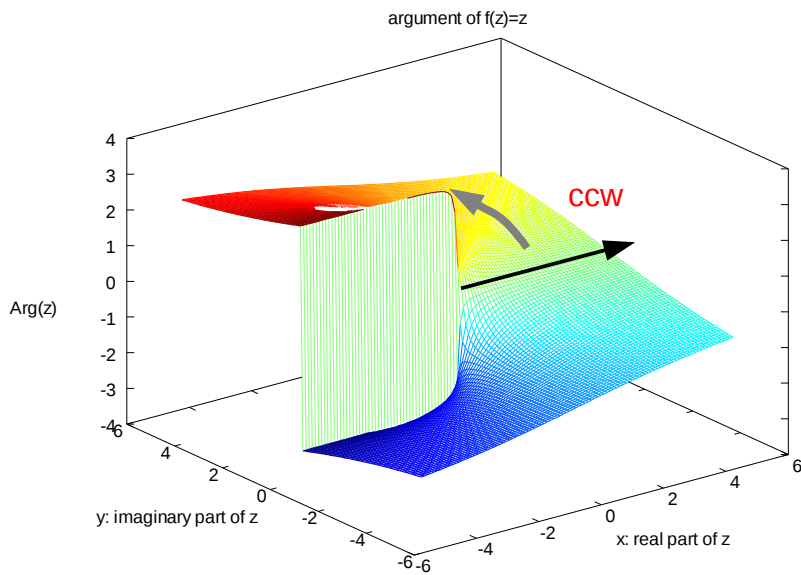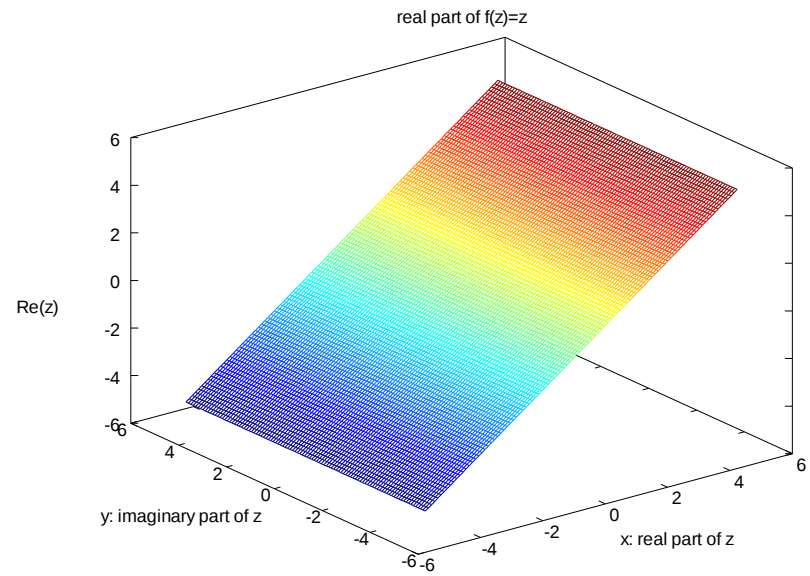
f(z) = z²

$$z = x + iy \quad \Longrightarrow \quad f(z) = u(x,y) + iv(x,y)$$

$$f(z) = z^2 = (x+iy)^2 = (x^2 + i2xy - y^2)$$
$$= (x^2 - y^2) + i(2xy)$$

$$u(x,y) = (x^2 - y^2) \qquad v(x,y) = (2xy)$$

$$\boxed{\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}}$$

$$\boxed{\dfrac{\partial v}{\partial x} = -\dfrac{\partial u}{\partial y}}$$

$$\frac{\partial u}{\partial x} = 2x \qquad\qquad \frac{\partial v}{\partial x} = 2y$$

$$\frac{\partial u}{\partial y} = -2y \qquad\qquad \frac{\partial v}{\partial y} = 2x$$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = 2x + i2y$$

