Signal Analysis

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This document was produced by using LibreOffice.

Based on

Signal Processing with Free Software : Practical Experiments
F. Auger

Octave Spectrogram Function

Function File: **specgram** (x)
Function File: **specgram** (x, n)
Function File: **specgram** (x, n, Fs)
Function File: **specgram** (x, n, Fs, window)
Function File: **specgram** (x, n, Fs, window, overlap)
Function File: [S, f, t] = **specgram** (...)

https://octave.sourceforge.io/signal/function/specgram.html

Input and Output Arguments

x	: the signal x.	
n	: the size of overlapping segments	(default: 256)
fs	: specifies the sampling rate of the input signal	
window	: specifies an alternate window	(default: hanning)
overlap	: specifies the number of samples overlap	(default: (window)/2)

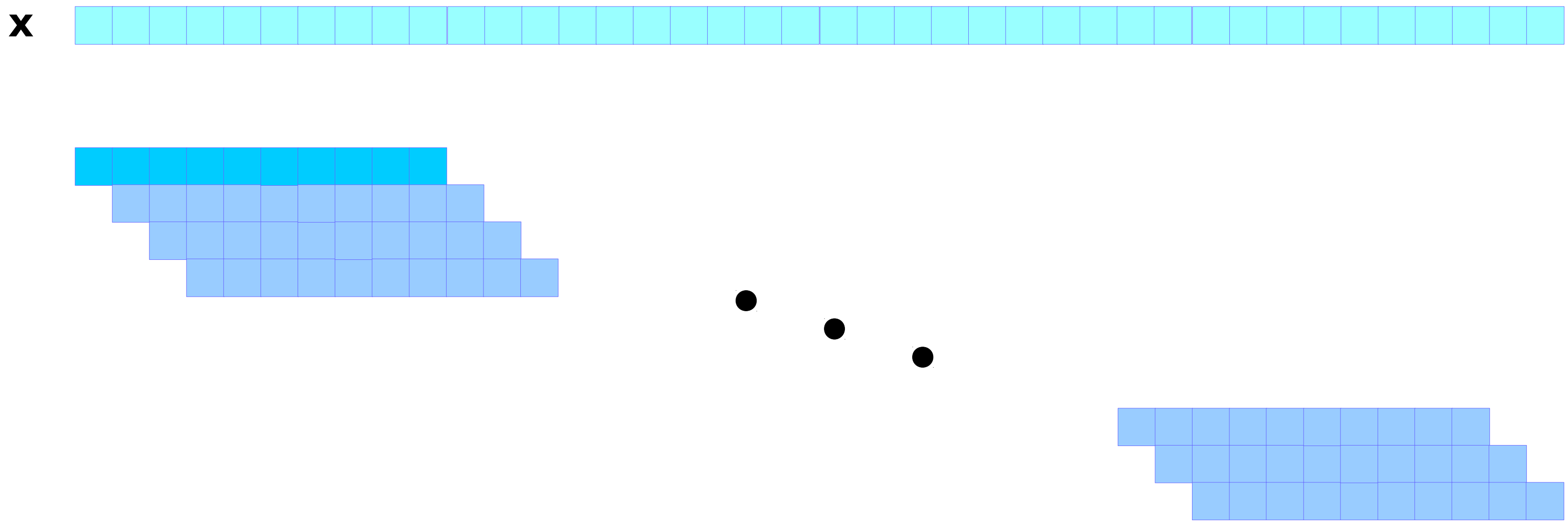
S	: the complex output of the FFT, one row per slice
f	: the frequency indices corresponding to the rows of S
t	: the time indices corresponding to the columns of S.

- if no output arguments are given,
  the spectrogram is displayed.
- otherwise,
  [**S**, **f**, **t**] will be returned

