

FFT (9A)

- Complex Phase Periodicity
- Complex Phase Symmetry
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N=8 DFT

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$X[0]$	W_8^0	W_8^0	W_8^0	W_8^0	W_8^0	W_8^0	W_8^0	W_8^0	$x[0]$
$X[1]$	W_8^0	W_8^1	W_8^2	W_8^3	W_8^4	W_8^5	W_8^6	W_8^7	$x[1]$
$X[2]$	W_8^0	W_8^2	W_8^4	W_8^6	W_8^8	W_8^{10}	W_8^{12}	W_8^{14}	$x[2]$
$X[3]$	W_8^0	W_8^3	W_8^6	W_8^9	W_8^{12}	W_8^{15}	W_8^{18}	W_8^{21}	$x[3]$
$X[4]$	W_8^0	W_8^4	W_8^8	W_8^{12}	W_8^{16}	W_8^{20}	W_8^{24}	W_8^{28}	$x[4]$
$X[5]$	W_8^0	W_8^5	W_8^{10}	W_8^{15}	W_8^{20}	W_8^{25}	W_8^{30}	W_8^{35}	$x[5]$
$X[6]$	W_8^0	W_8^6	W_8^{12}	W_8^{18}	W_8^{24}	W_8^{30}	W_8^{36}	W_8^{42}	$x[6]$
$X[7]$	W_8^0	W_8^7	W_8^{14}	W_8^{21}	W_8^{28}	W_8^{35}	W_8^{42}	W_8^{49}	$x[7]$

N=8 IDFT

$$x[n] = \frac{1}{N} \sum_{k=0}^7 W_8^{-kn} X[k] \quad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \frac{1}{N} \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^{-1} & W_8^{-2} & W_8^{-3} & W_8^{-4} & W_8^{-5} & W_8^{-6} & W_8^{-7} \\ W_8^0 & W_8^{-2} & W_8^{-4} & W_8^{-6} & W_8^{-8} & W_8^{-10} & W_8^{-12} & W_8^{-14} \\ W_8^0 & W_8^{-3} & W_8^{-6} & W_8^{-9} & W_8^{-12} & W_8^{-15} & W_8^{-18} & W_8^{-21} \\ W_8^0 & W_8^{-4} & W_8^{-8} & W_8^{-12} & W_8^{-16} & W_8^{-20} & W_8^{-24} & W_8^{-28} \\ W_8^0 & W_8^{-5} & W_8^{-10} & W_8^{-15} & W_8^{-20} & W_8^{-25} & W_8^{-30} & W_8^{-35} \\ W_8^0 & W_8^{-6} & W_8^{-12} & W_8^{-18} & W_8^{-24} & W_8^{-30} & W_8^{-36} & W_8^{-42} \\ W_8^0 & W_8^{-7} & W_8^{-14} & W_8^{-21} & W_8^{-28} & W_8^{-35} & W_8^{-42} & W_8^{-49} \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix}$$

N=8 DFT Matrix (1)

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$X[0]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$x[0]$
$X[1]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 5}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 7}$	$x[1]$
$X[2]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$x[2]$
$X[3]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 7}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 5}$	$x[3]$
$X[4]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$x[4]$
$X[5]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 5}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 7}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$x[5]$
$X[6]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$x[6]$
$X[7]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 7}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 5}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$x[7]$

N=8 IDFT Matrix (1)