$$f'(z) = -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = i2y + 2x$$
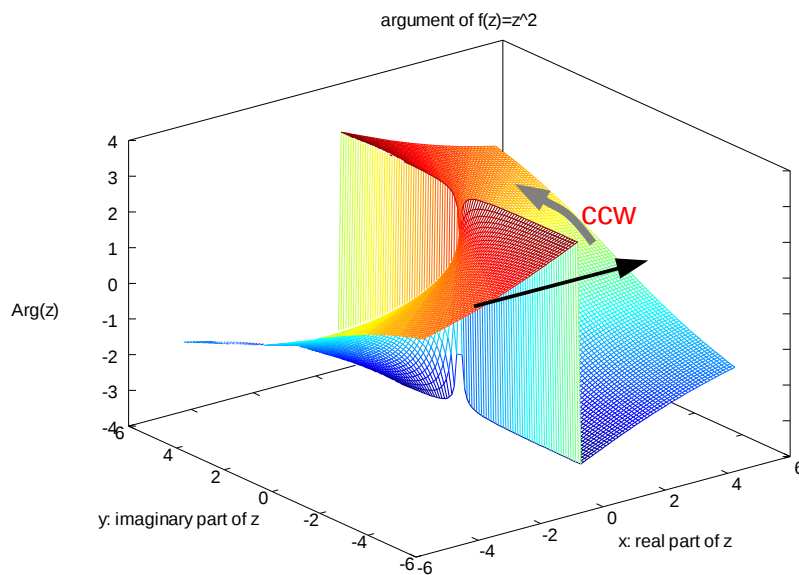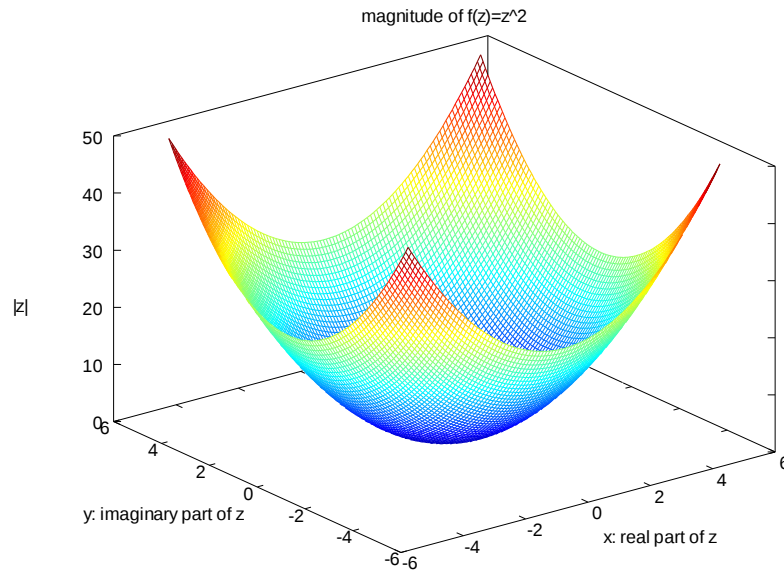
$$f'(z) = 2z = 2(x + iy)$$

Complex Numbers
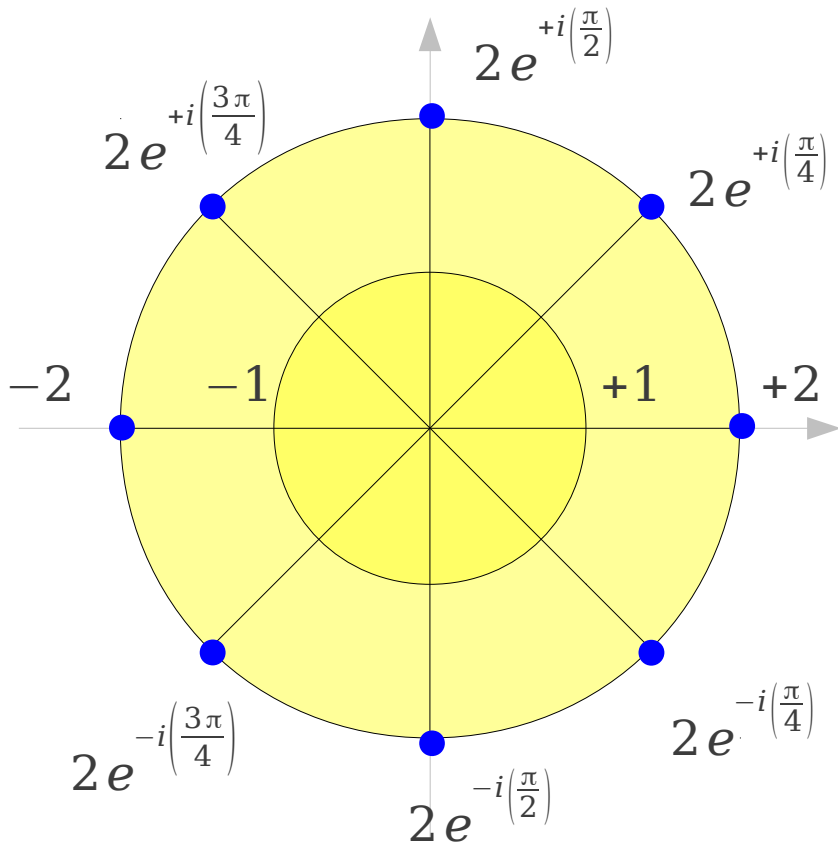
polar	rectangular
$+2$	$+2$
$2e^{+i\left(\frac{\pi}{4}\right)}$	$\sqrt{2}(+1+i)$
$2e^{+i\left(\frac{\pi}{2}\right)}$	$2i$
$2e^{+i\left(\frac{3\pi}{4}\right)}$	$\sqrt{2}(-1+i)$
-2	-2
$2e^{-i\left(\frac{3\pi}{4}\right)}$	$\sqrt{2}(-1-i)$
$2e^{-i\left(\frac{\pi}{2}\right)}$	$-2i$
$2e^{-i\left(\frac{\pi}{4}\right)}$	$\sqrt{2}(+1-i)$