https://octave.sourceforge.io/signal/function/specgram.html

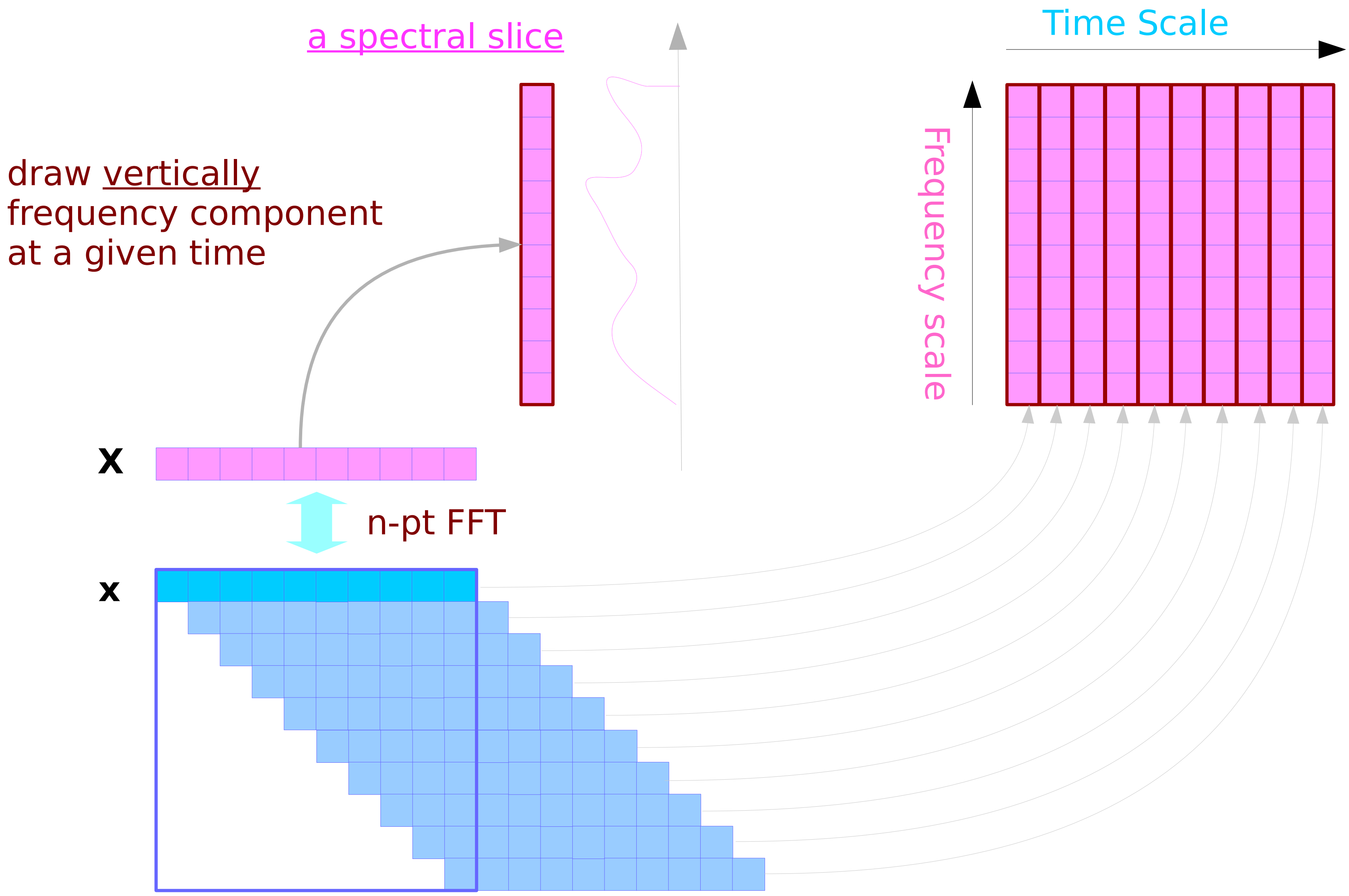
Spectrogram Operations

- the signal is chopped into overlapping segments of length **n**
- each segment is **windowed** and transformed by using the **FFT**
- if **fs** is given, it specifies the sampling rate of the input signal
- an alternate window to apply rather than the default of hanning (n)
- **overlap**: the number of samples overlap between successive segments

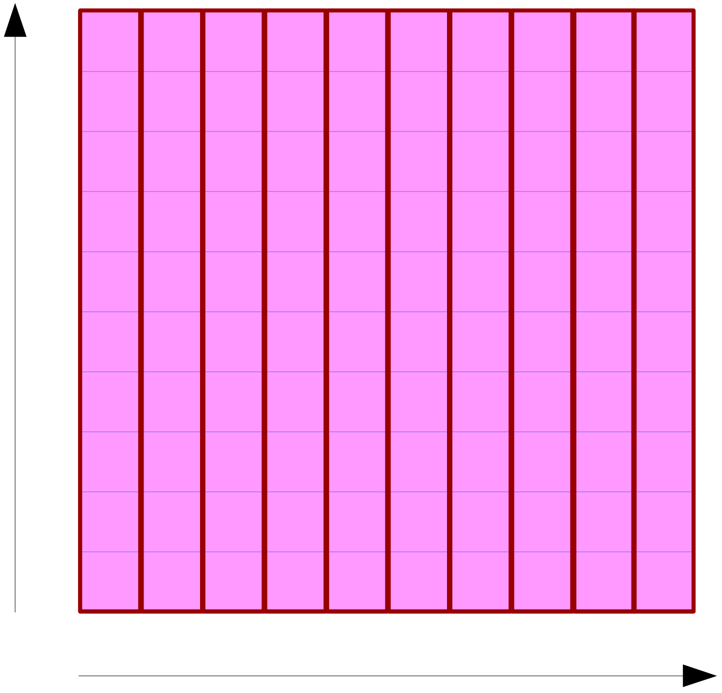


https://octave.sourceforge.io/signal/function/specgram.html

3D representation of spectrum over time-frequency domain



Time and Frequency Resolutions



Frequency scale

Frequency Resolution
$= f_0 = f_s/n = 1/nT_s$

Time Scale

Time Resolution = step

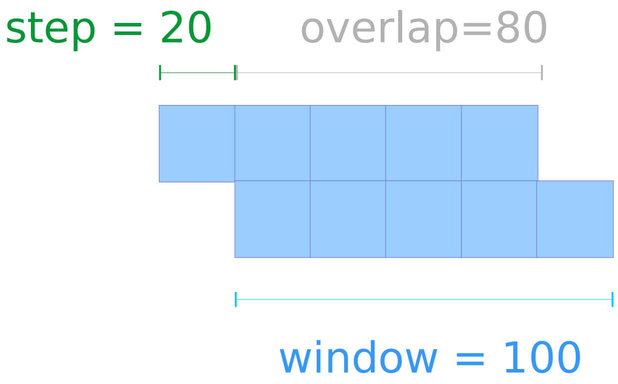
Step Size

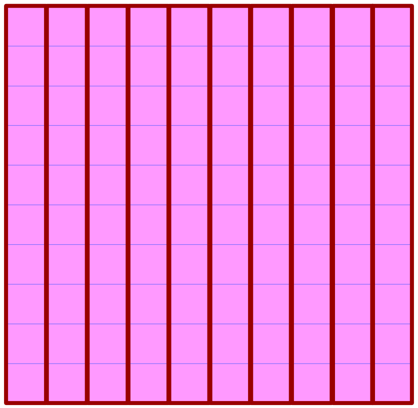
Step size

- window length minus overlap length
- controls the horizontal (time) scale of the spectrogram.