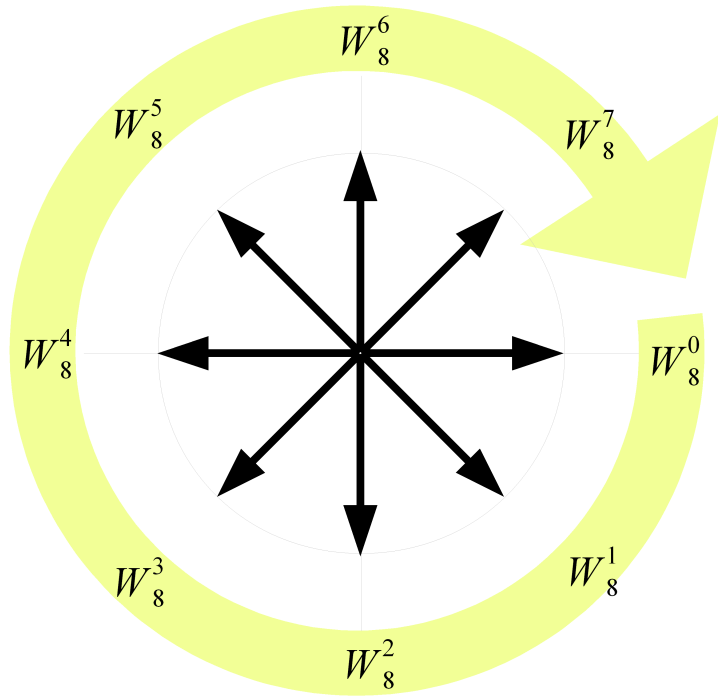
$$x[n] = \frac{1}{N} \sum_{k=0}^7 W_8^{-kn} X[k] \quad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$x[0]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$\frac{X[0]}{N}$
$x[1]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 1}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 3}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 5}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 7}$	$\frac{X[1]}{N}$
$x[2]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$\frac{X[2]}{N}$
$x[3]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 3}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 1}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 7}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 5}$	$\frac{X[3]}{N}$
$x[4]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$\frac{X[4]}{N}$
$x[5]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 5}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 7}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 1}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 3}$	$\frac{X[5]}{N}$
$x[6]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$\frac{X[6]}{N}$
$x[7]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 7}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 5}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 3}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 1}$	$\frac{X[7]}{N}$