Complex Functions

magnitude of f(z)=z^2

|z|

y: imaginary part of z

x: real part of z

argument of f(z)=z^2

Arg(z)

CCW

y: imaginary part of z

x: real part of z

$f(z) =$	z^2
$f(+2)$	4
$f\left(2e^{+i\left(\frac{\pi}{4}\right)}\right)$	$4e^{+i\left(\frac{\pi}{2}\right)}$
$f\left(2e^{+i\left(\frac{\pi}{2}\right)}\right)$	$4e^{+i(\pi)}$
$f\left(2e^{+i\left(\frac{3\pi}{4}\right)}\right)$	$4e^{-i\left(\frac{\pi}{2}\right)}$
$f(-2)$	4
$f\left(2e^{-i\left(\frac{3\pi}{4}\right)}\right)$	$4e^{+i\left(\frac{\pi}{2}\right)}$
$f\left(2e^{-i\left(\frac{\pi}{2}\right)}\right)$	$4e^{-i(\pi)}$
$f\left(2e^{-i\left(\frac{\pi}{4}\right)}\right)$	$4e^{-i\left(\frac{\pi}{2}\right)}$

f(z)=z^2
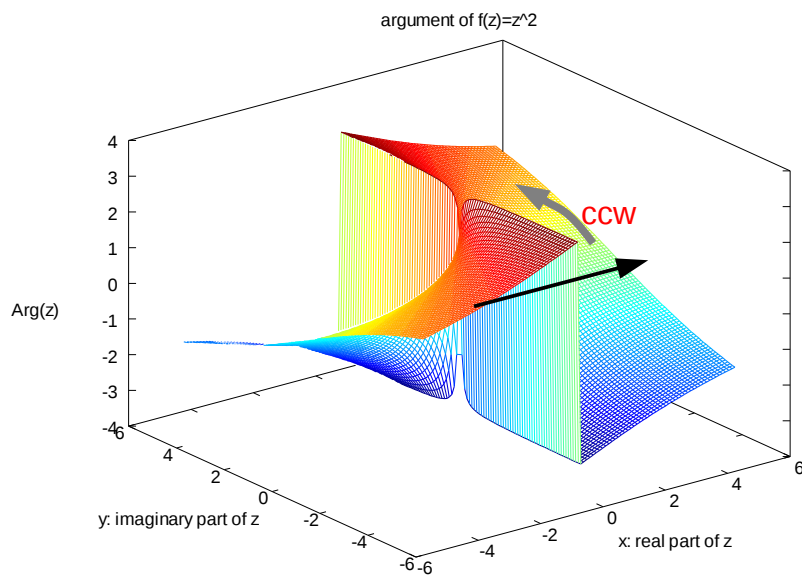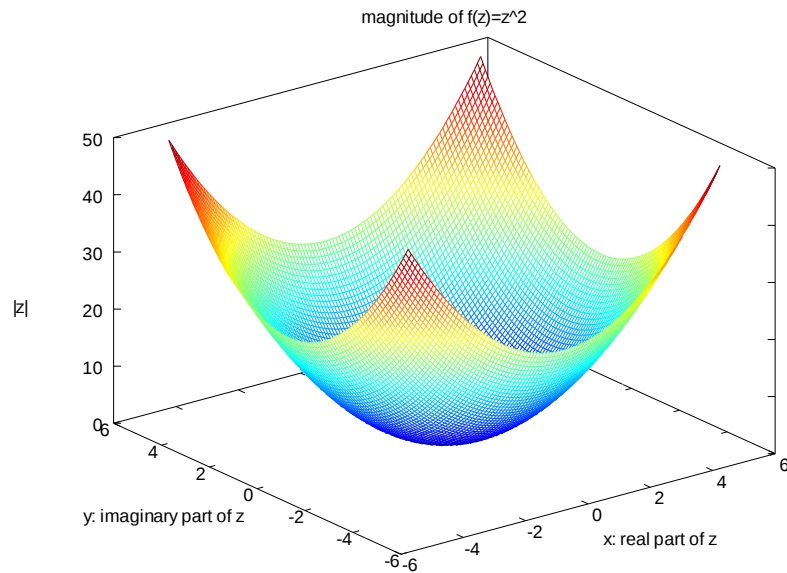
magnitude of f(z)=z^2