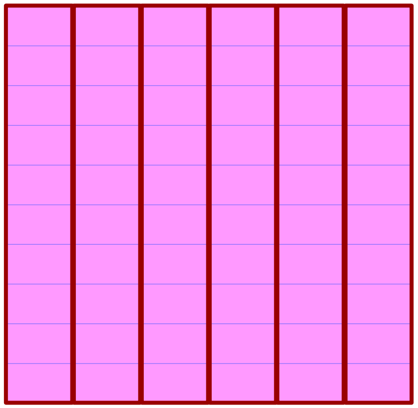
- the range 1-5 msec is good for speech.

window – overlap = step





small step size



large step size

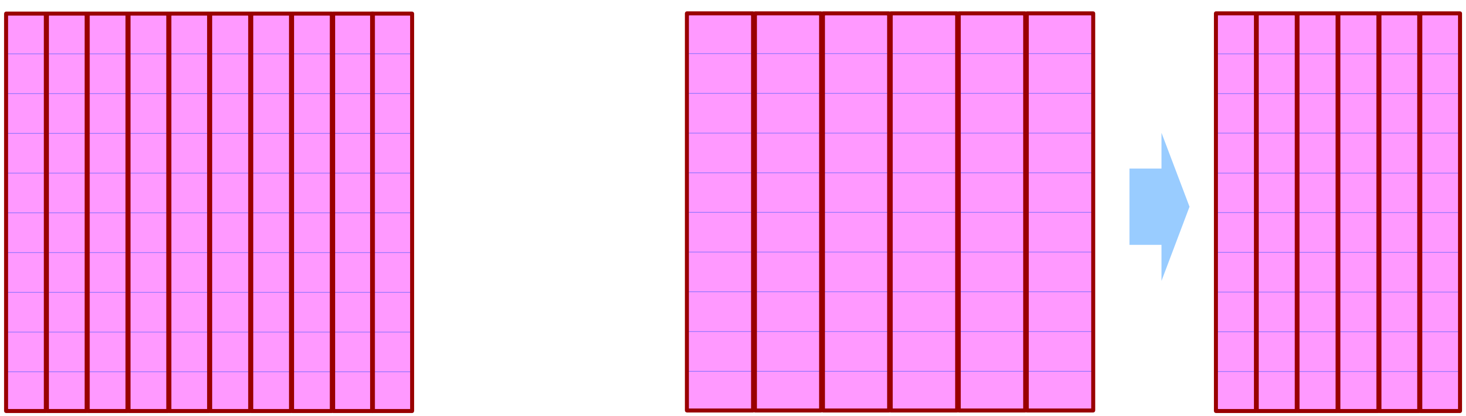
https://octave.sourceforge.io/signal/function/specgram.html

Step Size Effects

https://octave.sourceforge.io/signal/function/specgram.html

Step size

- *Increasing* step size to compress the spectrogram
- *Decreasing* step size to stretch the spectrogram
- *Increasing* step size will reduce time resolution,
- *Decreasing* it will not improve it much
  - beyond the limits imposed by the window size
  - gain a little bit, depending on the shape of your window
  - as the peak of the window slides over peaks in the signal energy



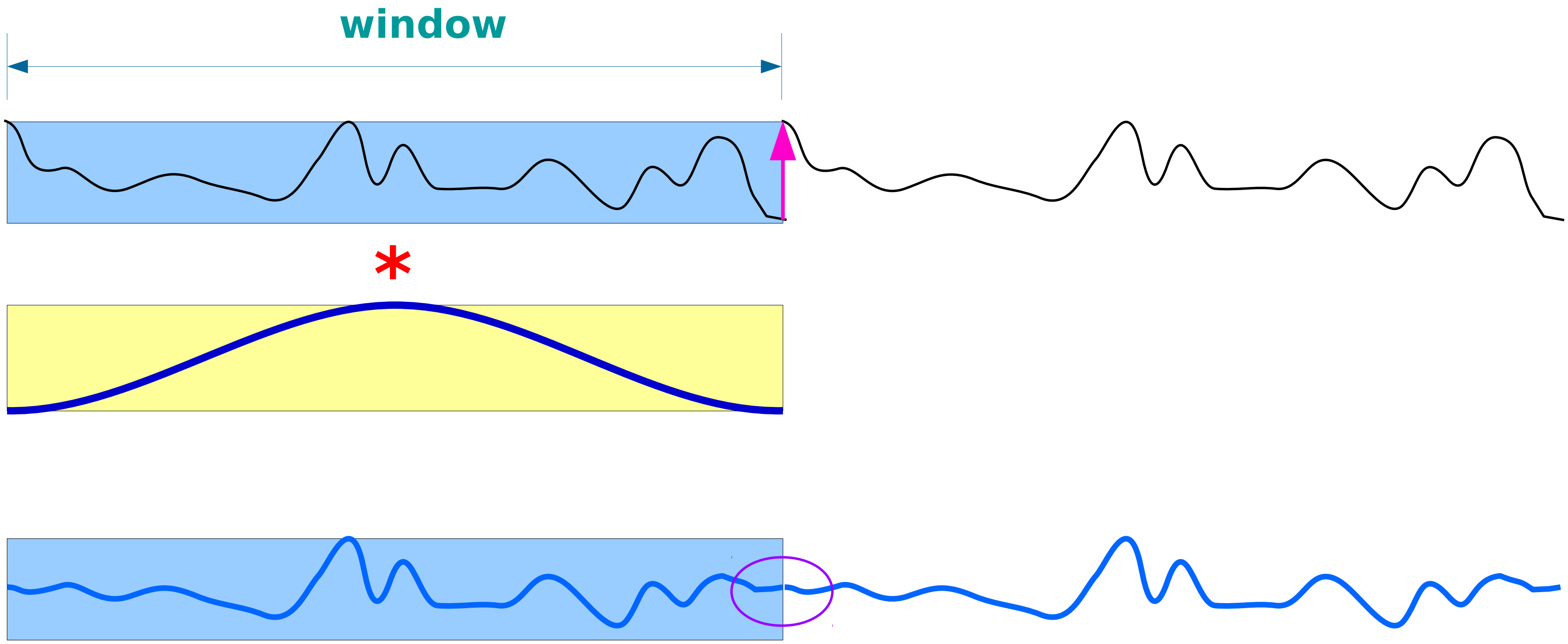
small step size
stretched
high resolution