Complex Phase Periodicity

$$W_8^k = e^{-j\left(\frac{2\pi}{8}\right)k}$$

$$W_8^{-k} = e^{+j\left(\frac{2\pi}{8}\right)k}$$



$$W_8^1 = W_8^{-7}$$

$$W_8^2 = W_8^{-6}$$

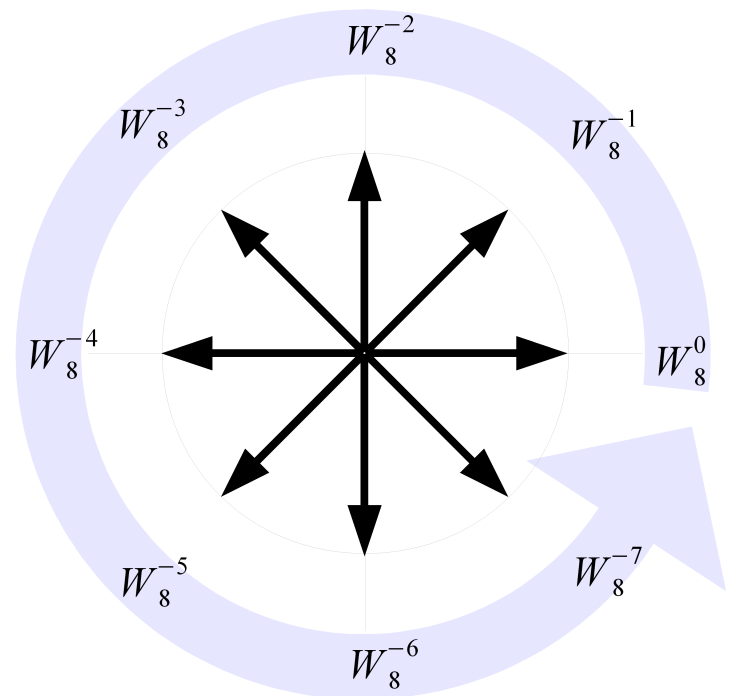
$$W_8^3 = W_8^{-5}$$

$$W_8^4 = W_8^{-4}$$

$$W_8^5 = W_8^{-3}$$

$$W_8^6 = W_8^{-2}$$

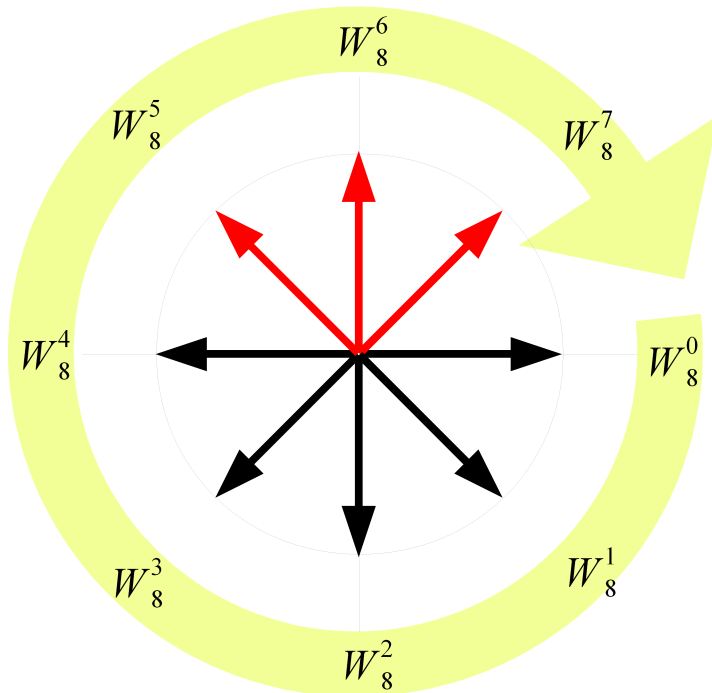
$$W_8^7 = W_8^{-1}$$



$$W_N^{k+N} = W_N^k$$

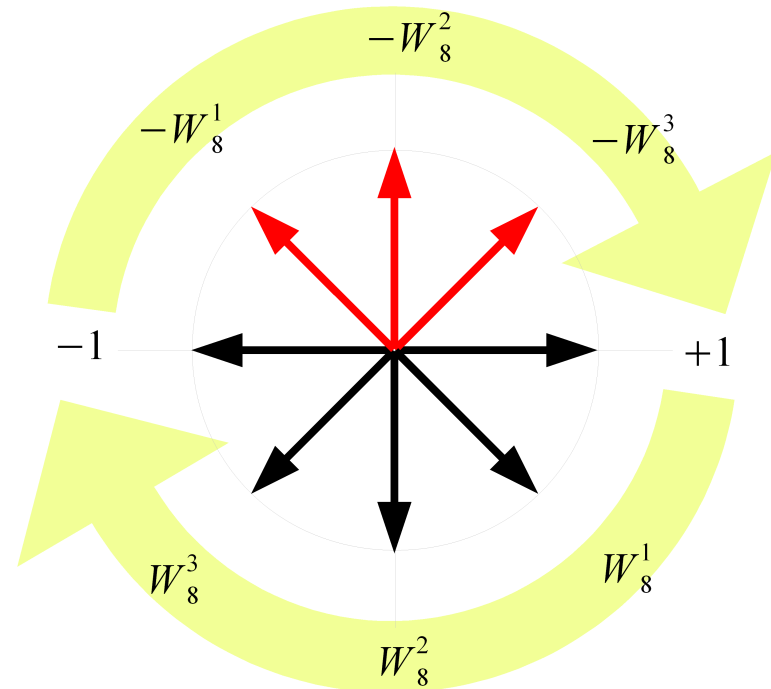
Complex Phase Symmetry (1)