argument of f(z)=z^2

```
%----------------------------------------------------------------------------
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% Modified: 2012.11.23
% Author: Young W. Lim
%----------------------------------------------------------------------------

x = linspace(-5, +5, 100);
y = linspace(-5, +5, 100);
[xx yy] = meshgrid(x, y);

z = xx + i* yy;


mesh(xx, yy, abs(z))
title("magnitude of f(z)=z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("|z|");
print -demf z.mag.emf

pause

mesh(xx, yy, arg(z))
title("argument of f(z)=z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("Arg(z)");
print -demf z.arg.emf
```
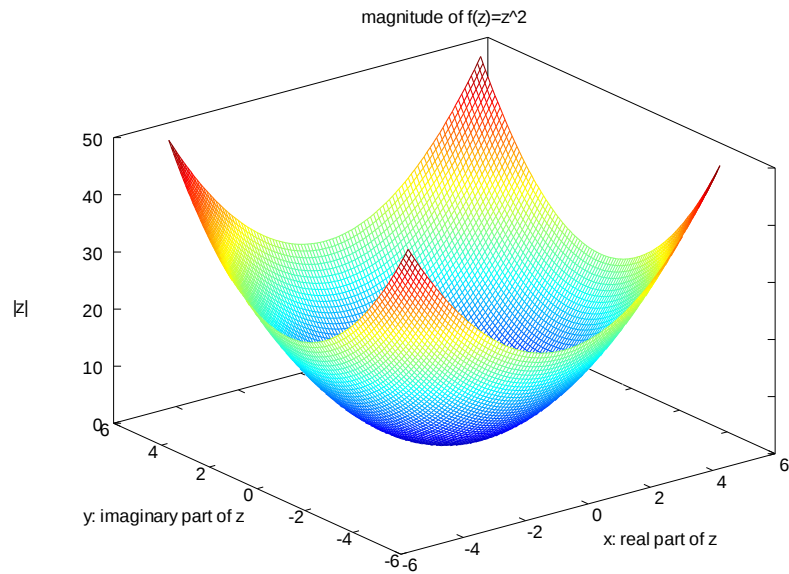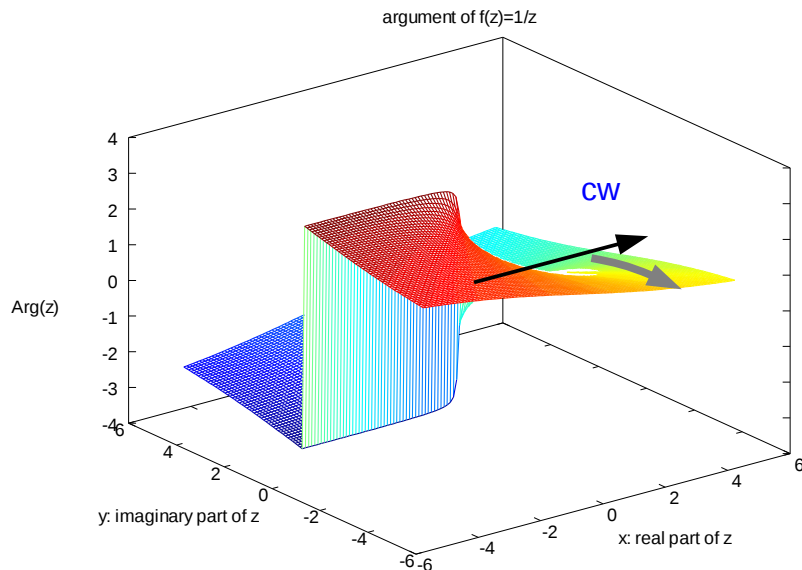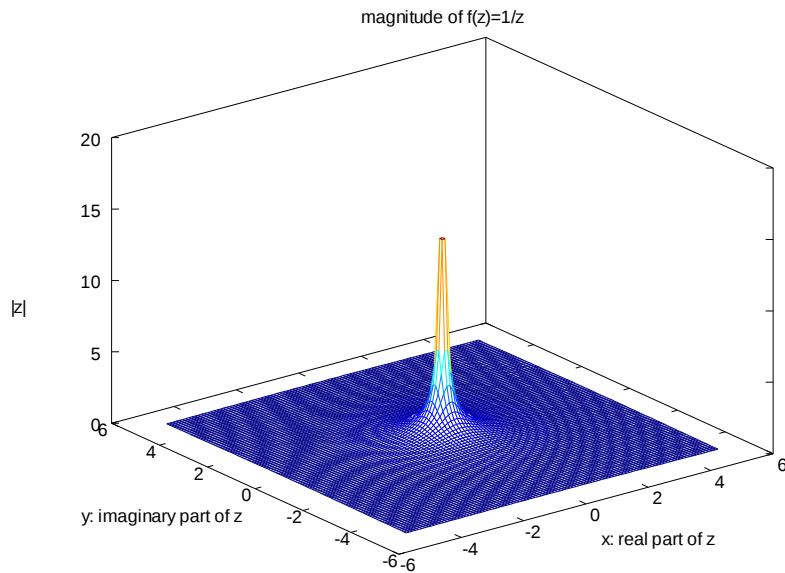
CCW