large step size

compressed
small resolution

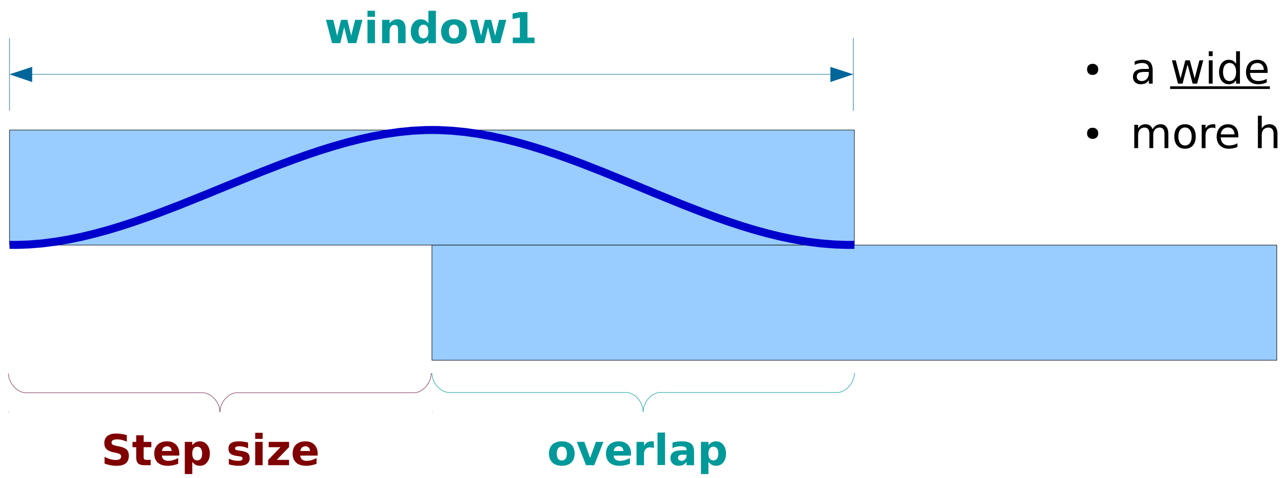
Windowing

- the <u>shape</u> of the window is <u>not</u> so <u>critical</u>
  so long as it goes gradually to zero on the ends.