$$W_8^k = e^{j\left(\frac{2\pi}{8}\right)k}$$



$$k = 0, 1, 2, 3, 4, 5, 6, 7$$

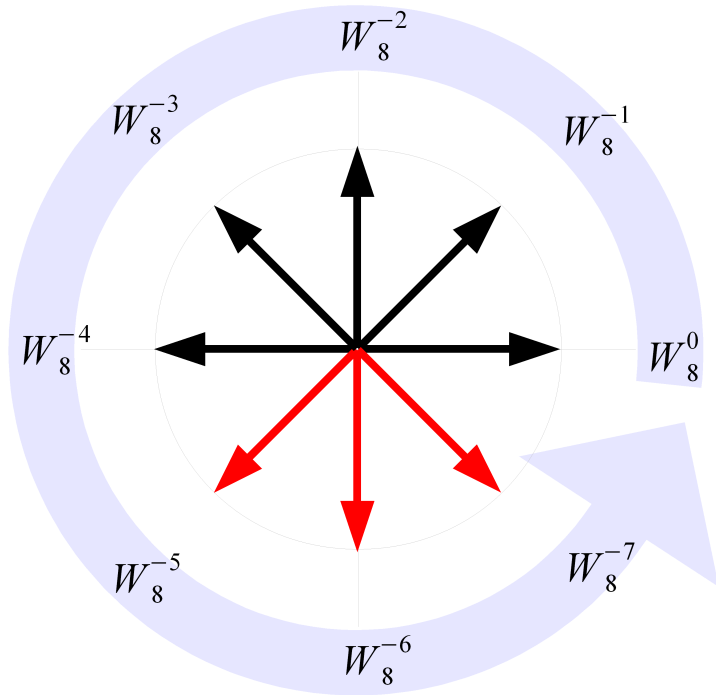
$$W_N^{k+\frac{N}{2}} = -W_N^k$$



$$k = 0, 1, 2, 3$$

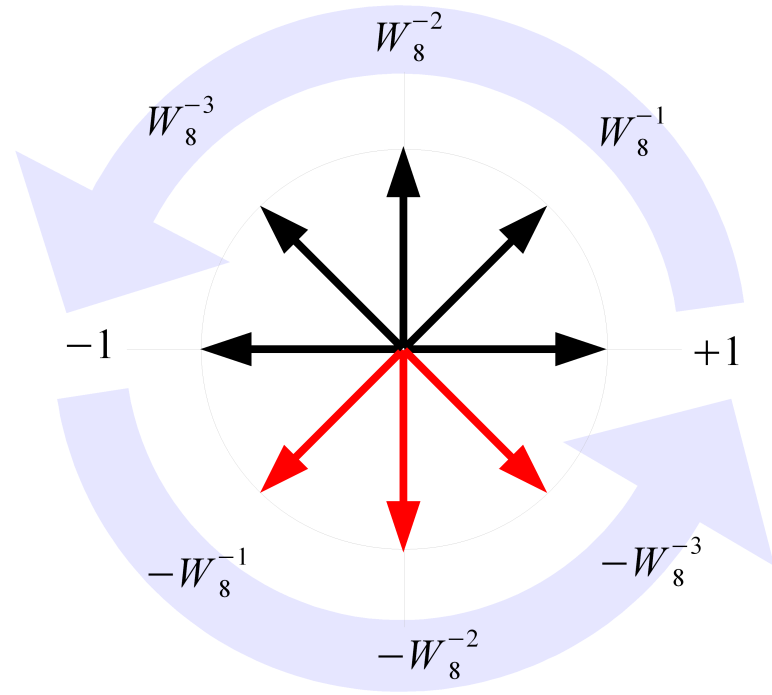
Complex Phase Symmetry (2)

$$W_8^{-k} = e^{+j\left(\frac{2\pi}{8}\right)k}$$



$$k = 0, 1, 2, 3, 4, 5, 6, 7$$

$$W_N^{-\left(k+\frac{N}{2}\right)} = -W_N^{-k}$$



$$k = 0, 1, 2, 3$$

Complex Phase - Periodicity

$$W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}$$

$$W_N^{-k} = e^{+j\left(\frac{2\pi}{N}\right)k}$$

$$W_N^{k-N} = W_N^k$$

$$W_N^{k+N} = W_N^k$$

$$W_N^{k-N} = e^{-j\left(\frac{2\pi}{N}\right)(k-N)}$$

$$W_N^{k+N} = e^{-j\left(\frac{2\pi}{N}\right)(k+N)}$$

$$\frac{W_N^{k-N}}{W_N^k} = \frac{e^{-j\left(\frac{2\pi}{N}\right)(k-N)}}{e^{-j\left(\frac{2\pi}{N}\right)k}} = e^{j2\pi} = 1$$

$$\frac{W_N^{k+N}}{W_N^k} = \frac{e^{-j\left(\frac{2\pi}{N}\right)(k+N)}}{e^{-j\left(\frac{2\pi}{N}\right)k}} = e^{-j2\pi} = 1$$

$$W_N^{kN} = 1$$

$$W_N^{-kN} = 1$$

$$W_N^{kN} = e^{-j\left(\frac{2\pi}{N}\right)kN} = e^{-j2\pi k} = 1$$

$$W_N^{-kN} = e^{+j\left(\frac{2\pi}{N}\right)kN} = e^{+j2\pi k} = 1$$

Complex Phase - Symmetry

$$W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}$$

$$W_N^{-k} = e^{+j\left(\frac{2\pi}{N}\right)k}$$

$$W_N^{k+\frac{N}{2}} = -W_N^k$$

$$W_N^{-\left(k+\frac{N}{2}\right)} = -W_N^{-k}$$

$$\begin{aligned} W_N^{k+\frac{N}{2}} &= e^{-j\left(\frac{2\pi}{N}\right)\left(k+\frac{N}{2}\right)} \\ &= e^{-j\frac{2\pi}{N}k} \cdot e^{-j\left(\frac{2\pi}{N}\right)\left(\frac{N}{2}\right)} \\ &= e^{-j\frac{2\pi}{N}k} \cdot e^{-j\pi} \\ &= e^{-j\frac{2\pi}{N}k} \cdot (-1) \end{aligned}$$

$$\begin{aligned} W_N^{-\left(k+\frac{N}{2}\right)} &= e^{+j\left(\frac{2\pi}{N}\right)\left(k+\frac{N}{2}\right)} \\ &= e^{+j\frac{2\pi}{N}k} \cdot e^{+j\left(\frac{2\pi}{N}\right)\left(\frac{N}{2}\right)} \\ &= e^{+j\frac{2\pi}{N}k} \cdot e^{+j\pi} \\ &= e^{+j\frac{2\pi}{N}k} \cdot (-1) \end{aligned}$$

$$W_N^{k+\frac{N}{2}} \cdot W_N^k = -1$$

$$W_N^{-\left(k+\frac{N}{2}\right)} \cdot W_N^{-k} = -1$$

N=8 DFT Matrix (2)