f(z)=z^2

magnitude of f(z)=z^2

real part of f(z)=z^2

argument of f(z)=z^2

imaginary part of f(z)=z^2

f(z)=1/z

magnitude of f(z)=1/z

argument of f(z)=1/z

CW

```
%----------------------------------------------------------------------------
% Plot f(z) = 1/z
% Licensing: This code is distributed under the GNU LGPL license.
% Modified: 2012.11.23
% Author: Young W. Lim
%----------------------------------------------------------------------------

x = linspace(-5, +5, 100);
y = linspace(-5, +5, 100);
[xx yy] = meshgrid(x, y);

z1 = xx + i* yy;
z = 1 ./ z1;


mesh(xx, yy, abs(z))
title("magnitude of f(z)=1/z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("|z|");
print -demf 1_z.mag.emf

pause

mesh(xx, yy, arg(z))
title("argument of f(z)=1/z");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("Arg(z)");
print -demf 1_z.arg.emf
```
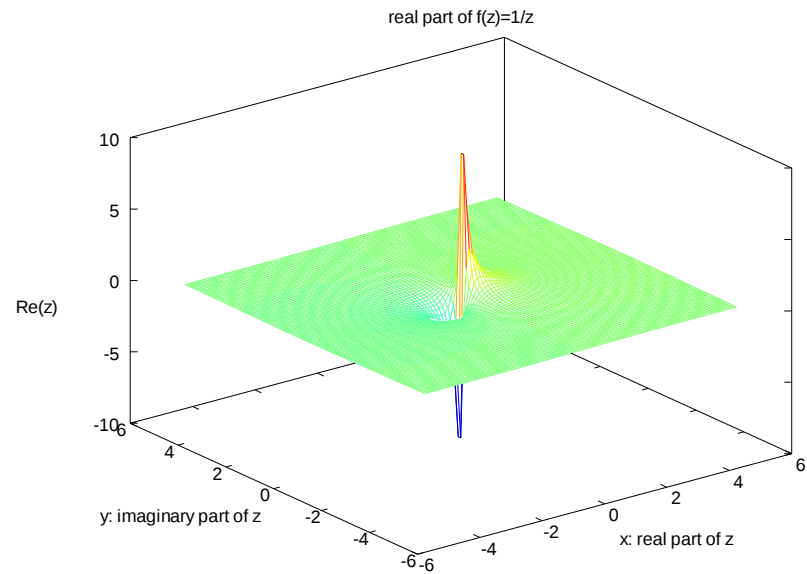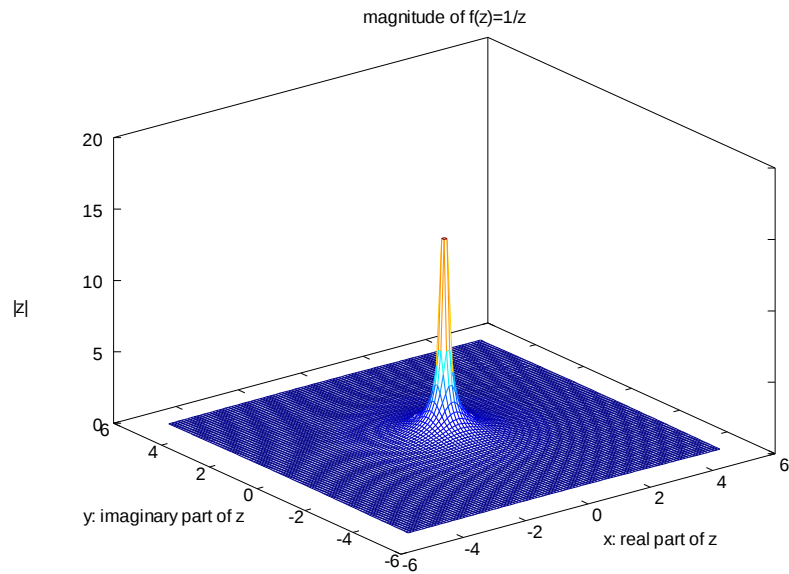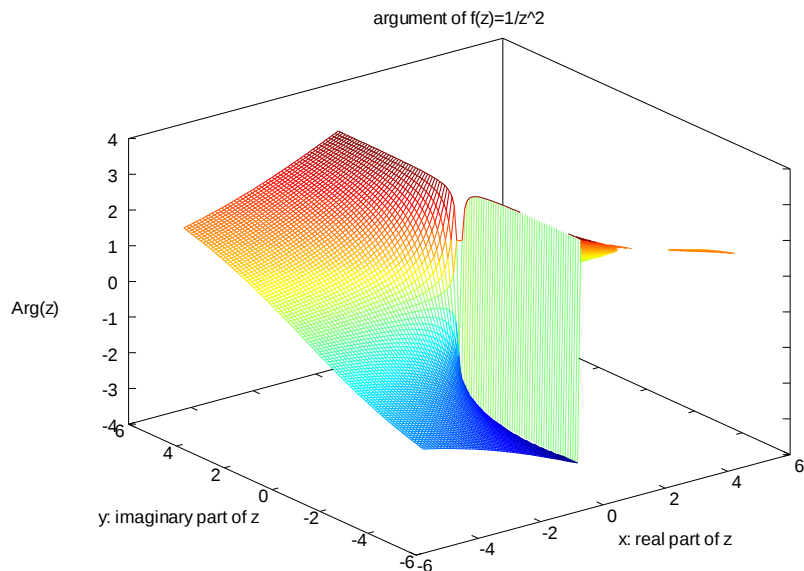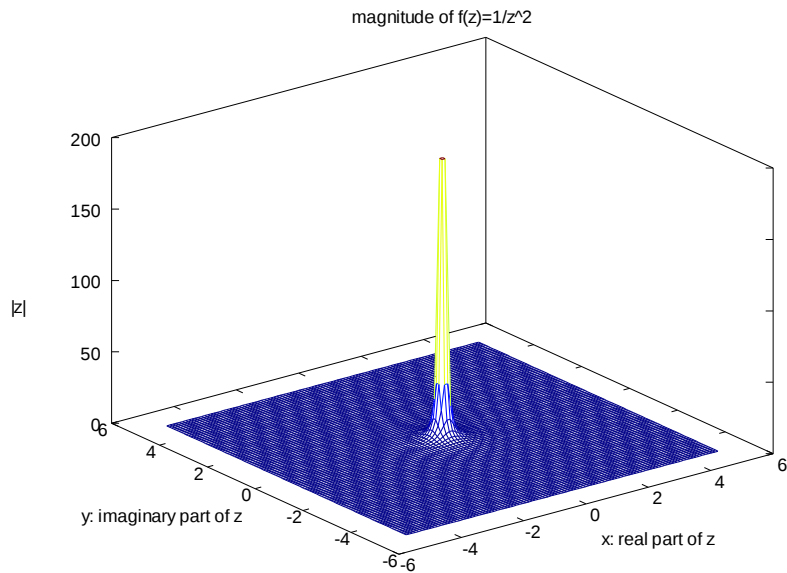
f(z)=1/z

Complex Functions (1A)