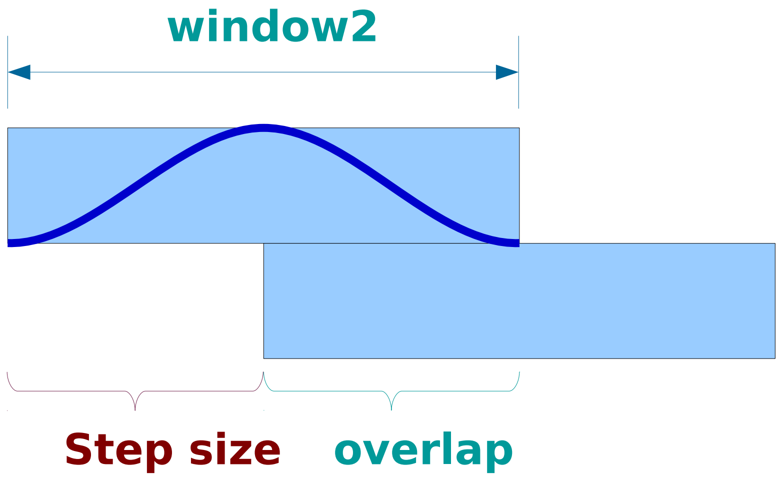


https://octave.sourceforge.io/signal/function/specgram.html

Window Size



- a <u>wide</u> window
- more harmonic <u>detail</u>



- a <u>narrow</u> window
- <u>averages</u> over the harmonic detail

https://octave.sourceforge.io/signal/function/specgram.html

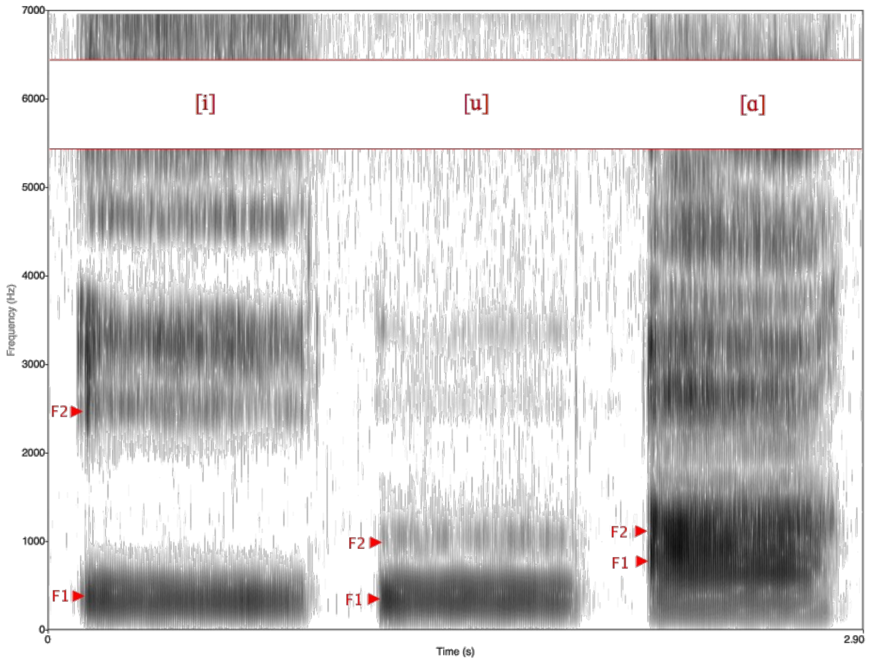
Formant Structure

The choice of window defines the time-frequency resolution.

- a wide window shows more harmonic detail
- a narrow window averages over the harmonic detail
  - shows more formant structure

  - "a range of frequencies in which there is an absolute or relative maximum in the sound spectrum"

  - Spectrogram of American English vowels [i, u, ɑ] showing the formants F1 and F2



https://octave.sourceforge.io/signal/function/specgram.html
https://en.wikipedia.org/wiki/Formant

FFT Length

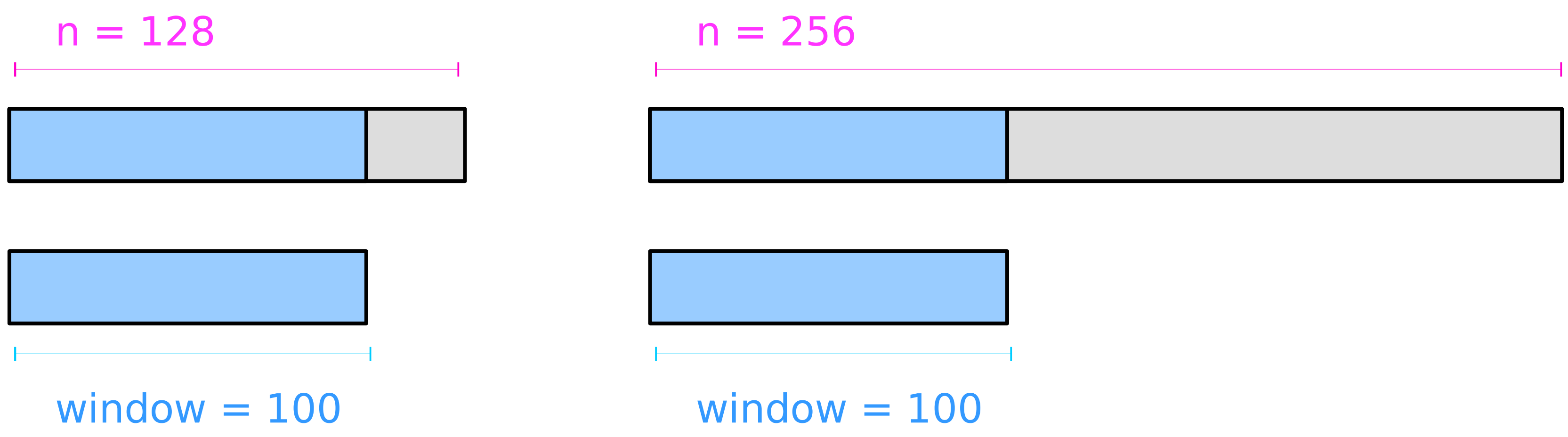
FFT length controls the vertical scale.

Selecting an FFT length *greater* than the window length

does not add any information to the spectrum

a good way to **interpolate** between frequency points

which can make for prettier spectrograms.



https://octave.sourceforge.io/signal/function/specgram.html

Normalization

After you have generated the **spectral slices**

- the phase information is discarded
- the energy **normalized**:

```
S = abs(S);
S = S/max(S(:));
```

https://octave.sourceforge.io/signal/function/specgram.html

Dynamic Range

then the **dynamic range** of the signal is chosen.

eliminate any dynamic range at the bottom end
max(the magnitude, minE=-40dB)
some minimum energy : minE=-40dB.
if (the magnitude < minE) then minE

eliminate any dynamic range in the very top of the range
min(the magnitude, maxE=-3dB)
some maximum energy : maxE=-3dB.
if (the magnitude > maxE) then maxE

```
S = max(S, 10^(minE/10));
S = min(S, 10^(maxE/10));
```

https://octave.sourceforge.io/signal/function/specgram.html

Frequency Range

the frequency range of the FFT is from [0, Fs/2]

for band limited signal,
no need to display the entire frequency range.

For the speech signal is below 4 kHz — [0, 4000]
so there is no reason to display
up to the Nyquist frequency of 10 kHz — fs/2 = 10
for a 20 kHz sampling rate. — fs =20

Only keep the first 40% of the rows
of the returned S and f. — [**S**, **f**, **t**]

to display the frequency range [minF, maxF],

```
idx = (f >= minF & f <= maxF);
```

https://octave.sourceforge.io/signal/function/specgram.html

Color Map

A **brightness** varying colormap such as copper or bone
gives good shape to the ridges and valleys.

A **hue** varying colormap such as jet or hsv
gives an indication of the steepness of the slopes.