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$X[0]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$x[0]$
$X[1]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 5}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 7}$	$x[1]$
$X[2]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$x[2]$
$X[3]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 7}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 5}$	$x[3]$
$X[4]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$x[4]$
$X[5]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 5}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 7}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$x[5]$
$X[6]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$x[6]$
$X[7]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 7}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 5}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$x[7]$

N=8 DFT Matrix (3)

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$X[0]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$x[0]$
$X[1]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$-e^{-j\frac{\pi}{4}\cdot 1}$	$-e^{-j\frac{\pi}{4}\cdot 2}$	$-e^{-j\frac{\pi}{4}\cdot 3}$	$x[1]$
$X[2]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$-e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$-e^{-j\frac{\pi}{4}\cdot 2}$	$x[2]$
$X[3]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$-e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$-e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$-e^{-j\frac{\pi}{4}\cdot 1}$	$x[3]$
$X[4]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$-e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$x[4]$
$X[5]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$-e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$-e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$-e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$x[5]$
$X[6]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$-e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$-e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$x[6]$
$X[7]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$-e^{-j\frac{\pi}{4}\cdot 3}$	$-e^{-j\frac{\pi}{4}\cdot 2}$	$-e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$x[7]$

N=8 IDFT Matrix (2)

$$x[n] = \frac{1}{N} \sum_{k=0}^7 W_8^{-kn} X[k] \quad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$x[0]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$\frac{X[0]}{N}$
$x[1]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 1}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 3}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 5}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 7}$	$\frac{X[1]}{N}$
$x[2]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$\frac{X[2]}{N}$
$x[3]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 3}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 1}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 7}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 5}$	$\frac{X[3]}{N}$
$x[4]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$\frac{X[4]}{N}$
$x[5]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 5}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 7}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 1}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 3}$	$\frac{X[5]}{N}$
$x[6]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$\frac{X[6]}{N}$
$x[7]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 7}$	$e^{+j\frac{\pi}{4}\cdot 6}$	$e^{+j\frac{\pi}{4}\cdot 5}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 3}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 1}$	$\frac{X[7]}{N}$

N=8 IDFT Matrix (3)

$$x[n] = \frac{1}{N} \sum_{k=0}^7 W_8^{-kn} X[k] \quad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$x[0]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$\frac{X[0]}{N}$
$x[1]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 1}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 3}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$-e^{+j\frac{\pi}{4}\cdot 1}$	$-e^{+j\frac{\pi}{4}\cdot 2}$	$-e^{+j\frac{\pi}{4}\cdot 3}$	$\frac{X[1]}{N}$
$x[2]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$-e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$-e^{+j\frac{\pi}{4}\cdot 2}$	$\frac{X[2]}{N}$
$x[3]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 3}$	$-e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 1}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$-e^{+j\frac{\pi}{4}\cdot 3}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$-e^{+j\frac{\pi}{4}\cdot 1}$	$\frac{X[3]}{N}$
$x[4]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$\frac{X[4]}{N}$
$x[5]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$-e^{+j\frac{\pi}{4}\cdot 1}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$-e^{+j\frac{\pi}{4}\cdot 3}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 1}$	$-e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 3}$	$\frac{X[5]}{N}$
$x[6]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$-e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 0}$	$-e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$\frac{X[6]}{N}$
$x[7]$	$e^{+j\frac{\pi}{4}\cdot 0}$	$-e^{+j\frac{\pi}{4}\cdot 3}$	$-e^{+j\frac{\pi}{4}\cdot 2}$	$-e^{+j\frac{\pi}{4}\cdot 1}$	$e^{+j\frac{\pi}{4}\cdot 4}$	$e^{+j\frac{\pi}{4}\cdot 3}$	$e^{+j\frac{\pi}{4}\cdot 2}$	$e^{+j\frac{\pi}{4}\cdot 1}$	$\frac{X[7]}{N}$

References

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