f(z)=1/z^2

magnitude of f(z)=1/z^2
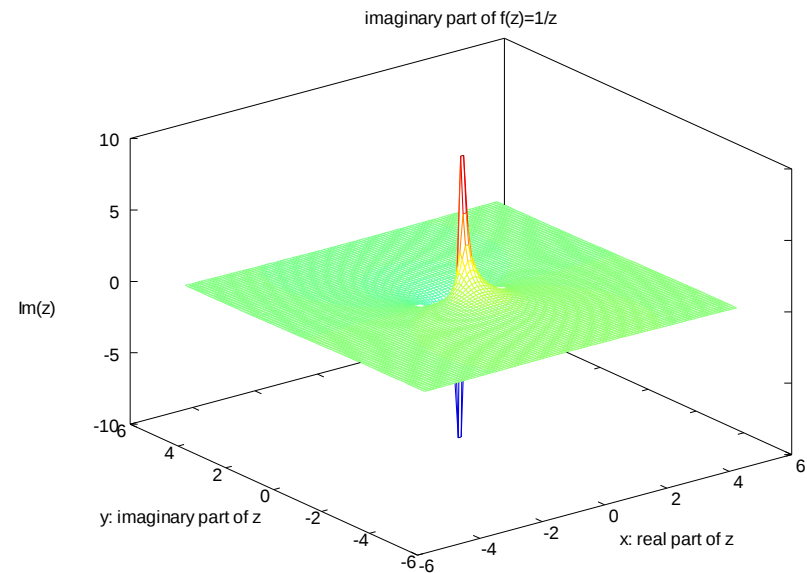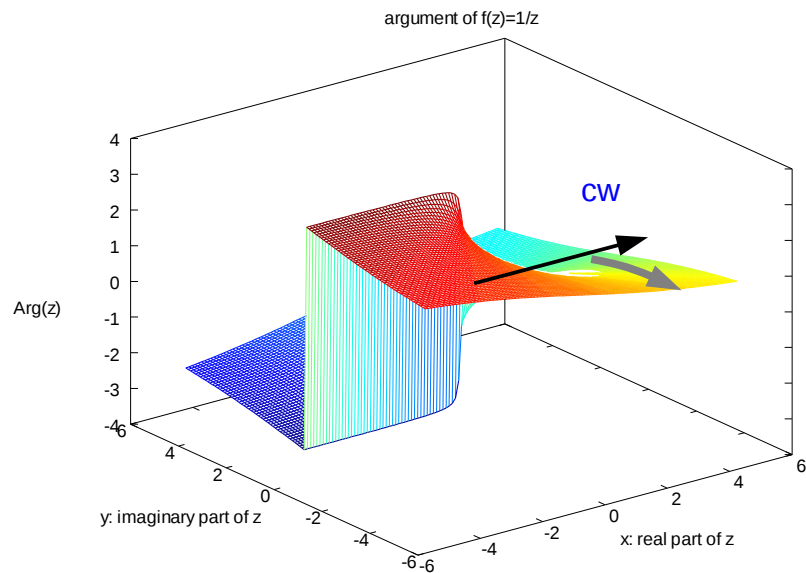
argument of f(z)=1/z^2


```
%---------------------------------------------------------------------------
% Plot f(z) = 1/z^2
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% Modified: 2012.11.23
% Author: Young W. Lim
%---------------------------------------------------------------------------

x = linspace(-5, +5, 100);
y = linspace(-5, +5, 100);
[xx yy] = meshgrid(x, y);

z1 = xx + i* yy;
z2 = z1 .* z1;
z = 1 ./ z2;


mesh(xx, yy, abs(z))
title("magnitude of f(z)=1/z^2");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("|z|");
print -demf 1_z2.mag.emf