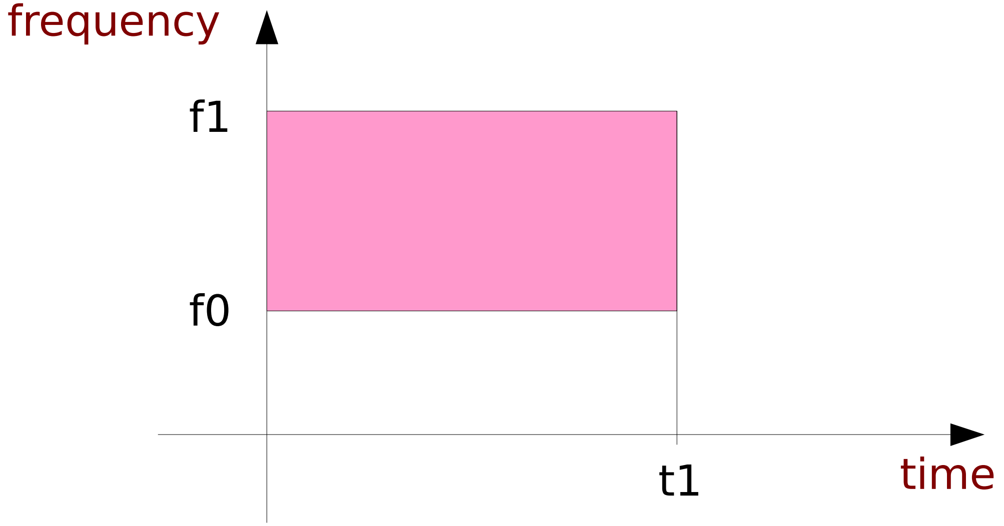
The final spectrogram is displayed in **log energy scale**
and by convention has low frequencies on the bottom of the image:

```
imagesc(t, f, flipud(log(S(idx,:))));
```

https://octave.sourceforge.io/signal/function/specgram.html

Chirp (1)

Function File: **chirp** (t)
Function File: **chirp** (t, f0)
Function File: **chirp** (t, f0, t1)
Function File: **chirp** (t, f0, t1, f1)
Function File: **chirp** (t, f0, t1, f1, form)
Function File: **chirp** (t, f0, t1, f1, form, phase)



form

$$f(t) = (f_1 - f_0) \cdot \left(\frac{t}{t_1}\right) + f_0$$

$$f(t) = (f_1 - f_0) \cdot \left(\frac{t}{t_1}\right)^2 + f_0$$

$$f(t) = (f_1 - f_0)^{\left(\frac{t}{t_1}\right)} + f_0$$

https://octave.sourceforge.io/signal/function/chirp.html

Chirp (2)

Evaluate a chirp signal at time t.

A chirp signal is a frequency swept cosine wave.

t	vector of times to evaluate the chirp signal	
f0	frequency at time t=0	[0 Hz]
t1	time t1	[1 sec]
f1	frequency at time t=t1	[100 Hz]
form	shape of frequency sweep	
	'linear' f(t) = (f1-f0)*(t/t1) + f0	
	'quadratic' f(t) = (f1-f0)*(t/t1)^2 + f0	
	'logarithmic' f(t) = (f1-f0)^(t/t1) + f0	
phase	phase shift at t=0	

https://octave.sourceforge.io/signal/function/specgram.html

Chirp (3)

Example

```
specgram(chirp([0:0.001:5]));                    # linear, 0-100Hz in 1 sec

specgram(chirp([-2:0.001:15], 400, 10, 100, 'quadratic'));

soundsc(chirp([0:1/8000:5], 200, 2, 500, "logarithmic"),8000);
```

If you want a different sweep shape f(t), use the following:

```
y = cos(2*pi*integral(f(t)) + 2*pi*f0*t + phase);

x = chirp([0:0.001:2],0,2,500);    # freq. sweep from 0-500 over 2 sec.
```

https://octave.sourceforge.io/signal/function/specgram.html

Example 1 (1)

```
x = chirp([0:0.001:2],0,2,500);   # freq. sweep from 0-500 over 2 sec.
Fs=1000;                          # sampled every 0.001 sec so rate is 1 kHz
step=ceil(20*Fs/1000);            # one spectral slice every 20 ms
window=ceil(100*Fs/1000);         # 100 ms data window
specgram(x, 2^nextpow2(window), Fs, window, window-step);
```

Fs = 1000 Hz = 1 kHz
Ts = 1/1000 sec = 1 msec

step	= 20 msec
window	= 100 msec

x	= x
n	= 2^nextpow2(100) = 2^7 = 128
Fs	= 1000
window	= 100
overlap	= 100-20 = 80

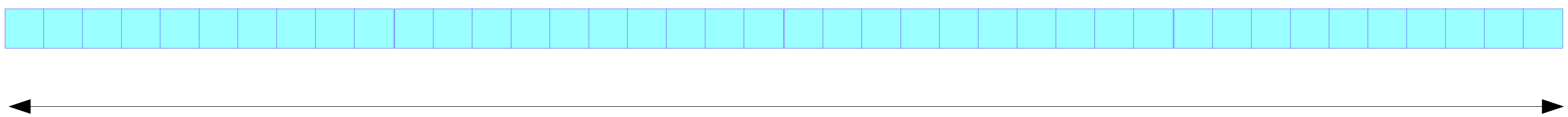
https://octave.sourceforge.io/signal/function/specgram.html

Example 1 (2)

Fs = 1000 Hz = 1 kHz
Ts = 1/1000 sec = 1 msec

step = 20 msec
window = 100 msec

x



2 sec

2 sec * 1000 samples /sec = 2000 samples

20 msec * 1 samples /msec = 20 samples

20 msec * (Fs samples/sec) / (1000 msec/sec)

