First Order Logic (3A)

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Alphabet of First-Order Logic

1. Constants : Socrates, John
2. Predicates : True, False, and married, love
3. Functions : mother, weight
4. Variables : a lower case letter x, y, z
5. Operators : $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$
6. Quantifiers : $\forall$, $\exists$
7. Grouping Symbols : ( ), comma

Non-logical Symbols

1. Constants : Socrates, John
2. Predicates : True, False, and married, love
3. Functions : mother, weight

Traditional philosophy / logic assumes

The existence of a fixed, infinite set of non-logical symbols

Only one language of first-order logic

AI application specifies non-logical symbols that are

appropriate to the application (signature)

Rules of Propositional Logic

1. a term
   - (a) a constant symbol
   - (b) a variable symbol
   - (c) a function symbol (comma separated terms)
2. a atomic formula
   - (a) a predicate symbol
   - (b) a predicate symbol (comma separated terms)
   - (c) two terms separated by =
3. a formula
   - (a) an atomic formula
   - (b) ¬ formula
   - (c) two formula separated by ∧, ∨, →, ↔
   - (d) {∀ or ∃} {variable} {formula}
4. a sentence : a formula without free variables

Semantics of Propositional Logic

A signature determines the language

Given a language, a model consists of

1. A nonempty set D of entities : a domain of discourse
2. An interpretation that consists of
   (a) an entity in D → each of the constant symbols
       Usually every entity is assigned
   (b) for each function, an entity → each possible input
   (c) the predicate true ← the value T
       the predicate false ← the value F
   (d) for every other predicate,
       the value T or F → each possible input of the entities to the predicate

The truth values of all sentences

1. ¬, ∧, ∨, →, ↔ in the same way in propositional logic
2. two terms separated by = symbol has T if both terms Refer to the same entity
3. ∀x p(x) has the value T if p(x) has value T for every assignment to x of an entity in D
4. ∃x p(x) has the value T if p(x) has value T for at least one assignment to x of an entity in D
5. the operator precedence ¬, ∧, ∨, →, ↔
6. the quantifiers precedes the operators
7. ( ) changes the precedence

Satisfied

If sentence s has T under interpretation I,

I satisfies s

A sentence is satisfiable if there is some interpretation

under which it has T

A formula that contains free variables and therefore not

sentence, then an interpretation alone does not determine

its truth value

A formula that contains free variables is

satisfied by any interpretation that assigns T to the formula

for every individual of its free variables in D

Valid

A formula is valid

If it is satisfied by every interpretation

A formula is contradict

If there is no interpretation that satisfies it

Given tow formulas A and *B*

If $A \rightarrow B$ is valid

A logically implies *B*

Given tow formulas A and *B*

If $A \leftrightarrow B$ is valid

A logically equivalent to *B*

Logical Arguments

An argument consists of

A set of formulas (premises) and

A formula (conclusion)

The premises entail the conclusion

If in every model in which all the premises are true,

the conclusion is also true

The argument is sound:

If the premises entail the conclusion

Otherwise, the argument is a fallacy

Universal Instantiation

Universal Generalization

Existential Generalization

Existential Instantiation

Modus Ponens

Unification

Generalized Modus Ponens

References

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