pause

mesh(xx, yy, arg(z))
title("argument of f(z)=1/z^2");
xlabel("x: real part of z");
ylabel("y: imaginary part of z");
zlabel("Arg(z)");
print -demf 1_z2.arg.emf
```
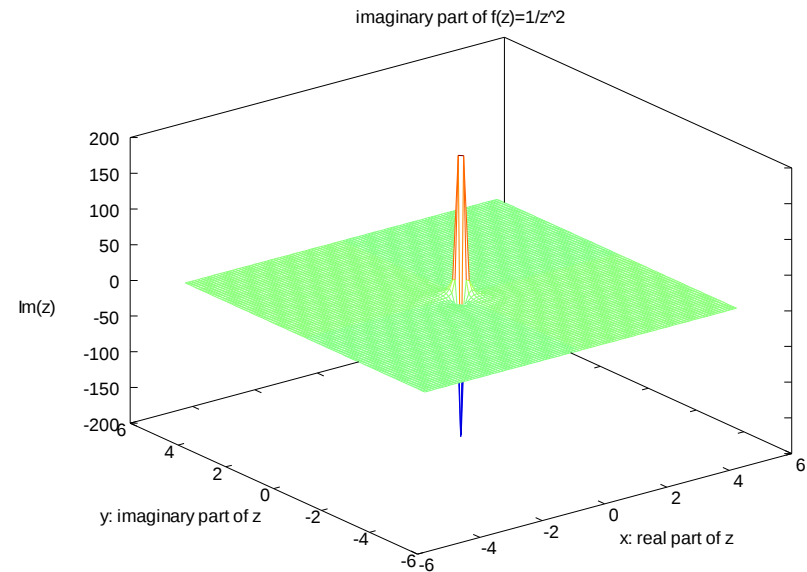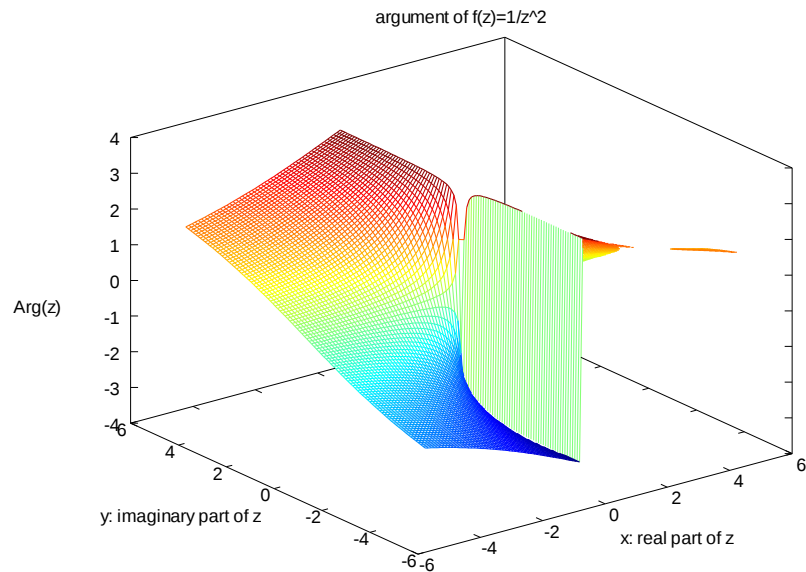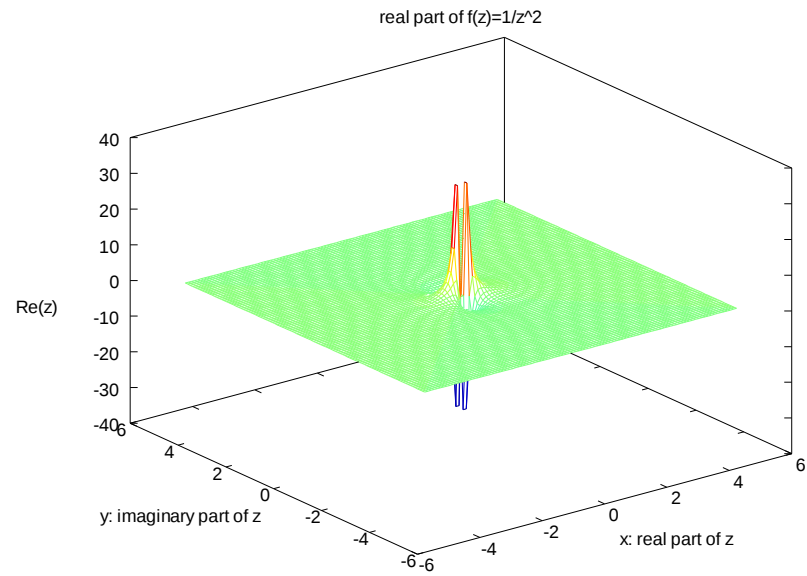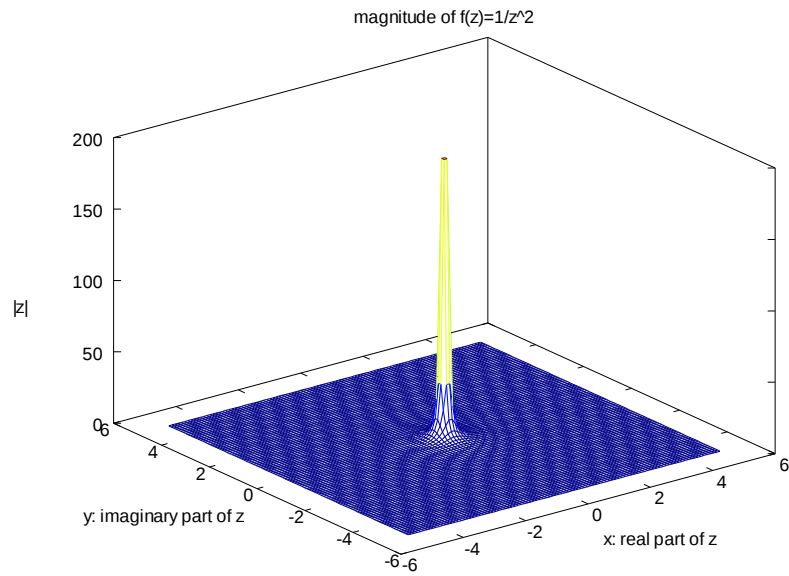
f(z)=1/z^2

magnitude of f(z)=1/z^2

real part of f(z)=1/z^2

argument of f(z)=1/z^2

imaginary part of f(z)=1/z^2

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Right Hand Rule

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] M.L. Boas, "Mathematical Methods in the Physical Sciences"
[4] D.G. Zill, "Advanced Engineering Mathematics"