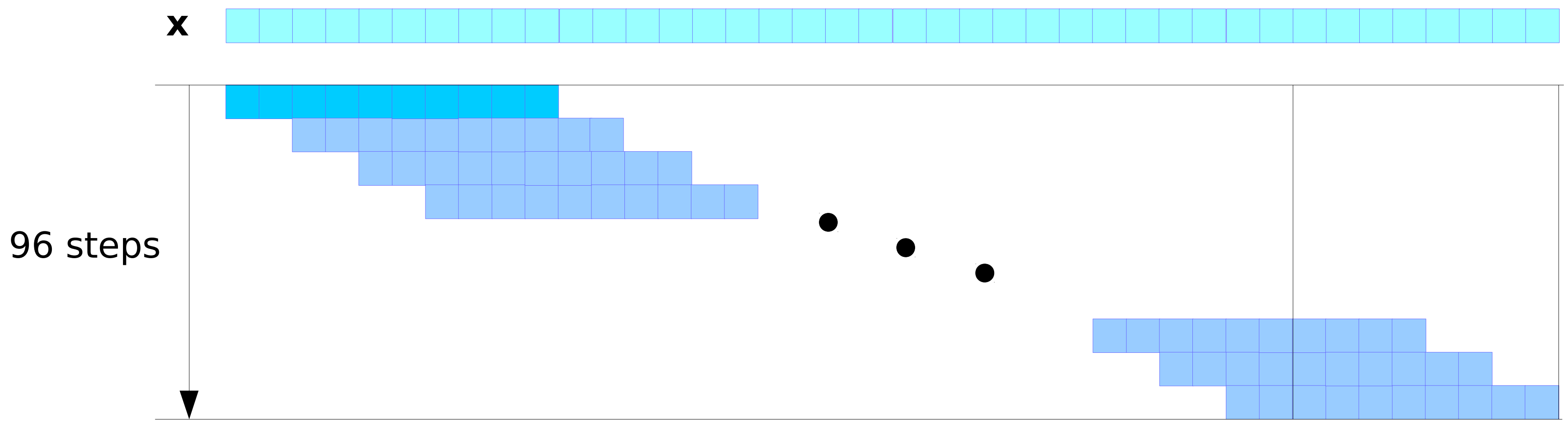
https://octave.sourceforge.io/signal/function/specgram.html

Example 1 (3)

Fs = 1000 Hz = 1 kHz
Ts = 1/1000 sec = 1 msec

step = 20 msec : 20 samples
window = 100 msec : 100 samples

2000 samples = 96 steps * 20 samples /step + 80 samples
= (1920 + 80) samples

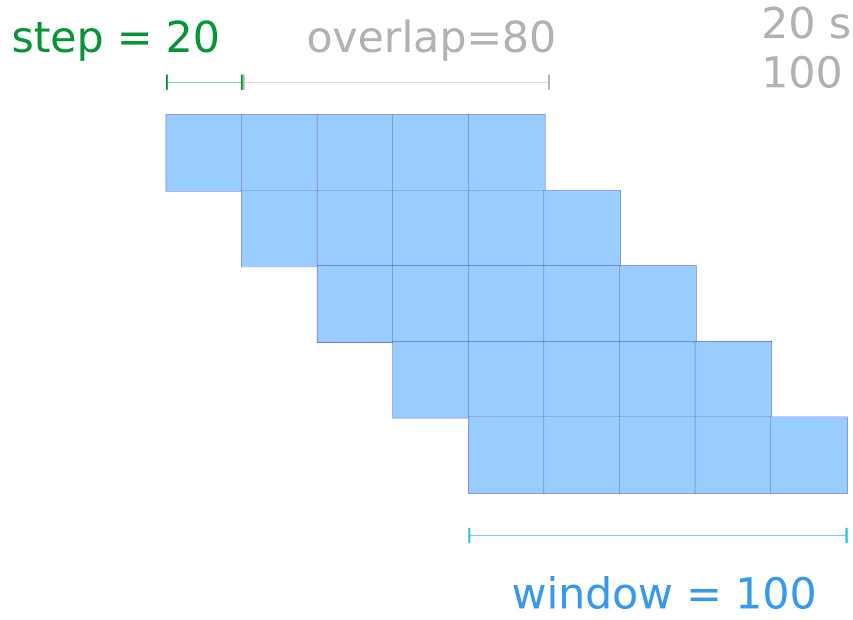


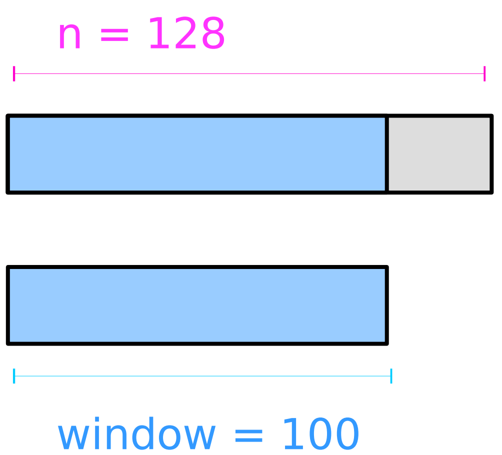
https://octave.sourceforge.io/signal/function/specgram.html

Example 1 (4)

```
x = chirp([0:0.001:2],0,2,500);  # freq. sweep from 0-500 over 2 sec.
Fs=1000;                         # sampled every 0.001 sec so rate is 1 kHz
step=ceil(20*Fs/1000);           # one spectral slice every 20 ms
window=ceil(100*Fs/1000);        # 100 ms data window
specgram(x, 128, Fs, 100, 80);
```

a sample : 0.001 sec = 1 msec
20 samples : 20 msec
100 samples : 100 msec



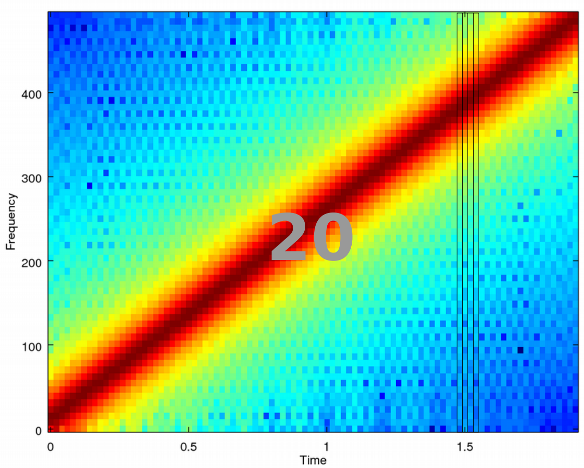


https://octave.sourceforge.io/signal/function/specgram.html

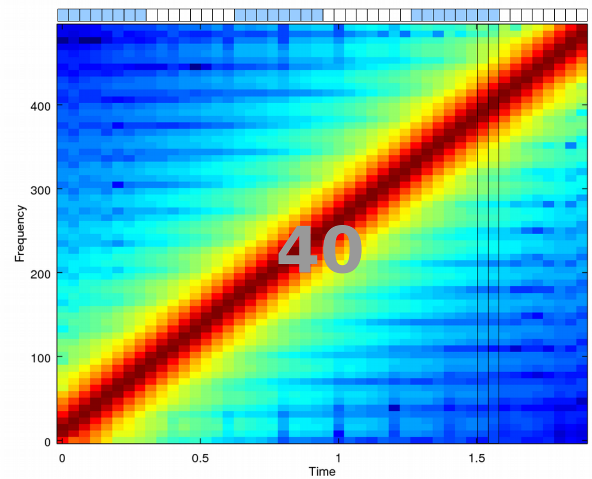
Example 1 (5)

https://octave.sourceforge.io/signal/function/specgram.html

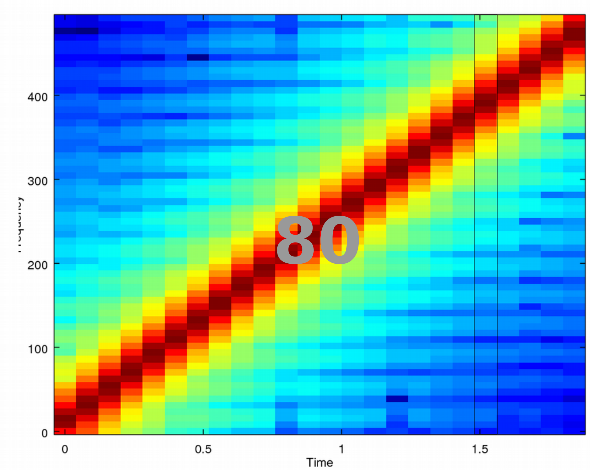
```
Fs=1000;
x = chirp([0:1/Fs:2],0,2,500);        # freq. sweep from 0-500 over 2 sec.
step=ceil(20*Fs/1000);                # one spectral slice every 20 ms
window=ceil(100*Fs/1000);             # 100 ms data window

## test of automatic plot
[S, f, t] = specgram(x);
specgram(x, 2^nextpow2(window), Fs, window, window-step);
```



step=20msec
96 steps



step=40msec
48 step



step=80msec
24 steps

Example 2

```
Fs=1000;
x = chirp([0:1/Fs:2],0,2,500);          # freq. sweep from 0-500 over 2 sec.
step=ceil(20*Fs/1000);                  # one spectral slice every 20 ms
window=ceil(100*Fs/1000);               # 100 ms data window

## test of automatic plot
[S, f, t] = specgram(x);
specgram(x, 2^nextpow2(window), Fs, window, window-step);
```

https://octave.sourceforge.io/signal/function/specgram.html

Example 2

```
x = chirp([0:0.001:2],0,2,500);  # freq. sweep from 0-500 over 2 sec.
Fs=1000;                         # sampled every 0.001 sec so rate is 1 kHz
step=ceil(20*Fs/1000);           # one spectral slice every 20 ms
window=ceil(100*Fs/1000);        # 100 ms data window
specgram(x, 2^nextpow2(window), Fs, window, window-step);

## Speech spectrogram
[x, Fs] = auload(file_in_loadpath("sample.wav")); # audio file
step = fix(5*Fs/1000);           # one spectral slice every 5 ms
window = fix(40*Fs/1000);        # 40 ms data window
fftn = 2^nextpow2(window);       # next highest power of 2
[S, f, t] = specgram(x, fftn, Fs, window, window-step);
S = abs(S(2:fftn*4000/Fs,:));    # magnitude in range 0<f<=4000 Hz.
S = S/max(S(:));                 # normalize magnitude so that max is 0 dB.
S = max(S, 10^(-40/10));         # clip below -40 dB.
S = min(S, 10^(-3/10));          # clip above -3 dB.
imagesc (t, f, log(S));              # display in log scale
set (gca, "ydir", "normal");     # put the 'y' direction in the correct direction
```

https://octave.sourceforge.io/signal/function/specgram.html

References

[1] F. Auger, Signal Processing with Free Software : Practical Experiments